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June 28, 1915

Historical remarks about
^{Arabic}
~~numbers~~ figures

Better give a sketch of
this history in Appendix
Relgate Roman Notation to Appendix

COMMON SCHOOL ARITHMETIC

ON THE

ANALYTIC SYSTEM.

WITH AN APPENDIX OF THE METRIC SYSTEM
OF WEIGHTS AND MEASURES.

BY

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PREFACE.

THE increasing demand for graded schools seems to require a gradation of works on Arithmetic. With a view to meet this demand, this book has been carefully prepared. The examples are new, and special pains have been taken to make the processes in the reasoning, as well as the examples themselves, succeed each other by uniform gradations.

A short treatise on Mensuration has been introduced toward the close of the work, which, the author trusts, will meet the practical wants of a large class of students.

S. P. S.

MERCER UNIVERSITY, GA.

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COMMON SCHOOL ARITHMETIC.

OF NUMBERS.

ARTICLE 1. A UNIT is any single thing, as an *orange*, an *apple*, a *dollar*, etc.

ART. 2. A NUMBER is a unit, or a collection of units; *one* pound is a unit; *five* pounds is a collection of units.

ART. 3. ARITHMETIC is the science of numbers. It teaches us how to use numbers in making calculations.

ART. 4. Numbers may be expressed in several ways—by words, by letters or by figures.

In arithmetic, numbers are usually expressed by figures.

ART. 5. NOTATION is the art of *writing* numbers.

ART. 6. NUMERATION is the art of *reading* numbers after they have been *written*.

ART. 7. Two methods of Notation are in general use, the Arabic and the Roman.

ARABIC NOTATION.

ART. 8. The ARABIC Notation was introduced into Europe from Arabia in the tenth century. To express numbers, it uses *ten* different characters, called *figures*, viz. :

1, which represents <i>one</i> unit.	6, which represents <i>six</i> units
2, " " <i>two</i> units	7, " " <i>seven</i> "
3, " " <i>three</i> "	8, " " <i>eight</i> "
4, " " <i>four</i> "	9, " " <i>nine</i> "
5, " " <i>five</i> "	0, " " <i>naught</i> .

ART. 9. The Arabic figure, 0, *naught*, is frequently called *cipher* or *zero*. By itself it has *no* value, but is just as necessary as any other figure. The other nine figures are called *significant* figures, because each one *signifies* or expresses a certain number of units.

ART. 10. By these *ten* figures *all* numbers can be expressed. *Nine* is the largest number that can be expressed by *one* figure. Numbers above nine are represented by combining two or more of these ten figures, thus :

<i>Ten</i> is expressed by 10.	<i>Fifteen</i> is expressed by 15.
<i>Eleven</i> " 11.	<i>Sixteen</i> " 16.
<i>Twelve</i> " 12.	<i>Seventeen</i> " 17.
<i>Thirteen</i> " 13.	<i>Eighteen</i> " 18.
<i>Fourteen</i> " 14.	<i>Nineteen</i> " 19.

ART. 11. As men had *ten* fingers, it was natural, when they wished to make calculations, to count on the fingers *one, two, three, four, five, six, seven, eight, nine, ten*, and then to begin again and count another *ten* ; and they would naturally count as many *tens* as they had fingers. In this way they would count a number of *tens*, viz. :

<i>One</i> ten, or <i>Ten</i> ,	expressed by	10
<i>Two</i> tens, or <i>Twenty</i> ,	"	20.
<i>Three</i> " <i>Thirty</i> ,	"	30.
<i>Four</i> " <i>Forty</i> ,	"	40.
<i>Five</i> " <i>Fifty</i> ,	"	50.
<i>Six</i> " <i>Sixty</i> ,	"	60

<i>Seven</i>	tens, or <i>Seventy</i> ,	expressed by	70.
<i>Eight</i>	" <i>Eighty</i> ,	"	80.
<i>Nine</i>	" <i>Ninety</i> ,	"	90.
<i>Ten</i>	" <i>One Hundred</i> ,	"	100

ART. 12. To express numbers between the *tens* we proceed thus :

<i>Twenty-one</i>	means 2 tens and one,	expressed by	21
<i>Twenty-two</i>	" 2 tens " two,	"	22
<i>Twenty-nine</i>	" 2 tens " nine,	"	29
<i>Thirty-five</i>	" 3 tens " five,	"	35
<i>Forty-six</i>	" 4 tens " six,	"	46
<i>Fifty-two</i>	" 5 tens " two,	"	52

ART. 13. After counting to *one hundred*, men would naturally proceed with the *hundreds* in the same way that they did with the *tens*, and they would count as many *hundreds* as they had fingers. In this way they would get a number of *hundreds*, viz. :

<i>One</i>	hundred,	expressed by	100.
<i>Two</i>	"	"	200.
<i>Three</i>	"	"	300.
<i>Four</i>	"	"	400.
<i>Five</i>	"	"	500.
<i>Six</i>	"	"	600.
<i>Seven</i>	"	"	700.
<i>Eight</i>	"	"	800.
<i>Nine</i>	"	"	900.
<i>Ten</i>	"	"	1000.

Ten hundred is called *one thousand*.

ART. 14. The particular position a figure occupies with respect to other figures is called its *place*; thus, in the number 253, counting from the *right* hand, 3 stands in the *first* place,

5 in the *second* place and 2 in the *third* place, and so on for any number of figures. The figure in the *first* place is called *units*, the figure in the *second* place is called *tens*, and in the *third* place *hundreds*. Thus, 253 would express 2 *hundreds* 5 *tens* and 3 *units*.

ART. 15. In Art. 13 the pupil was shown how to write *hundreds*. If we wish to supply intermediate numbers between 100 and 200, between 200 and 300, etc., we simply write the figure expressing *hundreds* in the *third* place, *tens* in the *second* place and *units* in the *first* place. Thus:

Three hundred and fifty-nine would be written 359.

Five hundred and sixty-seven " " " 567.

Nine hundred and thirty-five " " " 935.

NOTE.—The teacher would do well to pause just here and drill the pupil in writing numbers up to one thousand, until he can write them readily and accurately. "*Make haste slowly*" is the true secret of teaching Arithmetic.

ART. 16. Sometimes the *units* or the *tens*, or both, may be wanting. In such cases we supply their *places* with a *cipher*, 0. Thus:

Two hundred and seven would be written 207.

Four hundred and fifty " " " 450.

Let the pupil express the following in figures:

Eight hundred and three.		Six hundred and eighty.
Two hundred and forty.		Seven hundred and one.
Three hundred and seven.		Two hundred and thirty.

NOTE.—The foregoing examples illustrate the utility of the 0. Although it has no value of *itself*, it serves a useful purpose in *keeping other figures in their proper places*.

ART. 17. After counting to *one thousand*, as was explained in Art. 13, men would naturally proceed with the *thousands* as they did with the *hundreds*, counting as many *thousands* as

they had fingers, and thus would get *one thousand, two thousand, etc.*, up to *ten thousand*. After counting to *ten-thousands*, as just explained, men would proceed with the *ten-thousands* as they had done with the *thousands*, and thus would get *ONE ten-thousand, TWO ten-thousands, etc.*, up to *TEN ten-thousands*, now called *ONE hundred thousand*. Proceeding in the same way, they would finally reach *TEN hundred thousand*, which is called a *MILLION*.

ART. 18. After counting to *MILLIONS*, as just explained, men would count *ONE million, TWO millions, THREE millions, etc.*, until they reached *TEN millions*. They would then count *ten* of these *TEN millions*, making *ONE HUNDRED millions*. Counting on, they would reach *TEN HUNDRED millions*. This last number is now called a *BILLION*.

ART. 19. Counting successively by *tens*, we have learned the names of the following places of figures, viz.: *units, tens, hundreds, thousands, ten-thousands, hundred-thousands, millions, ten-millions, hundred-millions, billions*. We have thus formed a scale of units in which each new unit is *ten times greater* than the preceding one. Thus *thousands* are *ten times greater* than *hundreds*, and *hundreds* are *ten times greater* than *tens*.

ART. 20. The figures 1, 2, 3, 4, etc., when they stand alone, or when they occupy the first place, denote simply *units* or *ones*, and are called units of the *first order*. When they occupy the *second* place, they represent *tens*, and are called units of the *second order*. When found in the *third* place, they stand for *hundreds*, and are called units of the *third order*, and so on. This may be illustrated by the following table:

The 1st order of units is called *units*.

" 2d	"	"	"	<i>tens.</i>
" 3d	"	"	"	<i>hundreds.</i>
" 4th	"	"	"	<i>thousands.</i>

The 5th order of units is called *ten-thousands*.

"	6th	"	"	"	<i>hundred-thousands</i>
"	7th	"	"	"	<i>millions.</i>
"	8th	"	"	"	<i>ten-millions.</i>
"	9th	"	"	"	<i>hundred-millions.</i>
"	10th	"	"	"	<i>billions.</i>
"	11th	"	"	"	<i>ten-billions.</i>
"	12th	"	"	"	<i>hundred-billions.</i>

And we may extend this table to *trillions*, *quadrillions*, etc.

ART. 21. Suppose we wish to express in figures the number thirty-two thousand five hundred and nine.

In this example the highest order of units is *ten-thousands*, of which there are 3; the next order is *thousands*, of which there are 2; the next order is *hundreds*, of which there are 5; the next order is *tens*, of which there are 0; and the next order is *units*, of which there are 9. To write the number we first set down the units of the *highest* order, viz.: *ten-thousands*; then the *thousands*, *hundreds*, *tens* and *units* in succession. The number is therefore written thus: 32509. Hence we have the

RULE FOR NOTATION

Begin at the left hand and write the units of the HIGHEST order mentioned, and set down each order in succession in its proper place, filling the place of each absent order with a cipher.

EXERCISES IN NOTATION.

ART. 22. Let the pupil express the following numbers in figures

1. Eighty-seven.

Ans 87.

2. Three hundred and seventy-six.

3. Five hundred and seven.

4. Six thousand three hundred and seventy-five.

5. Nine thousand and forty-eight.
6. Five thousand and seven.
7. Forty-three thousand six hundred and fifty-nine.
8. Five hundred and thirty-six thousand three hundred and two.
9. Four hundred and five thousand three hundred and sixteen.
10. Five million forty-three thousand and thirty-seven.
11. Sixty-four thousand seven hundred and ninety-two.
12. Three hundred and fifty-six thousand and ninety-seven.
13. Nine million three hundred and forty-five thousand and twenty-seven.
14. Eighty thousand and fifty-six.
15. Seventeen thousand four hundred and seven.

ROMAN NOTATION.

ART. 23. In the *Roman* Notation seven capital letters were used to express numbers; these letters were:

I,	representing the number 1.
V,	" " 5.
X,	" " 10.
L,	" " 50.
C,	" " 100.
D,	" " 500.
M,	" " 1000.

All other numbers were expressed by means of these letters repeated or variously combined.

ART. 24. TABLE OF ROMAN NUMBERS.

I	stands for	1	V	stands for	5
II	"	2	VI	"	6
III	"	3	VII	"	7
IV	"	4	VIII	"	8

IX	stands for	9	XXIX	stands for	29
X	"	10	XXX	"	30
XI	"	11	XXXIV	"	34
XII	"	12	XXXVI	"	36
XIII	"	13	XL	"	40
XIV	"	14	XLII	"	42
XV	"	15	L	"	50
XVI	"	16	LVII	"	57
XVII	"	17	LX	"	60
XVIII	"	18	LXX	"	70
XIX	"	19	CCC	"	300
XX	"	20	MDCC	"	1700
XXV	"	25	MDCCLXV	"	1765

ART. 25. Whenever a letter is repeated, the value of that letter is repeated. Thus X stands for 10, and XXX for 30; C is 100, CC is 200.

When a letter of less value *follows* one of greater value, it *adds* its own value to the greater. Thus, V = 5, VII = 7, L = 50, LX = 60.

When a letter of less value *precedes* one of greater, it *takes away* its value from the greater. Thus, X = 10, IX = 9, L = 50, XL = 40.

When a letter of less value stands *between* two of greater value, the less is to be taken from the one that *follows* it, and the remainder is to be added to that which precedes it; as XIX = 19, CXL = 140, CXC = 190.

ART. 26. Arithmetical calculations *can* be made by the Roman Notation, but the method is tedious, and on this account is not used. The Roman characters are chiefly employed to indicate the hours on clocks and watches, to number the volumes and chapters of books, and to designate the values of coins and bank-bills.

NUMERATION.

ARTICLE 27. NUMERATION is the art of expressing in words numbers that are written in figures.

ART. 28. To read large numbers with facility, the figures are separated by commas into periods or groups, beginning at the right hand. By the French method of Numeration a period consists of *three* figures; by the English, of *six*. The French method is most convenient, and is in almost universal use.

ART. 29. The first or right-hand period contains units, tens and hundreds, and is called the *period of units*; the second period contains thousands, ten-thousands and hundred-thousands, and is called the *period of thousands*; the third period contains millions, ten-millions and hundred-millions, and is called the *period of millions*, etc. This will be more clearly illustrated by the following table:

NUMERATION TABLE.

Hundred-Trillions. Ten-Trillions. Trillions.			Hundred-Billions. Ten-Billions. Billions.			Hundred-Millions. Ten-Millions. Millions.			Hundred-Thousands. Ten-Thousands Thousands.			Hundreds. Tens. Units.		
8	9	6	5	4	3	2	7	5	4	5	6	1	4	5
5th period, TRILLIONS.			4th period, BILLIONS.			3d period, MILLIONS.			2d period, THOUSANDS.			1st period, UNITS.		

The value of the figures in the above table, expressed in words, is eight hundred and ninety-six *trillion*, five hundred and forty-three *billion*, two hundred and seventy-five *million*, four hundred and fifty-six *thousand*, one hundred and forty-five.

The above table can be extended to any number of places. The periods above trillions, in their regular order, are quadrillions, quintillions, sextillions, etc.

ART. 30. To read large numbers readily, we have the following

RULE FOR NUMERATION.

Point off the number into periods of three figures each, beginning at the right hand; then, commencing at the left hand, read the figures of each period separately, adding the name of each period, except that of units.

Let us read the number 3472856312. We first point off the figures into periods; this may be done by a comma, thus: 3,472,856,312. Our number will then be read,

3 billion, 472 million, 856 thousand, 312.

EXERCISES IN NUMERATION.

ART. 31. Let the pupil read the following numbers :

1.	95	12.	75905	23.	875426381
2.	487	13.	60308	24.	312507217
3.	708	14.	75027	25.	627453705
4.	540	15.	324508	26.	9435205654
5.	5897	16.	407246	27.	5273312205
6.	8409	17.	527075	28.	345670314
7.	9720	18.	752304	29.	27405259
8.	5043	19.	5257046	30.	45263107
9.	7008	20.	7803527	31.	8724565
10	25974	21.	45956352	32.	897357
11.	38205	22.	83402784	33.	80752

EXERCISES IN NOTATION.

ART. 32. Let the pupil express the following in figures :

1. Seven hundred and nine. Ans. 709
2. Eight thousand and thirty-seven. Ans. 8037

3. Nine hundred and four.
4. Four thousand three hundred and five.
5. Seven thousand four hundred and six.
6. Forty-six thousand three hundred and five.
7. Eighty-two thousand and seventy-six.
8. Two hundred and twelve thousand and ninety-seven.
9. Five million three hundred and twenty thousand and nine.
10. Ninety-seven million three hundred and four thousand and five.
11. Two million sixty-three thousand four hundred and forty-two.
12. Seventy thousand three hundred and twenty-seven.

ADDITION.

ARTICLE 33. Ex. 1. Thomas had 5 apples; his brother gave him 3 apples and his sister gave him 2 apples more. How many apples did he then have?

Analysis. To find the answer we must *put* these three numbers of apples together; thus 5 apples and 3 apples are 8 apples, and 2 apples are 10 apples. This operation is called *Addition*. Hence,

ART. 34. ADDITION is the uniting of two or more numbers so as to make ONE number.

The number obtained by addition is called the SUM or AMOUNT.

SIMPLE ADDITION consists in uniting numbers of the same kind or denomination, as pounds to pounds, dollars to dollars, etc.

ART. 35. To show that two or more numbers are to be added, we use the SIGN, +, called plus;* thus, $5 + 4$ means

* *Plus* is a Latin word, signifying more.

that 4 is to be *added* to 5, and it is *read*, 5 *plus* 4, which is equal to 9.

Instead of the words *equal to* we use this SIGN (=), which is called the SIGN OF EQUALITY. Thus, $5 + 4 = 9$, means that the *sum* of 5 and 4 is *equal* to 9

ADDITION TABLE.

1 and 1 are 2 2 " 3 3 " 4 4 " 5 5 " 6 6 " 7 7 " 8 8 " 9 9 " 10 10 " 11	2 and 1 are 3 2 " 4 3 " 5 4 " 6 5 " 7 6 " 8 7 " 9 8 " 10 9 " 11 10 " 12	3 and 1 are 4 2 " 5 3 " 6 4 " 7 5 " 8 6 " 9 7 " 10 8 " 11 9 " 12 10 " 13	4 and 1 are 5 2 " 6 3 " 7 4 " 8 5 " 9 6 " 10 7 " 11 8 " 12 9 " 13 10 " 14
5 and 1 are 6 2 " 7 3 " 8 4 " 9 5 " 10 6 " 11 7 " 12 8 " 13 9 " 14 10 " 15	6 and 1 are 7 2 " 8 3 " 9 4 " 10 5 " 11 6 " 12 7 " 13 8 " 14 9 " 15 10 " 16	7 and 1 are 8 2 " 9 3 " 10 4 " 11 5 " 12 6 " 13 7 " 14 8 " 15 9 " 16 10 " 17	8 and 1 are 9 2 " 10 3 " 11 4 " 12 5 " 13 6 " 14 7 " 15 8 " 16 9 " 17 10 " 18
9 and 1 are 10 2 " 11 3 " 12 4 " 13 5 " 14 6 " 15 7 " 16 8 " 17 9 " 18 10 " 19	10 and 1 are 11 2 " 12 3 " 13 4 " 14 5 " 15 6 " 16 7 " 17 8 " 18 9 " 19 10 " 20	11 and 1 are 12 2 " 13 3 " 14 4 " 15 5 " 16 6 " 17 7 " 18 8 " 19 9 " 20 10 " 21	12 and 1 are 13 2 " 14 3 " 15 4 " 16 5 " 17 6 " 18 7 " 19 8 " 20 9 " 21 10 " 22

CASE I.

ART. 36. To add when the sum of each column is less than 10.

Ex. 1. What is the sum of 422 cents, 342 cents and 134 cents?

Analysis. We write the numbers, placing *units* under *units*, *tens* under *tens* and *hundreds* under *hundreds*. We begin at the units' column and say: 4 units and 2 units are 6 units, and 2 units are 8 units, and we write the 8 in the units' column. Then, 3 tens and 4 tens are 7 tens, and 2 tens are 9 tens, and we write the 9 in the tens' column. Then, 1 hundred and 3 hundreds are 4 hundreds, and 4 hundreds are 8 hundreds, and we write the 8 in the hundreds' column. Hence, the entire sum is 8 *hundreds* 9 *tens* and 8 *units*, or 898.

OPERATION.

hundreds.	tens.	units.
4	2	2
3	4	2
1	3	4
898 cents.		

Ans. 898 cents.

In *practice* we shorten the work in this way:

Short Analysis. 4 and 2 are 6, and 2 are 8; 3 and 4 are 7, and 2 are 9; 1 and 3 are 4, and 4 are 8; making 898.

2. Add 3214 pounds, 2312 pounds and 3453 pounds.

Short Analysis. 3 and 2 are 5, and 4 are 9, which we place in units' column. Then, 5 and 1 are 6, and 1 are 7, which we place in tens' column. Then, 4 and 3 are 7, and 2 are 9, which we write in hundreds' column. Lastly, 3 and 2 are 5, and 3 are 8, which we set in thousands' column, making the sum 8979.

OPERATION.

3214 pounds.
2312
3453
8979 pounds.

Ans. 8979 pounds.

To add numbers they must be of the *same kind*. 5 *peaches* and 3 *apples* make neither 8 peaches nor 8 apples. Upon the same principle, 5 *tens* and 3 *units* make neither 8 *tens* nor

8 units. Hence, when we add we must always place numbers of the *same kind* in the *same column*; i. e., we must set *units* under *units*, *tens* under *tens*, etc.

EXAMPLES FOR PRACTICE.

(3.)	(4.)	(5.)	(6.)	(7.)
323	252	315	214	313
431	343	240	320	231
233	402	323	454	443
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
Ans. 987				

(8.)	(9.)	(10.)	(11.)	(12.)
3214	2324	3415	4023	5213
2342	3431	2340	3531	2534
3123	2204	3123	2325	2140
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
Ans. 8679				

(13.)	(14.)	(15.)	(16.)	(17.)
34214	23421	43241	32314	42312
52343	31345	34503	45232	32543
12340	43232	21243	21452	24104
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
Ans. 98897				

(18.)	(19.)	(20.)	(21.)	(22.)
423143	325142	415231	243541	323511
842314	432423	243423	324343	431234
224420	231332	140344	312015	234150
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
Ans. 989877				

PRACTICAL QUESTIONS.

1. Harry paid 25 cents for a melon, 32 cents for pears and 31 cents for oranges. How many cents did he spend?

Ans. 88 cents.

2. A farmer has 312 acres in cotton, 432 acres in corn, 132

acres in wheat and 103 in oats. How many acres does he cultivate?
Ans. 979 acres.

3. A man owns 4 lots of land; the first contains 1143 acres, the second 2320 acres, the third 3425 acres and the fourth 2010 acres. How many acres does he own?

Ans. 8898 acres.

4 Four men built a factory; the first furnished 13214 dollars, the second 31423 dollars, the third 43230 dollars and the fourth 10122 dollars. What was the cost of the factory?

Ans. 97989 dollars.

5. The population of 4 cities is as follows: the first contains 321213 persons, the second 213310 persons, the third 145245 persons and the fourth 220130 persons. What is the total population?

Ans. 899898 persons.

CASE II.

ART. 37. *To add when the sum of any column equals or exceeds 10.*

Ex. 1. What is the sum of 957 pounds, 783 pounds and 694 pounds?

OPERATION.

Analysis. Writing *units* under *units*, *tens* under *tens*, etc., we say: 4 units and 3 units are 7 units, and 7 units are 14 units, which are equal to 1 *ten* and 4 *units*. Set the 4 *units* under the *units*' column and carry the 1 *ten* to the next column. Then, 1 *ten* (which we carry) and 9 *tens* are 10 *tens*, and 8 *tens* are 18 *tens*, and 5 *tens* are 23 *tens* = 2 *hundreds* and 3 *tens*. Set the 3 *tens* under the *tens*' column and carry the 2 *hundreds* to the next column. Then, 2 *hundreds* (which we carry) and 6 *hundreds* are 8 *hundreds*, and 7 *hundreds* are 15 *hundreds*, and 9 *hundreds* are 24 *hundreds*, all of which we set down.

957 pounds.
783
694
2434 pounds.

Ans. 2434 pounds.

Short Analysis. 4 and 3 are 7, and 7 are 14; set down the

4 and carry the 1. Then, 1 and 9 are 10, and 8 are 18, and 5 are 23; set down the 3 and carry the 2. Then, 2 and 6 are 8, and 7 are 15, and 9 are 24, all of which we write down, making 2434. *Ans.* 2434 pounds.

From the foregoing explanations we derive the

RULE.

I. *Write the numbers to be added, placing units under units, tens under tens, etc.*

II. *Begin at the right hand and add each column separately. If the amount of any column be less than 10, place it under the column added; but if the amount be 10 or more, place the right-hand figure of the amount under the column added, and carry the left-hand figure or figures to the next column.*

III. *Proceed in the same way through all the columns, and set down the whole amount of the last column.*

PROOF. Begin at the top of the units' column and add the several columns downward; if the two results agree, the work may be *presumed* to be correct.

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)
756	857	945	897	764
895	675	873	545	878
784	654	564	627	584
<u>Ans.</u> 2435				

(7.)	(8.)	(9.)	(10.)	(11.)
7856	8546	7659	8293	7825
8943	6787	8542	3828	4274
6789	5428	5783	5482	3858
4584	6384	4205	6827	5245
<u>Ans</u> 28172				

(12.)	(13.)	(14.)	(15.)	(16.)
75687	48925	75485	275825	856483
43825	82380	4836	823496	75824
58476	46827	48692	485747	546782
85974	56784	5438	84295	84527
<u>78427</u>	<u>45328</u>	<u>75684</u>	<u>358715</u>	<u>856493</u>
<i>Ans.</i> 342389				

17 What is the sum of $428 + 5827 + 8095 + 78304 + 896827$? *Ans.* 989481.

18. Add six thousand and ninety-five, fifty-nine thousand three hundred and eighty-six, eight thousand and ninety-seven, six hundred and eighteen and sixty-nine thousand and seven. *Ans.* 143203.

PRACTICAL QUESTIONS.

1. A man paid 535 dollars for a carriage, 575 dollars for a pair of horses and 750 dollars for a piano. How much did he pay for all ? *Ans.* 1860 dollars.

2. A planter sold 4 bales of cotton ; the first weighed 543 pounds, the second 567 pounds, the third 499 pounds and the fourth 528 pounds. What was the whole weight ? *Ans.* 2137 pounds.

3. A farmer sold cotton for 9857 dollars, corn for 2583 dollars, wheat for 787 dollars and wool for 463 dollars. What was the amount of his sales ? *Ans.* 13690 dollars.

4. An army has 35876 infantry, 9285 cavalry and 1387 artillerymen. How many men belong to the army ? *Ans.* 46548 men.

5. A locomotive ran in one week 1973 miles, in the second week 1784 miles, the third week 2259 miles, the fourth week 1875 miles and the fifth week 1869 miles. What was the entire distance run ? *Ans.* 9760 miles.

6. Five cars were loaded with cotton. The first carried

17845 pounds, the second 21318 pounds, the third 19548 pounds, the fourth 23839 pounds and the fifth 17857 pounds.

What was the entire weight? *Ans.* 100407 pounds.

7. Five men built a factory. The first furnished 28375 dollars, the second 37935 dollars, the third 27875 dollars, the fourth 36295 dollars and the fifth 43289 dollars. What was the entire capital? *Ans.* 173769 dollars.

8. A gentleman owns 27875 dollars' worth of R. R. stock, 23425 dollars in bank stock, a mill valued at 12850 dollars, a house worth 9875 dollars, land worth 6885 dollars and furniture valued at 985 dollars. What is the value of his property? *Ans.* 81895 dollars.

9. A father left an estate to his son, his daughter and his wife. To the son he left 7845 dollars, to the daughter 450 dollars more than to the son, and to the wife 1275 dollars more than to the daughter. What was the amount of the estate? *Ans.* 25710 dollars.

10. Four men, A, B, C and D, engaged in business. A put in 3475 dollars, B put in 280 dollars more than A, C put in as much as A and B together, and D put in 225 dollars more than C. What was the entire amount put in? *Ans.* 21915 dollars.

11. In 1851 the population of England was 16921888, of Ireland 6515794, of Wales, 1005721, and of Scotland 2888742. What was the entire population of these countries? *Ans.* 27332145.

12. A manufacturer bought 23 bales of cotton for 2133 dollars, 27 bales for 2867 dollars, 135 bales for 13186 dollars, and 75 bales for 8127 dollars; how many bales did he buy and how many dollars did they cost? *Ans.* 260 bales; 26363 dollars.

13. For the year ending June 1, 1860, there were manufactured in Virginia, 7544297 pounds of cotton; in North Caro-

lina, 5540738 pounds; in South Carolina, 3978061 pounds; in Georgia, 13907904 pounds; and in Alabama, 5246800 pounds; how many pounds were used in these five States?

Ans. 36217800 pounds.

14. A merchant invested 5273 dollars in dry goods, 1827 dollars in hardware, 1285 dollars in crockery and 4357 dollars in groceries. He sold the dry goods at a profit of 756 dollars the hardware at a profit of 328 dollars, the crockery at a profit of 285 dollars and the groceries at a profit of 648 dollars; what did he sell the whole for?

Ans. 14759 dollars.

SUBTRACTION.

ARTICLE 38. Mary had 7 cherries, and gave 3 of them to Jane. How many has Mary left?

Analysis. To find out how many Mary has left, we must *take away* the 3 cherries from the 7 cherries, leaving 4 cherries. This operation is called a SUBTRACTION. Hence

ART. 39. SUBTRACTION is the operation of taking a less number from a greater, in order to find their difference.

The greater number is called the MINUEND.

The less number is called the SUBTRAHEND.

That which is left after subtracting is called the REMAINDER OR DIFFERENCE.

ART. 40. The SIGN of subtraction is a short horizontal line (—) called MINUS.* When placed between two numbers, it indicates that the one *after* it is to be taken from the one *before* it. Thus, $6 - 2 = 4$, shows that 2 is to be taken from 6, and is read, 6 *minus* 2 equals 4, or 6 *less* 2 equals 4. Here, 6 is the minuend, 2 is the subtrahend, and 4 the remainder.

* *Minus* is a Latin word, and signifies *less*.

SUBTRACTION TABLE.

1 from 1 leaves 0	2 from 2 leaves 0	3 from 3 leaves 0	4 from 4 leaves 0
2 " 1	3 " 1	4 " 1	5 " 1
3 " 2	4 " 2	5 " 2	6 " 2
4 " 3	5 " 3	6 " 3	7 " 3
5 " 4	6 " 4	7 " 4	8 " 4
6 " 5	7 " 5	8 " 5	9 " 5
7 " 6	8 " 6	9 " 6	10 " 6
8 " 7	9 " 7	10 " 7	11 " 7
9 " 8	10 " 8	11 " 8	12 " 8
10 " 9	11 " 9	12 " 9	13 " 9
11 " 10	12 " 10	13 " 10	14 " 10
5 from 5 leaves 0	6 from 6 leaves 0	7 from 7 leaves 0	8 from 8 leaves 0
6 " 1	7 " 1	8 " 1	9 " 1
7 " 2	8 " 2	9 " 2	10 " 2
8 " 3	9 " 3	10 " 3	11 " 3
9 " 4	10 " 4	11 " 4	12 " 4
10 " 5	11 " 5	12 " 5	13 " 5
11 " 6	12 " 6	13 " 6	14 " 6
12 " 7	13 " 7	14 " 7	15 " 7
13 " 8	14 " 8	15 " 8	16 " 8
14 " 9	15 " 9	16 " 9	17 " 9
15 " 10	16 " 10	17 " 10	18 " 10
9 from 9 leaves 0	10 from 10 leaves 0	11 from 11 leaves 0	12 from 12 leaves 0
10 " 1	11 " 1	12 " 1	13 " 1
11 " 2	12 " 2	13 " 2	14 " 2
12 " 3	13 " 3	14 " 3	15 " 3
13 " 4	14 " 4	15 " 4	16 " 4
14 " 5	15 " 5	16 " 5	17 " 5
15 " 6	16 " 6	17 " 6	18 " 6
16 " 7	17 " 7	18 " 7	19 " 7
17 " 8	18 " 8	19 " 8	20 " 8
18 " 9	19 " 9	20 " 9	21 " 9
19 " 10	20 " 10	21 " 10	22 " 10

NOTE.—Let the teacher drill the student thoroughly on the foregoing tables by asking questions in every possible variety. Time thus occupied will be more than regained by the expertness imparted to the pupil in performing more difficult *examples*.

CASE I.

ART. 41. *To subtract when no figure of the subtrahend is greater than its corresponding figure of the minuend.*

Ex. 1. A merchant bought 576 oranges, and sold 341 of them. How many were left?

Analysis. Here we are to find the difference between 576 and 341. We write the less number under the greater, placing *units* under *units*, *tens* under *tens*, etc. We begin with units and say: 1 unit from 6

OPERATION.

576	Minuend.
341	Subtrahend.
235	Remainder.

units leaves 5 units, and we write the 5 in the units' column below. Then 4 tens from 7 tens leave 3 tens, and we write the 3 under the tens' column. Lastly, 3 hundreds from 5 hundreds leave 2 hundreds, and we set the 2 under the hundreds' column. Hence, we have as the whole remainder 2 hundreds 3 tens and 5 units, or 235. *Ans.* 235 oranges.

In *actual practice* we *shorten* the work. We simply say: "1 from 6 leaves 5, 4 from 7 leaves 3, 3 from 5 leaves 2; answer, 235."

In order that the pupil may understand the *reason* of the operation, he should be required at first to give the *full analysis*.

2. A man had 6549 dollars, and after spending 4235 dollars how many had he left?

Short Analysis. 5 from 9 leaves 4, 3 from 4 leaves 1, 2 from 5 leaves 3, 4 from 6 leaves 2; answer, 2314.

OPERATION.

6549	Minuend.
4235	Subtrahend.
2314	Remainder

EXAMPLES FOR PRACTICE.

(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
From 75	89	65	87	98	87
Take 54	64	21	42	53	32
<u>Ans. 21</u>					

	(9.)	(10.)	(11.)	(12.)	(13.)	(14.)
From	456	765	854	954	847	675
Take	<u>241</u>	<u>243</u>	<u>323</u>	<u>710</u>	<u>302</u>	<u>213</u>
Ans.	215					

	(15.)	(16.)	(17.)	(18.)	(19.)	(20.)
From	827	987	859	785	879	759
Take	<u>412</u>	<u>210</u>	<u>232</u>	<u>253</u>	<u>343</u>	<u>323</u>
Ans.	415					

	(21.)	(22.)	(23.)	(24.)	(25.)	(26.)
From	5895	7895	5897	7928	9857	8795
Take	<u>2352</u>	<u>3235</u>	<u>2632</u>	<u>3224</u>	<u>3251</u>	<u>3251</u>
Ans.	3543					

	(27.)	(28.)	(29.)	(30.)	(31.)	(32.)
From	75853	89487	75659	87392	75285	88456
Take	<u>45213</u>	<u>32315</u>	<u>32417</u>	<u>43181</u>	<u>43151</u>	<u>32142</u>
Ans.	30640					

	(33.)	(34.)	(35.)	(36.)	(37.)	(38.)
From	546875	347985	973856	825944	756345	914756
Take	<u>513213</u>	<u>323415</u>	<u>951231</u>	<u>812512</u>	<u>713125</u>	<u>902314</u>
Ans.	33662					

PRACTICAL QUESTIONS.

1. Harry had 79 chestnuts, and gave his brother 23 of them. How many did he keep? *Ans.* 56 chestnuts.

2. A grocer bought 759 oranges, and sold 513 of them. How many remained unsold? *Ans.* 246 oranges.

3. A man bought a rockaway for 325 dollars, and sold it for 387 dollars. How much was his profit? *Ans.* 62 dollars.

4. A man started on a journey of 875 miles. After traveling 523 miles, how much farther had he to travel?

Ans. 352 miles

5. Two men jointly built a mill for 7856 dollars; one furnished 4520 dollars. What did the other furnish?

Ans. 3336 dollars.

6. The earnings of a factory for a year were 45689 dollars, and the expenses were 21352 dollars. What were the profits?

Ans. 24337 dollars.

7. The gross receipts of a railroad were 357845 dollars, and the running expenses for the same time were 213423 dollars. What were the net earnings?

Ans. 144422 dollars.

CASE II.

ART. 42. *To subtract when a figure in the subtrahend is larger than the corresponding figure in the minuend.*

Ex. 1. James had 82 walnuts, and gave his sister 25 of them. How many did he keep for himself?

Analysis. We begin at the right hand, but as we cannot take 5 units from 2 units, we borrow 1 ten from the 8 tens, and adding the 1 ten = 10 units to the 2 units, we have 12 units. Then, 5 units from 12

OPERATION.

82	Minuend.
25	Subtrahend.
57	Remainder.

units leave 7 units, which we write under the units' column. Now, as we borrowed 1 ten from the 8 tens, we left only 7 tens; hence we say, 2 tens from 7 tens leave 5 tens, which we also set down, making the remainder 5 tens and 7 units, or 57.

Ans. 57 walnuts.

The foregoing is the *true* method of performing subtraction. The plan *usually* employed differs from this, though the results are precisely the same. To illustrate: let us take the example just given. After borrowing the 1 ten as above, and adding it to the 2 units, thereby making 12 units, the usual practice is to say: 5 units from 12 units leave 7 units; then, *carry* 1 ten to the 2 tens, making 3 tens, and 3 tens from 8 tens

leave 5 tens, as before. In other words, we *borrow from the upper number, and pay back to the lower number.*

The *reason* of this is easily explained. It depends upon the following principle, viz.:

The difference between any two numbers remains the same when each of them is equally increased. For example, $7 - 3 = 4$. Now, if we add 10 to each, we have $17 - 13 = 4$, as before. In the example above, after we *borrow* 1 ten from the 8 tens, it is no longer 8 tens, but *really* 7 tens; hence, when we call it 8 tens, we make it 1 ten *larger* than it *really* is, and therefore we must call the 2 tens in the *subtrahend*, 3 tens; i. e., we must carry 1 to the 2, in order to compensate for the increase of the *minuend*.

Observation.—To most persons, the *true* and philosophic mode of subtraction seems *unnatural*, for the simple reason, that they have been accustomed, all their lives, to operate upon a different plan.

From the foregoing explanations we derive the

RULE.

I. *Write the less number under the greater, placing units under units, tens under tens, etc., and begin at the right to subtract.*

II. *Subtract each figure in the lower line from the one above it, and set the remainder below.*

III. *If any figure in the lower line is greater than the one above it, add 10 to the upper figure before subtracting, and carry 1 to the next left-hand figure in the lower line, and proceed as before.*

PROOF. Add the remainder to the *subtrahend*; the *sum* should be equal to the *minuend*.

EXAMPLES FOR PRACTICE.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
From 74	84	63	95	54	71
Take 59	29	28	59	27	23
<u>Ans. 15</u>					

(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
From 425	837	758	648	826	756
Take 282	492	283	375	574	481
<u>Ans. 143</u>					

(13.)	(14.)	(15.)	(16.)	(17.)	(18.)
From 572	683	781	980	752	590
Take 259	328	435	348	439	425
<u>Ans. 313</u>					

(19.)	(20.)	(21.)	(22.)	(23.)	(24.)
From 7580	4850	6840	7680	3750	5870
Take 4245	2325	2234	4348	2318	2382
<u>Ans. 3335</u>					

(25.)	(26.)	(27.)	(28.)	(29.)	(30.)
From 7238	8215	5236	4239	5105	7258
Take 4854	3592	3951	2780	2872	3764
<u>Ans. 2384</u>					

(31.)	(32.)	(33.)	(34.)	(35.)	(36.)
From 52836	41729	82537	95807	72517	42815
Take 28371	28394	47180	48253	45281	25292
<u>Ans. 24465</u>					

(37.)	(38.)	(39.)	(40.)	(41.)	(42.)
From 400500	500300	700800	500000	700000	800000
Take 215327	257185	253478	275387	283541	504207
<u>Ans. 185173</u>					

PRACTICAL QUESTIONS.

1. A drover bought 725 hogs, and sold 386 of them. How many were left? *Ans.* 339 hogs.

2. A man bought a house for 7238 dollars, and sold it for 6893 dollars. How much did he lose? *Ans.* 345 dollars.

3. A man deposited 3238 dollars in bank, and afterward drew out 1875 dollars. How much had he remaining? *Ans.* 1363 dollars.

4. A lot of cotton was bought for 23417 dollars, and afterward sold for 21895 dollars. How much was the loss? *Ans.* 1522 dollars.

5. The art of printing was invented in 1449. How many years since its invention? *Ans.* — years.

6. James and Reuben together had 3000 chestnuts. James claimed 1875 of them. How many did Reuben own? *Ans.* 1125 chestnuts.

7. The earnings of a paper-mill for a year are 23418 dollars, and the expenses are 17945 dollars. What is the net profit? *Ans.* 5473 dollars.

8. Subtract five thousand and seven from thirty thousand and twenty-two. *Ans.* 25015.

9. From forty-three thousand and four subtract seven thousand two hundred and nine. *Ans.* 35795.

10. Charles has a certain number of walnuts. If he had 253 more, he would have 1000. How many has he? *Ans.* 747 walnuts.

11. The gross earnings of a railroad were 421329 dollars, and the expenses were 285940 dollars. What were the net profits? *Ans.* 135389 dollars.

12. The telescope was invented in 1608. How many years since that time? *Ans.* — years.

QUESTIONS COMBINING ADDITION AND SUBTRACTION.

1. A man owed 725 dollars. After paying 225 dollars at one time and 353 dollars at another, how much had he still to pay? *Ans.* 147 dollars.

2. If a clerk has a salary of 1200 dollars a year, and if he pays 325 dollars for board, 275 dollars for clothing, and 285 dollars for other expenses, how much can he lay up at the end of the year? *Ans.* 315 dollars.

3. A shepherd had 325 sheep. The first year he sold 89 of them, and the next year he bought 157 more. How many had he then? *Ans.* 393 sheep.

4. A gentleman engaged in farming with a capital of 4500 dollars. The first year he *gained* 1975 dollars, the second year he *lost* 1225 dollars, and the third year he *lost* 525 dollars. How did his capital then stand? *Ans.* 4725 dollars.

5. A gentleman bought a 1000-mile ticket on the Georgia Railroad for the use of his wife, his daughter, his son and himself. His wife rode 285 miles, his daughter 225 miles, his son 235 miles and he himself rode the remainder. How many miles did the gentleman ride? *Ans.* 255 miles.

6. A lady went into a store, and after buying a cloak worth 32 dollars, a bonnet for 14 dollars, a pair of gaiters for 5 dollars and two handkerchiefs at 2 dollars apiece, she handed the clerk a 100-dollar greenback. How much change ought the clerk to return to her? *Ans.* 45 dollars.

MULTIPLICATION.

ARTICLE 43. Ex. 1. If a pound of rice cost 10 cents, what would be the cost of 4 pounds?

Analysis. To answer this question, we must repeat the 10

cents, or *add* it to itself as many times as there are pounds, thus :

$$\begin{array}{r} 10 \text{ cents,} \\ 10 \text{ " } \\ 10 \text{ " } \\ 10 \text{ " } \\ \hline \end{array}$$

Ans. 40 cents.

ART. 44. This *repeated addition* of a number to itself is called MULTIPLICATION.

We might have found the answer to the question above much more readily by saying: If 1 pound cost 10 cents, 4 pounds would cost 4 *times* as many cents, and 4 *times* 10 cents are 40 cents. The operation would have been thus :

OPERATION.

$$\begin{array}{r} 10 \text{ cents,} \\ 4 \\ \hline \end{array}$$

Ans. 40 cents.

We see that this latter operation is much *shorter* than the other. Hence,

ART. 45 MULTIPLICATION is a *short method of performing many ADDITIONS of the same number.*

ART. 46. *Three terms* are used in Multiplication ; these are, the MULTIPLICAND, the MULTIPLIER and the PRODUCT.

The MULTIPLICAND is the number to be repeated.

The MULTIPLIER is the number which shows *how many times* the *multiplicand* is to be repeated.

The PRODUCT is the number obtained by the multiplication.

In the example above, 10 cents is the *multiplicand*, 4 is the *multiplier*, and 40 cents is the *product*.

The *multiplier* and *multiplicand* are also called *factors*.

ART. 47. The SIGN of multiplication is an inclined cross, $\times$, somewhat like a Roman X ; this sign when placed between two numbers signifies that they are to be multiplied together ; thus, $5 \times 3 = 15$ is read, 5 multiplied by 3 equals 15.

ART. 48. Since the multiplication of all numbers, however large, depends upon the multiplication of one figure by another, the pupil should thoroughly master the multiplication table given below:

MULTIPLICATION TABLE.

ONCE			2 times			3 times			4 times		
1	is	1	1	are	2	1	are	3	1	are	4
2	"	2	2	"	4	2	"	6	2	"	8
3	"	3	3	"	6	3	"	9	3	"	12
4	"	4	4	"	8	4	"	12	4	"	16
5	"	5	5	"	10	5	"	15	5	"	20
6	"	6	6	"	12	6	"	18	6	"	24
7	"	7	7	"	14	7	"	21	7	"	28
8	"	8	8	"	16	8	"	24	8	"	32
9	"	9	9	"	18	9	"	27	9	"	36
10	"	10	10	"	20	10	"	30	10	"	40
11	"	11	11	"	22	11	"	33	11	"	44
12	"	12	12	"	24	12	"	36	12	"	48

5 times			6 times			7 times			8 times		
1	are	5	1	are	6	1	are	7	1	are	8
2	"	10	2	"	12	2	"	14	2	"	16
3	"	15	3	"	18	3	"	21	3	"	24
4	"	20	4	"	24	4	"	28	4	"	32
5	"	25	5	"	30	5	"	35	5	"	40
6	"	30	6	"	36	6	"	42	6	"	48
7	"	35	7	"	42	7	"	49	7	"	56
8	"	40	8	"	48	8	"	56	8	"	64
9	"	45	9	"	54	9	"	63	9	"	72
10	"	50	10	"	60	10	"	70	10	"	80
11	"	55	11	"	66	11	"	77	11	"	88
12	"	60	12	"	72	12	"	84	12	"	96

9 times			10 times			11 times			12 times		
1	are	9	1	are	10	1	are	11	1	are	12
2	"	18	2	"	20	2	"	22	2	"	24
3	"	27	3	"	30	3	"	33	3	"	36
4	"	36	4	"	40	4	"	44	4	"	48
5	"	45	5	"	50	5	"	55	5	"	60
6	"	54	6	"	60	6	"	66	6	"	72
7	"	63	7	"	70	7	"	77	7	"	84
8	"	72	8	"	80	8	"	88	8	"	96
9	"	81	9	"	90	9	"	99	9	"	108
10	"	90	10	"	100	10	"	110	10	"	120
11	"	99	11	"	110	11	"	121	11	"	132
12	"	108	12	"	120	12	"	132	12	"	144

ART. 49. The *product* must always be like the *multiplicand*; i. e., if we multiply *dollars* the product must be *dollars*, if we multiply *bushels* the product must be *bushels*. Adding a quantity to itself does not change the character or kind of the quantity; and as multiplication is only a *short* way of performing several additions, the resulting product must always be like the multiplicand.

MENTAL QUESTIONS.

1. What will 4 lemons cost at 6 cents apiece?

Analysis. Since 1 lemon costs 6 cents, 4 lemons will cost 4 times 6 cents, which will be 24 cents.

2. At 5 dollars a yard, what will 6 yards of broadcloth cost?

Analysis. Since 1 yard costs 5 dollars, 6 yards will cost 6 times 5 dollars, which will be 30 dollars.

3. If 1 orange costs 7 cents, how many cents must you pay for 4 oranges?

4. One gallon contains 4 quarts; how many quarts in 8 gallons?

5. One peck contains 8 quarts; how many quarts in 5 pecks?

6. What would 8 bananas cost at 6 cents apiece?

7. How much would 5 barrels of flour cost at 9 dollars a barrel?

8. At 10 cents a pint, how much would 7 pints of milk cost?

9. At 9 cents a pound, how much would 7 pounds of rice cost?

10. In one week there are 7 days; how many days in 5 weeks?

11. At 9 cents a quart, what would be the cost of 6 quarts of berries?

12. In one dime there are 10 cents; how many cents in 7 dimes?

13. At 3 dollars a bushel, what would 12 bushels of peaches cost?

14. There are 4 gills in one pint; how many gills in 9 pints?

15. If a car run 12 miles an hour, how far would it run in 5 hours?

16. In one yard there are 3 feet; how many feet in 9 yards?

17. At 7 cents a pound, how much must you pay for 9 pounds of rice?

18. In one foot there are 12 inches; how many inches in 6 feet?

19. How much would 8 pounds of iron cost at 7 cents a pound?

20. At 6 cents apiece, how much must you pay for 7 pencils?

21. If one barrel hold 3 bushels of apples, how many bushels would 10 barrels hold?

22. If a ton of coal cost 9 dollars, what would 7 tons cost?

23. At 10 cents an ounce, how much would 8 ounces of indigo be worth?

24. Six feet make a fathom; how many feet in 7 fathoms?

25. If a bunch of grapes cost 8 cents, what would 9 bunches cost?

26. In one mile there are 8 furlongs; how many furlongs in 8 miles?

27. At 9 cents a gill, how much would 9 gills of wine cost?

28. In one bushel there are 4 pecks; how many pecks in 9 bushels?

29. At 5 cents a box, how many cents must you pay for 7 boxes of matches?

Presuming that the pupil has *thoroughly* mastered the

Multiplication Table, we proceed to illustrate the several cases that occur in Multiplication.

CASE I.

ART. 50. *When the multiplier consists of only one figure.*

Ex. 1. Multiply 321 by 3.

Analysis. We place the multiplier under the units' figure of the multiplicand and say: 3 times 1 *unit* are 3 units, and we write the 3 under the units' column. Then, 3 times 2 *tens* are 6 tens, and we write the 6 under the tens' column. Lastly, 3 times 3 *hundreds* are 9 hundreds, and we write the 9 under the hundreds' column, making the whole product 9 hundreds 6 tens and 3 units, or 963.

OPERATION.

	hundreds.	tens.	units.
321	Multiplicand.		
3	Multiplier.		
963	Product.		

Ans. 963.

2. Multiply 476 by 6.

Analysis. We say: 6 times 6 units are 36 units = 3 tens and 6 units. We place the 6 under the units' column, and carry the 3 tens to the next product. Then, 6 times 7 tens are 42 tens, and the 3 tens we carry are 45 tens = 4 hundreds and 5 tens. We write the 5 under the tens' column, and carry the 4 hundreds to the next product. Lastly, 6 times 4 hundreds are 24 hundreds, and the 4 hundreds we carry are 28 hundreds = 2 thousands and 8 hundreds, which we write under their proper columns, making the entire product 2 thousands 8 hundreds 5 tens and 6 units, or 2856.

OPERATION

	hundreds.	tens.	units.
476	Multiplicand		
6	Multiplier.		
2856	Product.		

Ans. 2856.

The pupil should now be taught that in working examples we *practically* pay no attention to figures as being *units, tens, hundreds*, etc., but we regard them *all* as units. This very

much *shortens* the operation. The actual working of the last example would be about as follows: "6 times 6 are 36; set down the 6 and carry 3; 6 times 7 are 42, and 3 to carry make 45; set down the 5 and carry 4; 6 times 4 are 24, and 4 to carry are 28, which we set down." *Ans.* 2856.

A little examination of the *analysis* will show that when the multiplier consists of but *one* figure,

We begin at the right hand and multiply each figure of the multiplicand by the multiplier, setting down the right-hand figure of each product and carrying the left, just as in Addition.

3. Multiply 7085 by 5.

<i>Short Analysis.</i> 5 times 5 are 25; set down the 5 and carry 2; 5 times 8 are 40, and 2 to carry are 42; set down the 2 and carry 4; 5 times 0 is 0, but 4 we carry is 4; set down the 4; 5 times 7 are 35, which we set down.	OPERATION. <table border="0"> <tr> <td>7085</td> <td>Multiplicand.</td> </tr> <tr> <td>5</td> <td>Multiplier.</td> </tr> <tr> <td> </td> <td></td> </tr> <tr> <td>35425</td> <td>Product.</td> </tr> </table> <i>Ans.</i> 35425.	7085	Multiplicand.	5	Multiplier.			35425	Product.
7085	Multiplicand.								
5	Multiplier.								
35425	Product.								

EXAMPLES FOR PRACTICE.

4. At 324 dollars apiece, how much would 2 buggies cost?

Ans. 648 dollars.

5. If a bale of cloth contain 321 yards, how many yards in 3 bales?

Ans. 963 yards.

6. What would 4 horses cost at 212 dollars apiece?

Ans. —.

7. Thomas has 3213 chestnuts, and William has 3 times as many. How many has William?

Ans. —.

8. If one piano cost 756 dollars, how much would 2 pianos cost?

Ans. —.

9. What would be the weight of 3 bales of cotton, each weighing 576 pounds?

Ans. —.

10. What would be the cost of 4 carriages at 568 dollars apiece?

Ans. —.

11. If a cask of bacon weigh 937 pounds, how much would 5 casks weigh? *Ans.* —.

12. If a locomotive run 396 miles in one day, how far could it run in 6 days? *Ans.* —.

13. What would be the weight of 7 tierces of rice, each weighing 575 pounds? *Ans.* —.

14. How many stalks of corn in 8 rows, each row containing 748 stalks? *Ans.* —.

15. How many eggs in 9 boxes, each box containing 864 eggs? *Ans.* —.

16. In one mile there are 5280 feet; how many feet in 4 miles? *Ans.* 21120 feet.

17. If a factory make 9757 yards of cloth in one week, how many yards could it make in 5 weeks?

Ans. 48785 yards.

18. If a ship can carry 7856 barrels of flour, how many barrels could be carried in 6 ships? *Ans.* 47136 barrels.

19. If a wagon can carry 5837 shingles, how many shingles could be carried by 7 wagons? *Ans.* 40859 shingles.

20. What would 8 miles of plank-road cost at 3489 dollars per mile? *Ans.* 27912 dollars.

21. Nine men built a factory, each one furnishing 8457 dollars. What was the cost of the factory?

Ans. 76113 dollars.

22. If a car can carry 19857 pounds, how many pounds could 4 such cars carry? *Ans.* 79428 pounds.

23. There are 63360 inches in a mile. How many inches in 5 miles? *Ans.* 316800 inches.

24. How many miles would a ship sail in going around the earth 6 times, the earth being 24856 miles in circumference?

Ans. 149136 miles.

25. What would be the cost of constructing 7 miles of railway at 35248 dollars per mile? *Ans.* 246736 dollars

26. If a wheel revolve 18576 times in going a certain distance, how many times would it revolve in going 8 times the distance?
Ans. 148608 times.

27. If the expense of running a steam-mill for one year is 35678 dollars, how much would it be for 9 years?
Ans. 321102 dollars.

CASE II.

ART. 51. *When the multiplier consists of more than one figure.*

Ex. 1 If 46 bushels of corn grow on one acre of land, how many bushels would grow on 36 acres?

Analysis. If 1 acre produce 46 bushels, then 36 acres would produce 36 times as many bushels. We place the multiplier under the multiplicand, units under units, and tens under tens. Beginning with units, we say: 6 times 6 units are 36 units = 3 tens and 6 units. Write the

OPERATION.

46 Multiplicand
 36 Multiplier.

276
 138
 1656 Product.

6 units under the units' column, and carry the 3 tens to the next product. Then, 6 times 4 tens are 24 tens, and the 3 tens we carry are 27 tens = 2 hundreds and 7 tens, each of which we set in their proper column. Next, we are to multiply by the 3 tens. We say: 3 tens multiplied by 6 units are 18 tens = 1 hundred and 8 tens. We set the 8 tens under the tens' column, and carry the 1 hundred to the next product. Lastly, 3 tens multiplied by 4 tens are 12 hundreds, and the 1 hundred we carry makes 13 hundreds = 1 thousand and 3 hundreds, each of which we write in their proper column. Adding these two products, we have 1656 as the whole product.

Ans. 1656 bushels.

It is plain that when we multiplied 46 bushels by 6, we found the number of bushels that would grow on 6 acres; and

when we multiplied by the 3 *tens* = 30 *units*, we found the number that would grow on 30 acres. Adding these two results, we evidently have the number of bushels that would grow on 30 acres + 6 acres, or 36 acres.

2. A merchant bought 34 pieces of calico, each piece containing 43 yards. How many yards did he buy?

Analysis. Since 1 piece contains 43 yards, then 34 pieces will contain 34 *times* as many yards; the *operation* would be as follows:

OPERATION.

$$\begin{array}{rcl}
 43 & = & \text{number of yards in 1 piece.} \\
 34 & = & \text{number of pieces.} \\
 \hline
 172 & = & \text{number of yards in 4 pieces.} \\
 129 & = & \text{number of yards in 30 pieces.} \\
 \hline
 1462 & = & \text{number of yards in 34 pieces.}
 \end{array}$$

In multiplying 43 yards by 4 we get 172 yards as the number in 4 pieces; and multiplying 43 by 3 *tens*, we get 129 *tens* of yards = 1290 yards, the number in 30 pieces; and when we add the two products we get 1462 yards, the number in 34 pieces.

Ans. 1462 yards.

NOTE.—The products obtained by using the separate figures of the multiplier are called *partial* products.

3. Multiply 236 by 309.

Analysis. We first multiply by the units' figure, as in the preceding examples, and get 2124 as a *partial* product. We next pass to the *tens*. As 0 occupies the *tens'* place, its product into the multiplicand will be 0, and we write 0 under the *tens'* column. We then

OPERATION.

$$\begin{array}{r}
 236 \text{ Multiplicand} \\
 309 \text{ Multiplier} \\
 \hline
 2124 \\
 7080 \\
 \hline
 72924 \text{ Product}
 \end{array}$$

proceed to the 3, which represents *hundreds*, and multiply each figure of the multiplicand by this, placing the first figure of the product under the column of *hundreds*, because

units multiplied by *hundreds* will always give *hundreds*. Adding these partial products, we have 72924 as the entire product *Ans.* 72924.

What we have said may be summed up in the following

RULE.

I. *Write the multiplier under the multiplicand, placing units under units, tens under tens, etc., and begin at the right to multiply.*

II. *Multiply each figure in the multiplicand successively by each figure in the multiplier, and set the first figure of each partial product directly under the figure by which you multiply.*

III. *Add the partial products, and their sum will be the whole product required.*

PROOF. Multiply the multiplier by the multiplicand, and if the product is the same as before, the work may be *presumed* to be right.

ART. 52. When we have two factors, it is immaterial which of the two we use as the *multiplicand*, or which as the *multiplier*, for in either case the resulting product is the same: $5 \times 8 = 8 \times 5$.

EXAMPLES FOR PRACTICE.

4. If a man can travel 45 miles in a day, how far could he travel in 25 days?

Analysis. Since he travels 45 miles in 1 day, then in 25 days he could travel 25 *times* as many miles $= 45 \times 25 = 1125$ miles. *Ans.* 1125 miles.

5. How far would a locomotive run in 22 hours at 34 miles an hour? *Ans.* 748 miles.

6. How many yards of bagging in 63 pieces, each piece containing 78 yards? *Ans.* 4914 yards.

7. What would be the value of 96 acres of land at 67 dollars an acre? *Ans.* 6432 dollars.

8. In one hogshead there are 63 gallons. How many gallons in 87 hogsheads? *Ans.* 5481 gallons.

9. If an engine can run 375 miles in a day, how far could it run in 29 days? *Ans.* 10875 miles.

10. What would be the weight of 65 bales of cotton, each bale weighing 537 pounds? *Ans.* 34905 pounds.

11. If a carriage wheel revolve 347 times in running one mile, how often would it revolve in going 85 miles? *Ans.* 29495 miles.

12. If a cask of bacon weigh 983 pounds, what would be the weight of 79 casks at the same rate? *Ans.* 77657 pounds.

13. Multiply 786 by 48. *Ans.* 37728.

14. Multiply 958 by 67. *Ans.* 64186.

15. Multiply 879 by 86. *Ans.* 75594.

16. Multiply 937 by 75. *Ans.* 70275.

17. In one bushel there are 2150 cubic inches. How many cubic inches in 58 bushels? *Ans.* 124700 cubic inches.

18. What would 78 miles of plank-road cost at 2409 dollars per mile? *Ans.* 187902 dollars.

19. Multiply 3256 by 58. *Ans.* 188848.

20. Multiply 4807 by 76. *Ans.* 365332.

21. What would be the cost of constructing 86 miles of railroad at 28956 dollars per mile? *Ans.* 2490216 dollars.

22. How many pounds could be carried on 27 freight-cars, each car carrying 19785 pounds? *Ans.* 534195 pounds.

23. If one ship can carry 987654 pounds of cotton to Liverpool, how many pounds could 49 ships carry? *Ans.* 48395046 pounds.

24. If a box contain 237 oranges, how many oranges would 425 boxes contain? *Ans.* 100725 oranges.

25. Multiply 765 by 235. *Ans.* 179775.

26. Multiply 894 by 326. *Ans.* 291444.

27. Multiply 657 by 408. *Ans.* 268056.
28. Multiply 6258 by 346. *Ans.* 2165268.
29. Multiply 5679 by 507. *Ans.* 2879253.
30. Multiply 7856 by 658. *Ans.* 5169248.
31. Multiply 9008 by 784. *Ans.* 7062272.
32. Multiply 3207 by 2345. *Ans.* 7520415.
33. Multiply 6579 by 3506. *Ans.* 23065974.
34. Multiply 8579 by 4078. *Ans.* 34985162.
35. Multiply 7058 by 6007. *Ans.* 42397406.
36. Multiply 35768 by 3456. *Ans.* 123614208.
37. A certain volume contains 752 pages; how many pages in 608 such volumes? *Ans.* 457216 pages.
38. Multiply six thousand and ninety-seven by nine hundred and eight. *Ans.* 5536076.
39. Multiply fifty-four thousand and forty-nine by six thousand and seventy-five. *Ans.* 328347675.
40. The two factors of a certain number are 656 and 907. What is the number? *Ans.* 594992.
41. How many letters in a book containing 672 pages, each page containing 43 lines, and each line 47 letters? *Ans.* 1358112 letters.

CONTRACTIONS IN MULTIPLICATION.

CASE I.

ART. 53. *When the multiplier is a composite number.*

A COMPOSITE number is one produced by multiplying together two or more numbers, each greater than a unit; thus 12 is a *composite* number, since $6 \times 2 = 12$; or $3 \times 4 = 12$; or $2 \times 2 \times 3 = 12$.

ART. 54. The *component factors* of a number are the numbers which, multiplied together, will produce the given number. Thus, the component factors of 18 are 9 and 2, or 6 and 3, or 3 and 3 and 2.

Ex. 1. What would be the cost of 42 pieces of silk at 63 dollars per piece?

Analysis. Since 1 piece is worth 63 dollars, 42 pieces would be worth 42 times 63 dollars; hence, we must multiply 63 dollars by 42. But since 42 is equal to 6×7 , or to $3 \times 2 \times 7$, we may perform the operation by the following methods:

1st Method.	2d Method.	3d Method.
63 dollars.	63 dollars.	63 dollars.
<u>42</u>	<u>6</u>	<u>3</u>
126	378	189
<u>252</u>	<u>7</u>	<u>2</u>
Ans. 2646 dollars.	2646 dollars.	378
		<u>7</u>
		2646 dollars.

Hence, we have the following

RULE.

Separate the composite number into two or more factors. Multiply the multiplicand by one of the factors, and the product thus obtained by another factor, until all the factors have been used. The last product will be the result required.

EXAMPLES FOR PRACTICE.

2. How many yards of cloth in 24 pieces, each piece containing 47 yards?

Ans. 1128 yards.

3. If a ton of guano cost 58 dollars, what would be the value of 32 tons?

Ans. 1856 dollars.

4. If an acre of land produce 52 bushels of wheat, how many bushels will be produced on 28 acres?

Ans. 1456 bushels.

5. Multiply 3456 by 56.

Ans. 193536.

6. Multiply 6809 by 63.

Ans. 428967.

7. Multiply 987 by 54.

Ans. 53298.

CASE II.

ART. 55. *When the multiplier is 10, 100, or 1000, etc.*

Annexing *one* cipher to a number removes the number *one* place to the left, and therefore increases its value *ten* times; annexing *two* ciphers removes the number *two* places to the left, and increases its value 100 times, which is the same as multiplying it by 100, etc. Thus, if we annex *one* cipher to 37, it becomes 370; annex *two* ciphers, and it becomes 3700; annexing *three* ciphers, we make it 37000.

Hence, to multiply by 10, 100, or 1000, etc.,

Annex as many ciphers to the multiplicand as there are ciphers in the multiplier.

EXAMPLES FOR PRACTICE.

1. In one dollar there are 100 cents; how many cents in 225 dollars? *Ans.* —.

2. In one mile there are 320 rods; how many rods in 100 miles? *Ans.* —.

3. At 75 dollars per acre, what would 1000 acres of land be worth? *Ans.* —.

4. What would 10000 tons of guano cost, at 59 dollars per ton? *Ans.* —.

5. In one dime there are 10 cents; how many cents in 465 dimes? *Ans.* —.

6. How much would 100000 barrels of flour cost, at 12 dollars a barrel? *Ans.* —.

CASE III.

ART. 56. *When there are ciphers at the right hand of one or both of the factors.*

Ex. 1. At 500 dollars apiece, what would be the cost of 30 pianos?

Analysis. At 100 dollars apiece, it is evident that the 30 pianos would cost $100 \times 30 = 3000$ dollars; and at 500 dollars apiece they would cost 5 times as much as at 100 dollars each, or 5 times 3000 dollars = 15000 dollars.

OPERATION.

500 Multiplicand
 30 Multiplier.
 15000 Product.

Hence, we use the following

RULE.

Multiply the SIGNIFICANT figures together, and to the product annex as many ciphers as there are at the right hand of both the factors.

EXAMPLES FOR PRACTICE.

2. What would be the weight of 300 bales of cotton averaging 550 pounds per bale?

OPERATION.

550
 300

Ans. 165000 pounds.

Instead of writing *units* under *units*, *tens* under *tens*, etc., it is the usual practice, when ciphers are at the right of the multiplier, to place the multiplier under the multiplicand, as indicated in the foregoing "operation."

- | | |
|---------------------------|------------------------|
| 3 Multiply 570 by 80. | <i>Ans.</i> 45600. |
| 4 Multiply 75600 by 320. | <i>Ans.</i> 24192000. |
| 5 Multiply 4030 by 5600. | <i>Ans.</i> 22568000. |
| 6. Multiply 5009 by 6200. | <i>Ans.</i> 31055800. |
| 7 Multiply 70600 by 4007 | <i>Ans.</i> 282894200 |
| 8. Multiply 80900 by 5040 | <i>Ans.</i> 407736000. |

DIVISION.

ARTICLE 57. Ex. 1. James has 12 oranges, which he wishes to divide between his 3 brothers; how many oranges can he give to each?

Analysis. Here we are required to divide 12 oranges into 3 equal parts.

If James gives each brother *one* orange, it will require 3 oranges, and 9 oranges would be left. If now he gives each of

them another orange, it will require 3 oranges, and 6 oranges would be left.

If he gives them one apiece a *third* time, it is evident there would be only

3 oranges left. If he gives them one apiece a *fourth* time, there would be

none left. Hence, it is plain that he can give each of his brothers 4 oranges.

OPERATION.

$$\begin{array}{r}
 12 \text{ oranges.} \\
 3 \\
 \hline
 9 \text{ 1st remainder.} \\
 3 \\
 \hline
 6 \text{ 2d remainder.} \\
 3 \\
 \hline
 3 \text{ 3d remainder.} \\
 3 \\
 \hline
 0 \text{ 4th remainder.}
 \end{array}$$

Ans. 4 oranges.

In this example we see that 12 contains 3 *four* times, for if we subtract 3 from 12 *four* times, nothing is left. A number, therefore, may be divided into equal parts by *subtraction*.

We might have found the answer in a shorter way by saying that each brother would have as many oranges as 3 is contained *times* in 12; and 3 is contained in 12 *four* times. How do we know this? Because we know that 3 times 4 are 12. Hence, to find how many times one number is contained in a second, we must merely find what number multiplied by the first will produce the second.

2. 5 is contained in 20 how many times? *Ans.* 4 times
How do we know this? Because 4 times 5 are 20.

3. 3 is contained in 18 how many times? *Ans.* 6 times
Why? Because 6 times 3 are 18.

4. How many times is 7 contained in 21? *Ans.* —. Why?

5. How often does 4 go into 24? *Ans.* —. Why?

6. If 3 lemons cost 15 cents, what is the cost of 1 lemon?

7. If 5 boys eat 30 peaches, how many will that be for each one?

8. If 5 marbles be put in a ring, how many rings can you make out of 40 marbles?

The foregoing examples are all solved by *Division*. Hence,
ART. 58. DIVISION is the process of finding how many times one number is contained in another.

ART. 59. The DIVIDEND is the number to be divided.

The DIVISOR is the number by which we divide.

The QUOTIENT is the number which shows how many times the divisor is contained in the dividend.

ART. 60. The REMAINDER is that which is sometimes left after dividing. The remainder being a part of the dividend, is always of the same kind or denomination as the dividend. When there is no remainder, the division is said to be *exact*.

ART. 61. The SIGN OF DIVISION is a short horizontal line with a dot above and a dot below, $\div$; it signifies that the number *before* it is to be divided by the number *after* it; thus $12 \div 4$ denotes that 12 is to be divided by 4.

Division is frequently indicated by a line, with the dividend above it and the divisor below it; thus, $\overset{15}{\underset{5}{\div}}$ denotes that 15 is to be divided by 5. In practice, Division is usually indicated by placing the divisor on the left of the dividend, with a curved line between: thus, $6)24$ indicates that 24 is to be divided by 6.

ART. 62. Division is the *opposite* of Multiplication. In Multiplication we usually have *two* factors to find their *product*. In Division we have the *product* and *one* of the factors to find the other factor.

The table below should be thoroughly mastered by the pupil before proceeding farther :

DIVISION TABLE.

1 in	2 in	3 in	4 in
1, 1 time.	2, 1 time.	3, 1 time.	4, 1 time.
2, 2 times.	4, 2 times.	6, 2 times.	8, 2 times.
3, 3 "	6, 3 "	9, 3 "	12, 3 "
4, 4 "	8, 4 "	12, 4 "	16, 4 "
5, 5 "	10, 5 "	15, 5 "	20, 5 "
6, 6 "	12, 6 "	18, 6 "	24, 6 "
7, 7 "	14, 7 "	21, 7 "	28, 7 "
8, 8 "	16, 8 "	24, 8 "	32, 8 "
9, 9 "	18, 9 "	27, 9 "	36, 9 "
10, 10 "	20, 10 "	30, 10 "	40, 10 "
11, 11 "	22, 11 "	33, 11 "	44, 11 "
12, 12 "	24, 12 "	36, 12 "	48, 12 "
5 in	6 in	7 in	8 in
5, 1 time.	6, 1 time.	7, 1 time.	8, 1 time.
10, 2 times.	12, 2 times.	14, 2 times.	16, 2 times.
15, 3 "	18, 3 "	21, 3 "	24, 3 "
20, 4 "	24, 4 "	28, 4 "	32, 4 "
25, 5 "	30, 5 "	35, 5 "	40, 5 "
30, 6 "	36, 6 "	42, 6 "	48, 6 "
35, 7 "	42, 7 "	49, 7 "	56, 7 "
40, 8 "	48, 8 "	56, 8 "	64, 8 "
45, 9 "	54, 9 "	63, 9 "	72, 9 "
50, 10 "	60, 10 "	70, 10 "	80, 10 "
55, 11 "	66, 11 "	77, 11 "	88, 11 "
60, 12 "	72, 12 "	84, 12 "	96, 12 "
9 in	10 in	11 in	12 in
9, 1 time.	10, 1 time.	11, 1 time.	12, 1 time.
18, 2 times.	20, 2 times.	22, 2 times.	24, 2 times.
27, 3 "	30, 3 "	33, 3 "	36, 3 "
36, 4 "	40, 4 "	44, 4 "	48, 4 "
45, 5 "	50, 5 "	55, 5 "	60, 5 "
54, 6 "	60, 6 "	66, 6 "	72, 6 "
63, 7 "	70, 7 "	77, 7 "	84, 7 "
72, 8 "	80, 8 "	88, 8 "	96, 8 "
81, 9 "	90, 9 "	99, 9 "	108, 9 "
90, 10 "	100, 10 "	110, 10 "	120, 10 "
99, 11 "	110, 11 "	121, 11 "	132, 11 "
108, 12 "	120, 12 "	132, 12 "	144, 12 "

SHORT DIVISION.

ART. 63. When the divisor does not exceed 12, the operation of division is usually carried on in the mind, and the result is written down. This is called **SHORT DIVISION**.

Ex. 1. At 2 cents apiece, how many apples can you buy for 846 cents?

Analysis. Since 2 cents will buy 1 apple, 846 cents will buy as many apples as 2 cents are contained *times* in 846 cents. We place the divisor

OPERATION.

Divisor 2)846 Dividend.
423 Quotient.

at the left of the dividend, with a curved line between, and draw a line underneath the dividend. We then say: 2 is contained in 8 hundreds 4 hundreds times, and we write the 4 in the *hundreds'* place, under the figure divided. Then, 2 is contained in 4 tens 2 tens times, and we write the 2 in the *tens'* place in the quotient. Lastly, 2 is contained in 6 units 3 units times, and we write the 3 in the *units'* place. Hence, the quotient is 4 hundreds, 2 tens and 3 units, or 423.

Ans. 423 apples.

EXAMPLES.

- | | |
|------------------------|----------------|
| 2. Divide 428 by 2. | <i>Ans.</i> —. |
| 3. Divide 248 by 2. | <i>Ans.</i> —. |
| 4. Divide 693 by 3. | <i>Ans.</i> —. |
| 5. Divide 936 by 3. | <i>Ans.</i> —. |
| 6. Divide 840 by 4. | <i>Ans.</i> —. |
| 7. Divide 484 by 4. | <i>Ans.</i> —. |
| 8. Divide 550 by 5. | <i>Ans.</i> —. |
| 9. Divide 6428 by 2. | <i>Ans.</i> —. |
| 10. Divide 93630 by 3. | <i>Ans.</i> —. |
| 11. Divide 48084 by 4. | <i>Ans.</i> —. |
| 12. Divide 968 by 2. | |

Analysis. We say: 2 is contained in 9 hundred 4 hundreds times, and 1 hundred over. We set the 4 in the *hundreds'* place in the quotient. Now, the 1 hundred over is equal to 10 *tens*. Adding these 10 *tens* to the 6 *tens* of the example, we have 16 *tens*; then 2 is contained in 16 *tens* 8 *tens* times, and we write the 8 in the *tens'* place in the quotient. Lastly, 2 is contained in 8 units 4 units times, and we write the 4 in the *units'* place. Hence, the quotient is 4 hundreds, 8 *tens* and 4 units, or 484.

OPERATION.

$$\begin{array}{r} \text{Divisor } 2 \overline{)968} \text{ Dividend.} \\ \underline{484} \text{ Quotient.} \end{array}$$

13. Divide 526 by 2.

Ans. 484.

14. Divide 728 by 2.

Ans. 263.

15. Divide 7950 by 3.

Ans. —.

16. Divide 4650 by 3.

Ans. 2650.

17. Divide 9720 by 4.

Ans. —.

18. Divide 5720 by 4.

Ans. 2430.

19. Divide 6650 by 5.

Ans. —.

20. Divide 6150 by 5.

Ans. 1330.

21. Divide 228 by 3.

Ans. —.

Analysis. Since the divisor, 3, is not contained in the first figure of the dividend, we find how many times it is contained in the *first two* figures. We regard the 2 *hundreds* as being equal to 20 *tens*, and adding these 20 *tens* to the 2 *tens* of the example, thereby making 22 *tens*, we say: 3 is contained in 22 *tens* 7 *tens* times, with a remainder of 1 *ten*. We write the 7 in the *tens'* place in the quotient. Since the 1 *ten* remaining is equal to 10 *units*, we add these 10 *units* to the 8 *units* of the example, making 18 *units*, and say: 3 is contained in 18 *units* 6 *units* times, and we write the 6 in the *units'* place, thus making the entire quotient 7 *tens* and 6 *units*, or 76.

OPERATION.

$$\begin{array}{r} 3 \overline{)228} \text{ Dividend.} \\ \underline{76} \text{ Quotient.} \end{array}$$

Ans. 76.

- | | |
|--------------------------------------|--------------------------------------|
| 22. Divide 154 by 2. <i>Ans.</i> 77. | 34. Divide 225 by 5. <i>Ans.</i> 45 |
| 23. Divide 134 by 2. <i>Ans.</i> —. | 35. Divide 325 by 5. <i>Ans.</i> —. |
| 24. Divide 178 by 2. <i>Ans.</i> 89. | 36. Divide 324 by 6. <i>Ans.</i> 54. |
| 25. Divide 192 by 2. <i>Ans.</i> —. | 37. Divide 312 by 6. <i>Ans.</i> —. |
| 26. Divide 282 by 3. <i>Ans.</i> 94. | 38. Divide 378 by 7. <i>Ans.</i> 54. |
| 27. Divide 258 by 3. <i>Ans.</i> —. | 39. Divide 301 by 7. <i>Ans.</i> —. |
| 28. Divide 288 by 3. <i>Ans.</i> 96. | 40. Divide 432 by 8. <i>Ans.</i> 54. |
| 29. Divide 225 by 3. <i>Ans.</i> —. | 41. Divide 448 by 8. <i>Ans.</i> —. |
| 30. Divide 144 by 4. <i>Ans.</i> 36. | 42. Divide 396 by 9. <i>Ans.</i> 44. |
| 31. Divide 172 by 4. <i>Ans.</i> —. | 43. Divide 378 by 9. <i>Ans.</i> —. |
| 32. Divide 304 by 4. <i>Ans.</i> 76. | 44. Divide 225 by 9. <i>Ans.</i> 25. |
| 33. Divide 344 by 4. <i>Ans.</i> —. | 45. Divide 216 by 9. <i>Ans.</i> —. |
46. Divide 8328 by 4.

Analysis. We say: 4 is contained in 8 thousand 2 thousands times, and we write 2 in the thousands' place. Then, 4 is con-

tained in 3 hundreds *no* hundreds times, so we put a 0 in the hundreds' place, and calling the 3 hundreds 30 *tens*, we add these 30 *tens* to the 2 *tens* of the example, making 32 *tens*. Then, 4 is contained in 32 tens 8 tens times, and we set the 8 in the tens' place in the quotient. Then, 4 is contained in 8 units 2 units times, and we write the 2 in the units' place. We thus get for the entire quotient, 2 thousands, 0 hundreds, 8 tens and 2 units, or 2082.

OPERATION.

Divisor 4)8328 Dividend.
2082 Quotient.

Observation.—In the actual working of examples we disregard the *local* values of figures, and consider them not as *tens*, *hundreds*, etc., but *all* as units. The working of the last example would be about as follows:

"4 into 8, 2 times; 4 into 3, 0 times; 4 into 32, 8 times; 4 into 8, 2 times."

Ans. 2082.

NOTE.—To teach a pupil merely *how* to work examples is to make Arithmetic simply a series of *mechanical* operations; to

teach him the *principle* that underlies each operation is to train his *mental* powers. This latter object is the aim of *Analysis*

- | | |
|-----------------------|-------------------|
| 47. Divide 9255 by 3. | <i>Ans.</i> 3085. |
| 48. Divide 6138 by 3. | <i>Ans.</i> — |
| 49. Divide 8256 by 4. | <i>Ans.</i> 2064. |
| 50. Divide 8212 by 4. | <i>Ans.</i> —. |
| 51. Divide 5325 by 5. | <i>Ans.</i> 1065. |
| 52. Divide 5235 by 5. | <i>Ans.</i> —. |
| 53. Divide 6336 by 6. | <i>Ans.</i> 1056. |
| 54. Divide 6264 by 6. | <i>Ans.</i> —. |
| 55. Divide 7315 by 7. | <i>Ans.</i> 1045 |
| 56. Divide 7238 by 7. | <i>Ans.</i> — |

From the foregoing explanations we derive the

RULE.

I. *Write the divisor at the left of the dividend, with a curved line between.*

II. *Find how many times the divisor is contained in the left-hand figure or figures of the dividend, and write the quotient underneath, and so proceed with each figure.*

III. *If there is a remainder after any division, prefix* it to the next figure of the dividend, and divide as before.*

IV. *When the divisor is not contained in any partial dividend, write a cipher in the quotient, prefix it to the next figure, and divide as before.*

PROOF. Multiply the quotient by the divisor, and to the product add the remainder, if any; the result should be equal to the dividend.

EXAMPLES FOR PRACTICE.

- | | |
|---------------------|------------------|
| 1. Divide 768 by 3. | <i>Ans.</i> 256. |
| 2. Divide 592 by 3. | <i>Ans.</i> —. |

* *Prefix* means to place before; i. e., at the left.

3. Divide 896 by 4	<i>Ans.</i> 224
4. Divide 765 by 5.	<i>Ans.</i> —.
5. Divide 876 by 6.	<i>Ans.</i> 146.
6. Divide 875 by 7.	<i>Ans.</i> —.
7. Divide 892 by 8.	<i>Ans.</i> 124.
8. Divide 8202 by 3	<i>Ans.</i> —.
9. Divide 8224 by 4.	<i>Ans.</i> 2056.
10. Divide 6625 by 5.	<i>Ans.</i> —.
11. Divide 7458 by 6.	<i>Ans.</i> 1243.
12. Divide 8575 by 7.	<i>Ans.</i> —.
13. Divide 9872 by 8.	<i>Ans.</i> 1234.
14. Divide 9945 by 9.	<i>Ans.</i> 1105.
15. Divide 32268 by 2.	<i>Ans.</i> —.
16. Divide 89598 by 3.	<i>Ans.</i> —.
17. Divide 97372 by 4.	<i>Ans.</i> 24343.
18. Divide 25625 by 5.	<i>Ans.</i> —.
19. Divide 44736 by 6.	<i>Ans.</i> 7456.
20. Divide 42245 by 7.	<i>Ans.</i> —.
21. Divide 43272 by 8.	<i>Ans.</i> 5409.
22. Divide 46926 by 9.	<i>Ans.</i> —.
23. Divide 27024 by 3.	<i>Ans.</i> 9008.
24. Divide 32136 by 4.	<i>Ans.</i> —.
25. Divide 32535 by 5.	<i>Ans.</i> 6507.
26. Divide 31584 by 6.	<i>Ans.</i> —.
27. Divide 49392 by 7.	<i>Ans.</i> 7056
28. Divide 48344 by 8.	<i>Ans.</i> —.
29. Divide 74178 by 9.	<i>Ans.</i> 8242.
30. Divide 99468 by 4.	<i>Ans.</i> 24867

PRACTICAL QUESTIONS.

1. At 3 dollars a yard, how many yards of cassimere could be bought for 834 dollars? *Ans.* 278 yards.

2. At 4 dollars a bushel, how many bushels of peaches could you buy for 992 dollars? *Ans.* 248 bushels.

3. How many sacks would hold 925 bushels of meal, each sack holding 5 bushels? *Ans.* 185 sacks.

4. At 6 cents apiece, how many oranges could you buy for 972 cents? *Ans.* 162 oranges.

5. Seven days make a week. How many weeks in 2275 days? *Ans.* 325 weeks.

6. Eight quarts make a peck. How many pecks in 3456 quarts? *Ans.* 432 pecks.

7. At 9 dollars a barrel, how many barrels of flour could be bought for 4725 dollars? *Ans.* 525 barrels.

8. In one gallon there are 4 quarts. How many gallons in 3512 quarts? *Ans.* 878 gallons.

9. At 5 cents apiece, how many boxes of matches could you buy for 7225 cents? *Ans.* 1445 boxes.

10. If 6 casks of bacon weigh 7572 pounds, what is the average weight of each cask? *Ans.* 1262 pounds.

11. If a locomotive run 14525 miles in 7 weeks, what is the average distance per week? *Ans.* 2075 miles.

12. If 8 miles of plank-road cost 16536 dollars, what is the average cost per mile? *Ans.* 2067 dollars.

13. Nine men built a mill for 45405 dollars. What was the average amount paid by each? *Ans.* 5045 dollars.

14. At 3 cents apiece, how many postage-stamps could you buy for 29628 cents? *Ans.* 9876 stamps.

15. A gentleman left an estate of 35056 dollars to be divided equally between his wife and his three daughters. What was the share of each? *Ans.* 8764 dollars.

16. A cotton factory, costing 43458 dollars, was erected by six gentlemen, each contributing an equal amount of capital; how much did each man furnish?

Ans. 7243 dollars.

17. If 7 sacks contain 63175 chestnuts, what is the average number in each sack? *Ans.* 9025 chestnuts.

18. A factory worth 64192 dollars was owned in equal shares by 8 gentlemen. What was the value of a share? *Ans.* 8024 dollars.

19. At 9 dollars a ton, how many tons of coal could be bought for 36225 dollars? *Ans.* 4025 tons.

20. The product of two numbers is 22575, and one of the numbers is 7; what is the other number? *Ans.* 3225.

21. If 97 apples be divided equally between 3 boys, how many would each boy have?

Analysis. In performing this division, we see that each boy receives 32 whole apples, and there is one apple left. This apple left, being a part of the dividend, is also to be divided between the 3 boys. But when a thing is divided into three equal parts, one of the parts is called one third, and is written $\frac{1}{3}$. So, each boy will have $32\frac{1}{3}$ apples. Hence, when there is a remainder, we place it at the right of the quotient, and write the divisor under it.

22. If 5 bales of cotton weigh 2626 pounds, what is the average weight per bale? *Ans.* $525\frac{1}{5}$ pounds.

23. If 6 tierces of rice weigh 3257 pounds, what is the average weight of each? *Ans.* $542\frac{2}{3}$ pounds.

LONG DIVISION.

ART. 64. When the divisor exceeds 12, the process is called LONG DIVISION. The principle of LONG DIVISION is precisely the same as that of SHORT DIVISION, the only difference being that in SHORT DIVISION the necessary multiplications and subtractions are carried on in the mind; whereas in LONG DIVISION we write down all the figures employed, because the numbers are usually too large to be retained in the memory.

Ex. 1. Divide 7296 by 16.

Analysis. We see that 16 units are not contained in 7 thousands any number of *thousands* times, consequently there will be no figure to put in the *thousands'* place in the quotient. We therefore join the 7 thousands to the 2 hundreds which follow, making 72 hundreds, and we say: 16 units are contained in 72 hundreds 4 *hundred* times, and we set the 4 on the *right* in the quotient. We then multiply the divisor, 16, by the quotient figure, 4, and place the product, 64, under 72, and subtract. The remainder is 8 *hundreds*, to which

OPERATION.

thousands. hundreds.	tens. units.	hundreds. tens. units.
16)7296	(456	
	64	
	89	
	80	
	96	
	96	
	0	

we bring down the 9 *tens* of the dividend, making 89 *tens*. We again say: 16 units are contained in 89 *tens*, 5 *tens* times, and we place the 5 at the right of the figure previously placed in the quotient. We then multiply the 16 units by the 5 *tens*, and subtract the product, 80 *tens*, from 89 *tens*, and we have a remainder of 9 *tens*. To this remainder of 9 *tens* we bring down the 6 units of the dividend, making 96 units. Then, 16 units are contained in 96 units, 6 times, and we write the 6 in the quotient. We then multiply the 16 units of the divisor by the 6 of the quotient, and subtracting the product, 96 units, from the 96 units of the dividend, we have 0 for a remainder. The entire quotient is therefore, 4 hundreds, 5 tens and 6 units, or 456.

Ans. 456.

ART. 65. When the pupil begins to understand the *reason* of the operations employed, he may then disregard the *local* values of figures, and consider them not as *tens*, *hundreds*, etc., but all as *units*. This plan gives a *Short Analysis*.

2. Divide 5175 by 15.

Short Analysis. 15 is contained in 51, 3 times; 3 times 15 are 45; 45 from 51 leaves 6, and bringing down the

next figure, 7, we have 67. Then, 15 is contained in 67, 4 times; 4 times 15 are 60; 60 from 67 leaves 7, and bringing down the next figure, 5, we have 75. Then, 15 is contained in 75, 5 times; 5 times 15 are 75; then 75 from 75 leaves 0. Hence the quotient is 345.

3 Divide 8568 by 28.

Short Analysis. 28 goes into 85, 3 times; 3 times 28 are 84; 84 from 85 leaves 1, and bringing down the next figure, 6, we have 16. Then, 28 goes into 16, 0 times, and we write 0 in the quotient. Bring down the 8, and we have 168. Then, 28 goes into 168, 6 times; 6 times 28 are 168; 168 from 168 leaves 0. Hence, the quotient is 306. *Ans.* 306.

By inspecting the foregoing *Analyses* the pupil will see that there are *four* steps in the operation:

1st, *Find how many times*; 2d, *Multiply*; 3d, *Subtract*; 4th, *Bring down*.

4. Divide 96768 by 224.

Short Analysis. 224 goes into 967, 4 times; 4 times 224 are 896; 896 from 967 leaves 71. Bring down the 6, and we have 716. Then, 224 goes into 716, 3 times; 3 times 224 are 672; 672 from 716 leaves 44. Bring

OPERATION.

$$\begin{array}{r}
 \text{Dividend.} \\
 \text{Divisor } 15 \overline{) 5175} \text{ (345 Quotient)} \\
 \underline{45} \\
 67 \\
 \underline{60} \\
 75 \\
 \underline{75} \\
 0
 \end{array}$$

Ans. 345.

OPERATION.

$$\begin{array}{r}
 \text{Dividend.} \\
 \text{Divisor } 28 \overline{) 8568} \text{ (306 Quotient)} \\
 \underline{84} \\
 168 \\
 \underline{168} \\
 0
 \end{array}$$

OPERATION.

$$\begin{array}{r}
 \text{Dividend.} \\
 \text{Divisor } 224 \overline{) 96768} \text{ (432 Quotient)} \\
 \underline{896} \\
 716 \\
 \underline{672} \\
 448 \\
 \underline{448} \\
 0
 \end{array}$$

down the 8, and we have 448. Then, 224 goes into 448, 2 times; 2 times 224 are 448; 448 from 448 leaves 0.

Ans. 432

From the foregoing illustrations we derive the

RULE.

I. *Draw a curved line on each side of the dividend, and write the DIVISOR on the left.*

II. *Find how many times the divisor is contained in the fewest left-hand figures of the dividend that will contain it, and place the quotient on the right.*

III. *Multiply the divisor by this quotient figure, and subtract the product from the figures of the dividend used, and to the remainder annex* the next figure of the dividend.*

IV. *Divide as before, and continue the operation until all the figures of the dividend have been brought down.*

V. *If any partial dividend will not contain the divisor, place a cipher in the quotient, and bring down the next figure of the dividend, and divide as before.*

PROOF. The same as in SHORT DIVISION.

NOTE 1.—When there is a remainder after all the figures of the dividend have been brought down and divided, it may either be set off by itself, or it may be written over the divisor and annexed to the quotient.

NOTE 2.—If the product of the divisor by the quotient figure is *larger* than the partial dividend, it shows that the quotient figure is too *large*, and must be diminished.

NOTE 3.—If the remainder at any time is *equal to*, or *greater than*, the divisor, it shows that the quotient figure is too *small*, and must be increased.

Observation. The teacher will find it profitable to make the pupil give the *full* analysis for a few times, until he becomes familiar with the operation.

* *Annex* means to place after; i. e., on the right.

EXAMPLES FOR PRACTICE

5. Divide 528 by 22.	<i>Ans.</i> 24
6. Divide 575 by 25.	<i>Ans.</i> —.
7. Divide 1056 by 32.	<i>Ans.</i> 33
8. Divide 1085 by 35.	<i>Ans.</i> —.
9. Divide 972 by 36.	<i>Ans.</i> 27.
10. Divide 924 by 42.	<i>Ans.</i> —.
11. Divide 3276 by 14.	<i>Ans.</i> 234.
12. Divide 5472 by 16.	<i>Ans.</i> —.
13. Divide 6210 by 18.	<i>Ans.</i> 345.
14. Divide 7245 by 21.	<i>Ans.</i> —.
15. Divide 5472 by 24.	<i>Ans.</i> 228.
16. Divide 6318 by 26.	<i>Ans.</i> —.
17. Divide 6789 by 31.	<i>Ans.</i> 219.
18. Divide 7755 by 33.	<i>Ans.</i> —.
19. Divide 7942 by 38.	<i>Ans.</i> 209.
20. Divide 9492 by 42.	<i>Ans.</i> —.
21. Divide 30452 by 23.	<i>Ans.</i> 1324.
22. Divide 77253 by 33.	<i>Ans.</i> —.
23. Divide 98304 by 48.	<i>Ans.</i> 2048.
24. Divide 86408 by 56.	<i>Ans.</i> 1543.
25. Divide 92625 by 75.	<i>Ans.</i> 1235.
26. Divide 85936 by 82.	<i>Ans.</i> 1048.
27. Divide 56133 by 231.	<i>Ans.</i> 243.
28. Divide 82944 by 256.	<i>Ans.</i> 324.
29. Divide 98595 by 313.	<i>Ans.</i> 315.
30. Divide 524944 by 1526.	<i>Ans.</i> 344.
31. Divide 926492 by 2842.	<i>Ans.</i> 326.

PRACTICAL QUESTIONS.

1. At 35 dollars per month, how many months could a man board for 875 dollars? *Ans.* 25 months

2. A drover bought 45 head of cattle for 2025 dollars; what was the price per head? *Ans.* 45 dollars.

3. In one hogshead there are 63 gallons; how many hogsheads in 20412 gallons? *Ans.* 324 hogsheads.

4. If 75 shares of bank stock sell for 9225 dollars, what is the price per share? *Ans.* 123 dollars.

5. The product of two numbers is 53284, and one of the numbers is 154; what is the other? *Ans.* 346.

6. If 235 bushels of wheat weigh 14335 pounds, what is the average weight per bushel? *Ans.* 61 pounds.

7. A farmer has a field of cotton in which there are 155952 stalks in 342 rows; how many stalks in each row?

Ans. 456 stalks.

8. A man bought a farm of 524 acres for 24104 dollars; what was the average price per acre? *Ans.* 46 dollars.

9. If 645 tons of railroad-iron cost 40635 dollars, what is the average price per ton? *Ans.* 63 dollars.

10. The earth moves around the sun about 68000 miles in an hour, or 60 minutes; what is the speed per minute?

Ans. $1133\frac{1}{3}$ miles

11. The wages paid in a certain factory amount in a year, or 52 weeks, to 67849 dollars; what is the average amount per week? *Ans.* $1304\frac{1}{4}$ dollars.

12. The earth being 24856 miles around, how many miles of daily travel would it require for a person to go around it in a year, or 365 days? *Ans.* $68\frac{3}{5}$ miles.

13. How many bales could be made out of 281765 pounds of cotton, allowing 517 pounds to the bale? *Ans.* 545 bales.

14. A drover paid 44225 dollars for 160 horses and 145 mules; what was the average cost of each? *Ans.* 145 dollars.

15. How many bales could be made out of 67704 yards of cloth, allowing 42 yards to a piece, and 31 pieces to a bale?

Ans. 52 bales.

16. If a ship's crew of 756 men be allowed 338688 pounds of biscuit for a cruise, how much can be allowed to each man?

Ans. 448 pounds.

17. At the rate of 307 miles a day, how many days would a steamship occupy in sailing 9824 miles?

Ans. 32 days.

18. Divide one million forty-three thousand six hundred and twenty-five by six hundred and five.

Ans. 1725.

CONTRACTIONS IN DIVISION.

ART. 66. *When the divisor is a COMPOSITE number.*

Ex. 1. If 18 acres of land cost 864 dollars, what would be the cost per acre?

We may obtain the answer by the ordinary method of LONG DIVISION, or

Analysis. Since 18 is equal to 9×2 , or 6×3 , we may use two other methods:

$$\begin{array}{r} \text{1st Method.} \\ 18)864(48 \text{ dollars.} \\ \underline{72} \\ 144 \\ \underline{144} \end{array}$$

$$\begin{array}{r} \text{2d Method.} \\ 9)864 \\ \underline{2)96} \\ 48 \text{ dollars.} \end{array}$$

$$\begin{array}{r} \text{3d Method.} \\ 6)864 \\ \underline{3)144} \\ 48 \text{ dollars.} \end{array}$$

From the *Analysis* we derive the following

RULE.

Divide the dividend by one of the factors of the divisor, then divide the resulting quotient by another factor, and so on until every factor has been used as a divisor. The last quotient will be the answer required.

EXAMPLES.

2. If 24 men equally divide 7488 dollars, how many dollars will be the share of each?

Ans. 312 dollars.

- | | |
|------------------------|------------------|
| 3. Divide 5184 by 36. | <i>Ans.</i> 144. |
| 4. Divide 9765 by 45. | <i>Ans.</i> 217. |
| 5. Divide 9632 by 56. | <i>Ans.</i> 172. |
| 6. Divide 9792 by 72. | <i>Ans.</i> 136. |
| 7. Divide 18144 by 81. | <i>Ans.</i> 224. |

ART. 67. In using the foregoing method of Division pupils sometimes find a difficulty in determining what is the *true* remainder when it happens that remainders are left after dividing by the *factors* of the *composite* number.

8. Divide 757 by 32.

By the Common Method.

$$\begin{array}{r}
 32 \overline{)757} \text{ (23 Quotient.} \\
 \underline{64} \\
 117 \\
 \underline{96} \\
 21 \text{ Remainder.}
 \end{array}$$

By the Method of Factors.

$$\begin{array}{r}
 8 \overline{)757} \\
 \underline{4)94} \text{ Remainder, 5.} \\
 23 \text{ Remainder, 2.}
 \end{array}$$

By the common method, we get 23 for a quotient, with a remainder of 21. By the method of factors, we also get a quotient, 23, but it does not so readily appear what is the *true* remainder. If, however, we multiply the *last remainder*, 2, by the *first divisor*, 8, making 16, and to the product add the *first remainder*, 5, we shall have the *true* remainder, 21.

We may explain the *reason* of the last operation thus :

We know that 8 *quarts* make a *peck*, and 4 *pecks* make a *bushel* ; therefore, a bushel contains 32 *quarts*. Suppose now we wished to find how many bushels there are in 757 *quarts*. We should do this by dividing 757 by 32, or by its factors, 8 and 4. When we divide 757 *quarts* by 8, we evidently get 94 *pecks*, with a remainder of 5 *quarts*. Again, as it requires 4 *pecks* to make a bushel, when we divide 94 *pecks* by 4, it is plain that we get 23 *bushels*, with a remainder of 2 *pecks*. Now, since 1 *peck* is equal to 8 *quarts*, 2 *pecks* will be equal

to 2 times 8 *quarts* = 16 *quarts*, and adding the 5 *quarts* which remained after the first division, we have 21 *quarts* for the *true* remainder.

The same mode of reasoning may be extended to *any number* of factors.

Hence, when there are *two* factors, we use the following general

RULE.

Multiply the first divisor by the last remainder, and to the product add the first remainder; the result will be the TRUE remainder.

When there are more than two factors :

Multiply the last remainder by the divisor preceding the last, and add the preceding remainder; multiply this sum by the next preceding divisor, and add the next preceding remainder, and so continue.

9 Divide 2837 by 42.	<i>Ans.</i> 67; Rem. 23.
10. Divide 3664 by 27.	<i>Ans.</i> 135; Rem. 19.
11. Divide 3971 by 54.	<i>Ans.</i> 73; Rem. 29.
12 Divide 2967 by 35	<i>Ans.</i> 84; Rem. 27.
13. Divide 4642 by 63.	<i>Ans.</i> 73; Rem. 43.

CASE II.

ART. 68. *To divide when the divisor is 10, 100, or 1000, etc*

In ART. 55 it was shown that to *annex* a cipher to a number is to *multiply* its value by 10; to annex *two* ciphers is to multiply by 100, and so on. As Division is the *reverse* of Multiplication, it follows that to *cut off* a cipher from the right of a number is to *divide* by 10; to cut off *two* ciphers is to divide by 100, and so on. Hence, when the divisor is 1, with ciphers annexed, we use the

RULE.

Cut off from the right of the dividend as many figures as there are ciphers at the right of the divisor; the remaining figures will be the quotient; those cut off will be the remainder

EXAMPLES.

- | | |
|-----------------------------|--------------------------------------|
| 1 Divide 3750 by 10. | <i>Ans.</i> 375. |
| 2 Divide 456 by 10 | <i>Ans.</i> 45 $\frac{6}{10}$. |
| 3. Divide 5875 by 100. | <i>Ans.</i> 58 $\frac{75}{100}$. |
| 4. Divide 4207 by 100. | <i>Ans.</i> 42 $\frac{7}{100}$. |
| 5. Divide 45875 by 1000. | <i>Ans.</i> 45 $\frac{875}{1000}$. |
| 6. Divide 37045 by 1000. | <i>Ans.</i> 37 $\frac{45}{1000}$. |
| 7. Divide 79006 by 1000. | <i>Ans.</i> 79 $\frac{6}{1000}$. |
| 8. Divide 7450085 by 10000. | <i>Ans.</i> 745 $\frac{85}{10000}$. |

CASE III.

ART. 69. *When the divisor is any number with ciphers annexed.*

Ex. 1. If 700 tons of guano sell for 44100 dollars, what is the price per ton?

Analysis. Here 700 may be regarded as a composite number, of which the two factors are 100 and 7. Now, cutting off two ciphers from the right of the dividend is equivalent to dividing by 100, which gives a quotient, 441; and dividing 441 by the other factor, 7, we have 63 as the quotient.

OPERATION.

$$\begin{array}{r} 7,00 \overline{)441,00} \\ \underline{63 \text{ dollars.}} \end{array}$$

Ans 63 dollars.

2. If 90 acres of land produce 3278 bushels of wheat, what would be the average yield per acre?

Analysis. Resolving 90 into the two factors 10 and 9, we first divide by 10, by cutting off the right-hand figure of the dividend, and we have 327 as a quotient, with 8 remainder.

OPERATION.

$$\begin{array}{r} 9,0 \overline{)327,8} \\ \underline{36.} \quad 3 \end{array}$$

Then, dividing the quotient, 327, by the other factor, 9, we have a quotient, 36, with a *remainder* of 3. To get the *true* remainder, we join the *last* remainder, 3, to the *first* remainder, 8, and we have the *true* remainder, 38. *Ans.* $36\frac{38}{9}$ bushels.

The *reason* of this last operation may be thus explained. When we divided the 3278 *units* by 10, the quotient was 327 *tens*, with a remainder of 8 *units*; and when we divided these 327 *tens* by 9, there was a remainder of 3 *tens*, or 30 *units*, which, added to the *first* remainder, 8 *units*, gave 38 as the *true* remainder.

From the foregoing illustrations we derive the

RULE.

I. *Cut off the ciphers at the right of the divisor, and also cut off the same number of figures from the right of the dividend.*

II. *Divide the remaining figures of the dividend by the remaining figures of the divisor, and to the remainder, if any, annex the figures cut off from the dividend for the TRUE remainder.*

EXAMPLES.

- | | |
|---------------------------|-------------------------------------|
| 3. Divide 34300 by 700. | <i>Ans.</i> 49. |
| 4. Divide 7856 by 40. | <i>Ans.</i> $196\frac{1}{5}$. |
| 5. Divide 87284 by 600. | <i>Ans.</i> $145\frac{284}{600}$. |
| 6. Divide 425375 by 800. | <i>Ans.</i> $531\frac{575}{800}$. |
| 7. Divide 320743 by 1200. | <i>Ans.</i> $267\frac{343}{1200}$. |

PROMISCUOUS QUESTIONS INVOLVING THE PREVIOUS RULES.

1. A gentleman bought a rockaway for 425 dollars, and a pair of horses at 225 dollars apiece; how much did he pay for the rockaway and horses? *Ans.* 875 dollars.

2. A gentleman left an estate worth 13275 dollars to his wife, his son, his daughter and his servant. To his son he left

3000 dollars, and to the daughter 525 dollars more than to the son. To the wife he gave as much as to the son and daughter together, and to the servant he left the remainder. What was the servant's share?

Ans. 225 dollars.

3. A gentleman bought a piano and 3 bureaus for 845 dollars. For the bureaus he paid 65 dollars apiece; what was the cost of the piano?

Ans. 650 dollars.

4. If a man has a salary of 1500 dollars a year, and his expenses average 3 dollars a day, how much could he lay up in a year, or 365 days?

Ans. 405 dollars.

5. Four men, A, B, C and D, own stock in a mill. A's capital is 3275 dollars; B's capital is 350 dollars more than A's; C's capital is 300 dollars less than B's, and D's capital is equal to that of B and C together; what is the value of the mill?

Ans. 17175 dollars.

6. If the driving-wheel of a locomotive makes 4 revolutions per second, and at each revolution moves 16 feet, how far would it move in an hour, or 3600 seconds?

Ans. 230400 feet.

7. A merchant bought 14 bales of cloth, each bale containing 28 pieces, and each piece measuring 36 yards; what would be the value of the whole at 16 cents a yard?

Ans. 225792 cents.

8. A merchant buys 288 barrels of flour at 9 dollars a barrel, and pays for it with tobacco at 36 dollars a box; how many boxes of tobacco must he give?

Ans. 72 boxes.

9. If a mechanic earns 52 dollars a month, and his expenses are 34 dollars a month, how long will it take him to pay for a farm of 36 acres, worth 12 dollars an acre?

Ans. 24 months.

10. What is the quotient of 43296 divided by 8 times the difference between 345 and 323?

Ans. 246.

11. How many pounds of tobacco at 45 cents per pound

would be required to pay for 9 tierces of rice, each weighing 576 pounds, and worth 10 cents a pound?

Ans. 1152 pounds.

12. A certain product is 27648, and one of the factors is 144; what is the other factor?

Ans. 192.

13. A farmer sold 48 bales of cotton at 112 dollars per bale. With the money he bought 360 acres of land at 12 dollars an acre; the remainder of the money he invested in the purchase of 8 mules; what was the price of the mules apiece?

Ans. 132 dollars.

14. If 15 horses are worth 3375 dollars, and one horse is equal in value to 5 cows, what is the value of one cow?

Ans. 45 dollars.

15. If a man earns 2500 dollars in a year, or 12 months, and if his expenses per month are 162 dollars, how many years will it require for him to lay up 4448 dollars?

Ans. 8 years.

GENERAL PRINCIPLES OF DIVISION.

ART. 70. In Division the value of the quotient depends on both the dividend and the divisor. If one of these remains the same, a change in the other produces a change in the quotient.

ART. 71. If the divisor remains the same, multiplying the dividend by any number is, in effect, multiplying the quotient by the same number; thus,

$$18 \div 6 = 3,$$

$$36 \div 6 = 6;$$

i. e., multiplying the dividend by 2 multiplies the quotient by 2.

ART. 72. If the divisor remains the same, dividing the dividend by any number is dividing the quotient by the same number; thus,

$$\begin{aligned} 32 \div 8 &= 4, \\ 16 \div 8 &= 2; \end{aligned}$$

i. e., dividing the dividend by 2 divides the quotient by 2.

ART. 73. If the dividend remains the same, multiplying the divisor divides the quotient; thus,

$$\begin{aligned} 24 \div 2 &= 12, \\ 24 \div 6 &= 4; \end{aligned}$$

i. e., multiplying the divisor by 3 divides the quotient by 3

ART. 74. If the dividend remains the same, dividing the divisor is multiplying the quotient; thus,

$$\begin{aligned} 36 \div 9 &= 4, \\ 36 \div 3 &= 12; \end{aligned}$$

i. e., dividing the divisor by 3 multiplies the quotient by 3.

ART. 75. Since multiplying the dividend multiplies the quotient (Art. 71), and multiplying the divisor divides the quotient (Art. 73), it follows, that multiplying both dividend and divisor by the same number does not alter the quotient.

ART. 76. Since dividing the dividend divides the quotient (Art. 72), and dividing the divisor multiplies the quotient (Art. 74), it follows, that if the dividend and divisor are both divided by the same number the quotient is not altered.

CANCELLATION.

ARTICLE 77 Ex. 1. What is the quotient of $12 \times 8 \times 5 \times 3$ divided by $8 \times 3 \times 2$?

OPERATION.

$$\frac{12 \times \cancel{8} \times 5 \times \cancel{3}}{\cancel{8} \times \cancel{3} \times 2} = \frac{12 \times 5}{2} = \frac{60}{2} = 30.$$

Analysis. Since the factors 8 and 3 are common both to the divisor and the dividend, we may strike out or *cancel* these two factors from each, since it is equivalent to dividing both

dividend and divisor by these factors (Art. 76); we then have 12×5 to be divided by 2, or 60 divided by 2, giving the quotient 30. *Ans.* 30.

2. Divide the product of 18, 10 and 5 by the product of 6, 5 and 2.

OPERATION.

$$\begin{array}{r} 3 \quad 5 \\ 18 \times 10 \times 5 \\ \hline 6 \times 5 \times 2 \end{array} = 15 \text{ Ans.}$$

Analysis. The factor 5, being common to both divisor and dividend, may be *canceled*. Since 6 in the divisor is a factor of 18 in the dividend, we may divide 18 by 6, *canceled* these two numbers, and substituting 3 in place of them; and since 2 in the divisor is a factor of 10 in the dividend, we may *cancel* 10 and 2, and put 5 in their place. The product of the remaining factors, 3 and 5, will be the quotient. *Ans.* 15.

The above operations are called CANCELLATION. Hence,

ART. 78. CANCELLATION is the method of shortening arithmetical operations by striking out equal factors from the divisor and the dividend.

From the *Analysis* we derive the following

RULE.

Cancel all the factors common to the dividend and divisor; then divide the product of the remaining factors of the dividend by the remaining factors of the divisor

NOTE 1.—When a factor is entirely canceled, it does not leave 0, but the quotient 1 takes its place; because any quantity divided by itself gives 1 for a quotient. Hence, for every factor entirely canceled, either in the dividend or in the divisor, the factor 1 will remain.

NOTE 2.—It should be remembered that *one* factor in the divisor can cancel *only one equal* factor in the dividend.

EXAMPLES.

3. Divide the product of 4, 3, 9 and 7 by the product of 10, 4, 15 and 7. *Ans.* $\frac{2}{15}$.

4. Divide the product of 63, 105, 20 and 17 by the product of 15, 30, 21 and 7. *Ans.* 34.

5. What is the quotient of $18 \times 36 \times 29$, divided by 27×24 ? *Ans.* 29.

6. Divide the product of 12, 32, 5 and 20 by the product of 16, 24 and 8. *Ans.* $12\frac{1}{2}$.

7. What is the quotient of $7 \times 42 \times 5 \times 72$, divided by $21 \times 14 \times 12$? *Ans.* 30.

8. What is the quotient of $10 \times 27 \times 9 \times 20 \times 19$, divided by $15 \times 18 \times 12 \times 15$? *Ans.* 19.

9. What is the quotient of 3, 5, 7 and 20, divided by $5 \times 6 \times 10$? *Ans.* 7.

10. Divide the product of 6, 5, 7, 8 and 66 by the product of 7, 8, 9 and 11. *Ans.* 20.

11. Divide the product of 8, 7, 12, 15 and 17 by the product of 3, 4, 5, 14 and 4. *Ans.* 51.

12. How many pieces of cloth, each containing 36 yards, at 15 cents a yard, would be required to pay for 6 boxes of tobacco, each weighing 72 pounds, at 30 cents a pound?

Analysis. The value of the tobacco will evidently be equal to $6 \times 72 \times 30$; and the value of one piece of cloth will equal 36×15 ; hence, dividing the

OPERATION.

$$\begin{array}{r} 2 \quad 2 \\ 6 \times 72 \times 30 \\ \hline 36 \times 15 \end{array} = 24 \text{ pieces}$$

whole value of the tobacco by the value of 1 piece of cloth, the quotient must be the number of pieces. *Ans.* 24 pieces.

13. How many pounds of sugar, at 16 cents a pound, would pay for 19 pounds of coffee, at 32 cents per pound?

Ans. 38 pounds.

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14. How many boxes of soda, each weighing 24 pounds, at 12 cents a pound, would be required to pay for 8 sacks of rice, each containing 84 pounds, at 9 cents per pound?

Ans. 21 boxes.

NOTE. Cancellation may be applied to Fractions, Proportion and various other Rules of Arithmetic. It teaches no new *principles*, but simply serves to shorten the *mechanical* operations in working examples.

PROPERTIES AND RELATIONS OF NUMBERS.

ARTICLE 79. One number is said to be a **DIVISOR** or **MEASURE** of another when the former is contained in the latter without a remainder. Thus, 6 is a *divisor* of 12; 5 is a *divisor* of 15.

Name a divisor of 21; of 30; of 42; of 45; of 72; of 84.

An **EVEN** number can be divided by 2 without a remainder; 2, 4, 6, etc. are *even*.

An **ODD** number *cannot* be divided by 2 without a remainder; thus 1, 3, 5, 7, etc. are *odd* numbers.

Give all the *even* numbers up to 40. Give all the *odd* numbers to 43.

ART. 80. All numbers are either *prime* or *composite*.

A **PRIME** number is one that can be divided only by itself and 1. Thus, 2, 3, 5, 7 and 11 are *prime* numbers.

A **COMPOSITE** number is one that can be divided by some whole number besides itself and 1. Thus, 12 is a *composite* number, because it can be divided by 2, 3, 4 and 6.

Mention all the *prime* numbers up to 43. Give all the *composite* numbers up to 50.

All *prime* numbers are *odd*, with the exception of the number 2, which is *even*.

ART. 81. A *prime* factor of a number is a prime number that will exactly divide it; thus, 3 is a *prime* factor of 15; 7 is a *prime* factor of 21.

Give a *prime* factor of 9; of 15; of 18; of 21; of 25; of 30; of 35.

ART. 82. To *resolve* a number into its *prime* factors is to find what prime numbers multiplied together will produce the given number. Thus, the *prime* factors of 30 are 2, 3 and 5, for each of these numbers is *prime*, and their product is 30

Ex. 1. Find the prime factors of 462.

Analysis. We first divide 462 by 2, the least *prime* factor above 1, which gives 231 for a quotient; we then divide 231 by 3, the next greater prime number, which gives 77 for a quotient; lastly, dividing 77 by 7, the next greater prime number, we obtain the quotient 11, which is itself a *prime* number. Hence, the prime factors of 462 are 2, 3, 7 and 11. Hence, we have the following

OPERATION.

$$\begin{array}{r} 2)462 \\ 3)231 \\ 7)77 \\ 11 \end{array}$$

RULE.

Divide the given number by the least prime number greater than 1 that will divide it without a remainder, and continue to divide each quotient in the same way until the quotient becomes a prime number. The last quotient and the several divisors will be all the prime factors.

EXAMPLES.

2. Give the prime factors of 126. *Ans.* 2, 3, 3 and 7.

3. What are the prime factors of 105? *Ans.* 3, 5 and 7.

4. What are the prime factors of 252?

Ans. 2, 2, 3, 3 and 7

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5. Resolve 210 into its prime factors. *Ans.* —.
6. Resolve 225 into its prime factors. *Ans.* —.
7. Find the prime factors of 675. *Ans.* 3, 3, 3, 5 and 5.
8. Give the prime factors of 693. *Ans.* 3, 3, 7 and 11.
9. Find the prime factors of 546. *Ans.* 2, 3, 7 and 13.
10. Give the prime factors of 855. *Ans.* 3, 3, 5 and 19.
11. What are the prime factors of 3906?
Ans. 2, 3, 3, 7 and 31.

GREATEST COMMON DIVISOR.

ART. 83. Any number that will divide two or more numbers without a remainder is called their **COMMON DIVISOR** or **COMMON MEASURE**. Thus, 5 is a *common measure* of 15 and 20; 4 is a *common measure* of 12 and 16.

Mention a common divisor of 16 and 24; of 20 and 30; of 24 and 36; of 32 and 40.

ART. 84. The *greatest* number that will divide two or more numbers without a remainder is their **GREATEST COMMON MEASURE**. Thus, 12 is the greatest common measure of 24 and 36.

Give the greatest common measure of 15 and 25; of 14 and 21; of 18 and 27; of 22 and 33; of 35 and 42; of 40 and 50; of 48 and 60.

ART. 85. When numbers are small a little inspection will show us what is their greatest common measure, but in the case of large numbers we must resort to other methods.

Ex. 1. What is the greatest common measure of 72 and 84?

Analysis. Resolving 72 and 84 into their *prime factors*, we have

$$\begin{aligned} 72 &= 2 \times 2 \times 2 \times 3 \times 3, \\ 84 &= 2 \times 2 \times 3 \times 7. \end{aligned}$$

The *common* factors are 2, 2 and 3; and their product, 12, is the greatest common measure. *Ans.* 12.

2. What is the greatest common measure of 144 and 168?

Analysis. Resolving 144 and 168 into their prime factors, we have

$$\begin{aligned} 144 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3, \\ 168 &= 2 \times 2 \times 2 \times 3 \times 7. \end{aligned}$$

The *common* factors are 2, 2, 2 and 3; and their product, 24, is the greatest common measure. *Ans.* 24.

It will be observed that in resolving 144 into factors we found *four* 2's, and in resolving 168 we found only *three* 2's; hence *one* of the 2's is not *common* to both numbers. In the same way, in 144 we found *two* 3's, and in 168 we found only *one* 3. Hence, only *one* 3 was *common* to both numbers.

From the above illustrations we derive the

RULE.

Resolve the numbers into their prime factors; then the product of all the prime factors COMMON to ALL the numbers will be their greatest common measure.

EXAMPLES.

Find the greatest common measure of the following:

- | | |
|------------------------------------|-------------------------------------|
| 3. Of 64 and 80. <i>Ans.</i> 16. | 8. Of 252 and 216. <i>Ans.</i> 36. |
| 4. Of 105 and 90. <i>Ans.</i> 15. | 9. Of 192 and 224. <i>Ans.</i> 32. |
| 5. Of 126 and 162. <i>Ans.</i> 18. | 10. Of 135 and 162. <i>Ans.</i> 27. |
| 6. Of 132 and 154. <i>Ans.</i> 22. | 11. Of 175 and 245. <i>Ans.</i> 35. |
| 7. Of 125 and 175. <i>Ans.</i> 25. | 12. Of 210 and 252. <i>Ans.</i> 42. |

The above method is sufficient for finding the greatest common measure of *all* numbers, but in the case of large numbers its application becomes tedious. We will give another method, which is more convenient.

Before giving the Rule we will state two general Principles on which it is founded:

Principle 1st. If a number measures two others, it likewise measures their SUM, and their DIFFERENCE. Thus, 5 measures

25 and 15. It also measures 40, the *sum* of 25 and 15; and it also measures 10, the *difference* of 25 and 15.

Principle 2d. If one number measures another, it measures the latter number repeated any number of times. For example, 8 measures 16; 8 also measures *twice* 16, or 32; or it measures 3 times 16, or 48, and so on.

Ex. 1. What is the greatest common measure of 170 and 102?

Analysis. We first divide 170 by 102, and obtain the *remainder* 68; we then divide 102 by 68, and we get the *remainder* 34; lastly, we divide 68 by 34, which leaves no *remainder*.

OPERATION.

$$\begin{array}{r}
 102 \overline{)170} 1 \\
 \underline{102} \\
 68 \overline{)102} 1 \\
 \underline{68} \\
 34 \overline{)68} 2 \\
 \underline{68} \\
 0
 \end{array}$$

It is evident that 34 is a *common measure* of 102 and 170; for $102 \div 34 = 3$; and $170 \div 34 = 5$. It only remains to show that it is their *greatest common measure*. Any number that measures 170 and 102 must likewise measure their *difference*, 68 (Principle 1st), and hence is a *common measure* of 170, 102 and 68. Again, whatever number measures 102 and 68 must likewise measure their *difference*, 34, and hence must be a *common measure* of 170, 102, 68 and 34. Therefore, the common measure of the original numbers, 170 and 102, must, at the same time, be a measure of 34. But the *greatest measure* of 34 is 34 itself. Hence 34 is the *greatest common measure* of 170 and 102.

Hence we derive the following

RULE.

Divide the greater number by the less; if there be a remainder, divide the less number by it, and continue to divide the last divisor by the last remainder until nothing remains. The last divisor will be the greatest common measure.

NOTE.—To find the greatest common measure of more than two numbers, first find the greatest common measure of any *two* of them; then find the greatest common measure of the number thus found, and a *third* number; and so on, through all the numbers.

EXAMPLES.

Find the greatest common measure of the following :

- | | |
|--------------------------|-------------------|
| 2 Of 147 and 210. | <i>Ans.</i> 21. . |
| 3 Of 154 and 198. | <i>Ans.</i> 22 |
| 4. Of 252 and 324. | <i>Ans.</i> 36. |
| 5. Of 245 and 525. | <i>Ans.</i> 35. |
| 6. Of 301 and 430. | <i>Ans.</i> 43. |
| 7. Of 392 and 672. | <i>Ans.</i> 56. |
| 8. Of 525 and 1175. | <i>Ans.</i> 25. |
| 9. Of 405 and 900. | <i>Ans.</i> 45. |
| 10. Of 252, 324 and 432. | <i>Ans.</i> 36. |
| 11. Of 378, 648 and 918. | <i>Ans.</i> 54. |

12. A drayman agrees to haul 450 bags of guano for one merchant, and 486 for another. What is the greatest number of bags he can haul at each load, so as to carry the same number each time ?

Ans. 18 bags.

13. A grocer has 136 quarts of strawberries, and 152 quarts of plums, which he wishes to put into boxes, each box to hold the same number of quarts, and the largest number possible. How many quarts may he put in each box ?

Ans. 8 quarts.

14. A miller has 2310 pounds of white corn-meal, and 2730 pounds of yellow corn-meal, which he wishes to put into bags, each bag to hold the same number of pounds, and this the greatest number possible. How many pounds must each bag hold ?

Ans. 210 pounds.

15. What is the greatest number of pears you could buy with 180 cents, or 225 cents, or 315 cents, so as to get the same number each time ?

Ans. 45 pears.

LEAST COMMON MULTIPLE.

ART. 86. A MULTIPLE of a number is a number that can be divided by it without a remainder; thus, 18 is a multiple of 6, because it can be exactly divided by 6.

Mention a multiple of 7; of 9; of 12; of 15; of 20, of 25.

ART. 87. A COMMON MULTIPLE of two or more numbers is any number that can be divided by each of them without a remainder. Thus, 12 is a common multiple of 2, 3, 4 and 6, because it can be divided by each of these numbers without a remainder.

Give a common multiple of 3 and 5; of 4 and 7; of 2, 3 and 4; of 3, 5 and 6; of 5 and 15; of 15 and 20; of 5, 6 and 10.

ART. 88. The LEAST COMMON MULTIPLE of two or more numbers is the least number that can be divided by each of them without a remainder. Thus, 20 is the least common multiple of 4, 5 and 10, because it is the *least* number that can be divided by each of them without a remainder.

Mention the least common multiple of 4 and 6; of 2, 3 and 8; of 5, 3 and 6; of 2, 3, 4 and 6; of 6, 9 and 12.

ART. 89. From the definition of a *multiple* above given, it is obvious that a multiple of a number must be a *product*, and this product must be divisible by *each* of the factors of the given number. Thus, 28 is a *multiple* of 14; it is divisible by the factors of 14. The factors of 14 are 7 and 2. Now, 7 and 2 will each divide 14 without a remainder; and so also will 7 and 2 divide 28, which is a multiple of 14.

ART. 90. Upon the same principle, the *least common multiple* of two or more numbers is the *least product* that can be formed by the *prime* factors that compose each number. Each prime factor must enter into this product *once*, and *only* once;

for if any prime factor did not enter into it, it could not be divisible by the number, and therefore could not be a *multiple* at all; and if it entered into it *more* than once, the resulting product might be a common multiple, but it could not be the *least* common multiple. Hence, *the least common multiple must contain all the prime factors that are common to all the numbers, or to any two of them, and such prime factors as each as are not common.*

Ex. 1. What is the least common multiple of 12, 20 and 30?

Analysis. Resolving each number into its prime factors, we have

$$\begin{aligned} 12 &= 2 \times 2 \times 3, \\ 20 &= 2 \times 2 \times 5, \\ 30 &= 2 \times 3 \times 5. \end{aligned}$$

We see that the factor 2 is *common* to *all* the numbers, and therefore it will be a factor in the least common multiple. The factor 3 is common to *two* of the numbers, and therefore must be a factor of the least common multiple. The factor 5 being common to *two*, must also be another factor; and the separate factor 2, which is common only to *two* of the numbers, is another factor of the least common multiple. Hence, our factors will stand thus: $2 \times 3 \times 5 \times 2 = 60$; which is the least common multiple of 12, 20 and 30. *Ans.* 60.

2. What is the least common multiple of 12, 18 and 24?

Analysis. Resolving each number into its prime factors, we have

$$\begin{aligned} 12 &= 2 \times 2 \times 3, \\ 18 &= 2 \times 3 \times 3, \\ 24 &= 2 \times 2 \times 3 \times 2. \end{aligned}$$

The first factor, 2, being *common* to all the numbers, will be a factor of the least common multiple. The second factor, 2, being common to *two* of the numbers, will be a factor also

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One of the 3's, being common to *all* the numbers, will also be a factor in the multiple. There is another factor, 3, which is *not* common, and therefore must be a factor also. There is also another 2, which is *not* common, and therefore will constitute another factor. Forming a product by using the factors designated, we have, $2 \times 2 \times 3 \times 3 \times 2 = 72$, which is the least common multiple. Ans. 72.

For the sake of convenience the following plan is adopted :

Let us take the first example given above. We arrange the numbers in a horizontal line and divide by 2, because 2 is a *prime* number, and because it is a factor of *all* the numbers, and we write the quotients, 6, 10 and 15, in the line underneath. We then divide the

OPERATION.

$$2 \overline{)12, 20, 30}$$

$$3 \overline{)6, 10, 15}$$

$$5 \overline{)2, 10, 5}$$

$$2 \overline{)2, 2, 1}$$

$$1, 1, 1$$

numbers in the line by 3, because 3 is a *prime* number, and because it is a factor of *two* of the numbers, and we write the quotients 2 and 5, together with the undivided number 10, in the line below. We then divide the numbers in this line by 5, another *prime* number, for the same reason as before ; and finally, we divide by 2, another prime number, for the same reason. The product of the four prime divisors, 2, 3, 5 and 2, gives 60 for the least common multiple, as before.

$$2 \times 3 \times 5 \times 2 = 60 \text{ Ans.}$$

What we have said may be summed up in the following

RULE.

I. Arrange the numbers in a horizontal line ; divide by any *prime* number that will divide two or more of them without a remainder, and write the quotients and the undivided numbers in a line below.

II. Then divide, and write the results as before, until there

is no prime number greater than 1 that will exactly divide any two of the numbers.

III. Multiply together the divisors and the numbers in the last line, and the product will be the least common multiple.

EXAMPLES.

Find the least common multiple of the following:

1. 15, 18 and 30. Ans. 90.
2. 12, 16 and 24. Ans. 48.
3. 7, 14, 21 and 42. Ans. 42.
4. 18, 27, 36 and 54. Ans. 108.
5. 10, 12, 15 and 20. Ans. 60.
6. 6, 9, 12 and 18. Ans. 36.
7. 4, 6, 8 and 12. Ans. 24.
8. 5, 8, 10, 20 and 40. Ans. 40.
9. 3, 5, 9, 15 and 30. Ans. 90.
10. 4, 6, 8, 12 and 24. Ans. 24.
11. 4, 6, 8, 12 and 16. Ans. 48.
12. 3, 7, 14, 21 and 28. Ans. 84.
3. 8, 12, 15, 20 and 30. Ans. 120
14. 6, 10, 15, 12 and 20. Ans. 60.

15. What is the least number of bushels of wheat that would make an exact number of full loads for three drays, hauling respectively 24, 30 or 36 bushels at a load?

Ans. 360 bushels.

16. What is the least number of dollars that would purchase an exact number of calves at 12 dollars; or cows at 40 dollars; or hogs at 16 dollars; or mules at 96 dollars?

Ans. 480 dollars.

17. What is the least number of cents with which you could buy an exact number of lemons at 6 cents apiece; or oranges at 8 cents; or bananas at 10 cents; or pineapples at 16 cents?

Ans. 240 cents.

FRACTIONS.

ARTICLE 91. When a melon is divided into *two* equal parts, each of the parts is called *one half*; when it is divided into *three* equal parts, each of the parts is called *one third*; when it is divided into *four* equal parts, one of the parts is called *one fourth*; two of the parts are called *two fourths*; three of the parts, *three fourths*, etc.

ART. 92. When a number or thing is divided into *five* equal parts, one of the parts is called *one fifth*; two of the parts are called *two fifths*; three of the parts are called *three fifths*, and so on.

ART. 93. The *value* of one of the equal parts will depend upon the *number* of parts into which the number or thing is divided. The *greater* the number of parts, the *less* will be each part. Thus, *one half* of a thing is greater than a *third* of it, and a *third* is greater than a *fourth*.

ART. 94. When a number or thing is divided into equal parts, these equal parts are called FRACTIONS.* Hence,

A FRACTION is an expression representing a part or parts of a unit, or whole. Thus, *one sixth* represents a *part* of a unit; *four sixths* represent *parts* of a unit.

ART. 95. FRACTIONS are divided into two classes—COMMON and DECIMAL.

COMMON FRACTIONS.

ART. 96. A *common* fraction is expressed by two numbers—one placed above the other, with a line between them.

* *Fraction*, from the Latin word, *fractus*, signifying *broken*, because the number is, as it were, *broken* into equal parts or pieces.

Thus—

One-half is written	$\frac{1}{2}$	One-seventh is written	$\frac{1}{7}$
One-third “	$\frac{1}{3}$	Four-fifths “	$\frac{4}{5}$
One-fourth “	$\frac{1}{4}$	Six-ninths “	$\frac{6}{9}$
One-fifth “	$\frac{1}{5}$	Seven-tenths “	$\frac{7}{10}$
One-sixth “	$\frac{1}{6}$	Five-eighths “	$\frac{5}{8}$
Two-thirds “	$\frac{2}{3}$	Nine-twelfths “	$\frac{9}{12}$
Three-fourths “	$\frac{3}{4}$	Ten-fifteenths “	$\frac{10}{15}$

The number *below* the line is called the **DENOMINATOR**; the number *above* the line is called the **NUMERATOR**.

ART. 97. The **DENOMINATOR*** shows into how many equal parts the number or thing is divided. In the fraction $\frac{1}{4}$ the unit is supposed to be divided into *four* equal parts; in $\frac{1}{8}$, it is divided into *eight* parts.

The **NUMERATOR**† shows how many of the equal parts are used. Thus, in the fraction $\frac{4}{7}$, *four* parts are used; in $\frac{5}{8}$, *five* parts are used.

ART. 98. The numerator and the denominator taken together are called the **TERMS** of the fraction. Thus, $\frac{5}{7}$ is a fraction—5 and 7 are its terms; 7 is the *denominator*, and shows that the unit is divided into *seven* equal parts, making each part *one seventh*; 5 is the *numerator*, and shows that *five* of the equal parts are used. It is read *five-sevenths*.

ART. 99. When we tell the names of the parts of a fraction and explain their meaning, we are said to *analyze* the fraction.

Ex. 1. Analyze the fraction $\frac{5}{8}$.

Analysis. 5 and 8 are the *terms*; 8 is the *denominator*, and

* *Denominator*, from the Latin word, *denomino*, signifying *to name*, because it gives *name* or *denomination* to the fraction.

† *Numerator*, from the Latin, *numero*, signifying *to number*, because it *numbers* the parts, or tells *how many* there are.

shows that the unit is divided into *eight* equal parts, making each part *one-eighth*; 5 is the *numerator*, and shows that *five* of the equal parts are used. It is read *five-eighths*.

Let the pupil analyze the following fractions, viz.:

2.	$\frac{2}{3}$	7.	$\frac{4}{5}$	12.	$\frac{4}{12}$	17.	$\frac{14}{17}$
3.	$\frac{3}{4}$	8.	$\frac{7}{8}$	13.	$\frac{4}{13}$	18.	$\frac{12}{18}$
4.	$\frac{5}{8}$	9.	$\frac{5}{10}$	14.	$\frac{12}{14}$	19.	$\frac{7}{19}$
5.	$\frac{3}{8}$	10.	$\frac{4}{7}$	15.	$\frac{7}{15}$	20.	$\frac{14}{20}$
6.	$\frac{7}{7}$	11.	$\frac{2}{11}$	16.	$\frac{9}{20}$	21.	$\frac{27}{21}$

ART. 100. The *value* of a fraction is the *quotient* of the numerator divided by the denominator. The value of $\frac{4}{2}$ is 2; of $\frac{12}{4}$ is 3; of $\frac{20}{4}$ is 5. From these examples we see that every fraction indicates *division*; the numerator being the *dividend*, the denominator being the *divisor*, and the value of the fraction being the *quotient*. All the rules of division are therefore applicable to fractions.

When the numerator is *equal* to the denominator, the value of the fraction is a unit, as $\frac{4}{4}$, $\frac{7}{7}$, $\frac{8}{8}$, etc.

When the numerator is *less* than the denominator, the value of the fraction is *less* than a unit, as $\frac{5}{8}$, $\frac{7}{9}$, etc.

When the numerator is *greater* than the denominator, the value of the fraction is *greater* than a unit, as $\frac{4}{3}$, $\frac{7}{5}$, etc.

ART. 101. A **PROPER** fraction is one whose numerator is *less* than its denominator, as $\frac{3}{8}$, $\frac{4}{5}$, etc.

An **IMPROPER** fraction is one whose numerator is *equal to* or *greater* than its denominator, as $\frac{5}{5}$, $\frac{7}{7}$, etc.

A **MIXED NUMBER** consists of a whole number and a fraction, as $2\frac{1}{2}$, $4\frac{1}{3}$, $5\frac{2}{3}$, etc.

ART. 102. Any whole number may be expressed as a fraction by writing 1 below it for a denominator; thus 4 may be written $\frac{4}{1}$; 6 may be written $\frac{6}{1}$.

ART. 103. A **COMPOUND** fraction is a fraction of a fraction as $\frac{2}{3}$ of $\frac{4}{5}$; $\frac{4}{5}$ of $\frac{7}{8}$.

A **COMPLEX** fraction is one which has a fraction either in its numerator or in its denominator, or in both; as $\frac{2\frac{1}{2}}{5}$; $\frac{3}{4\frac{1}{2}}$; $\frac{2\frac{3}{4}}{3\frac{1}{2}}$, etc.

QUESTIONS TO ILLUSTRATE FRACTIONS.

Ex. 1. How many *halves* in 2?

Analysis. In 1 unit there are 2 *halves*, and in 2 units there are 2 times as many *halves*, which will be 4 *halves*; or, expressed thus, $\frac{4}{2}$.

Ans. $\frac{4}{2}$.

2. How many *thirds* in 3?

Analysis. In 1 unit there are 3 *thirds*, and in 3 units there are 3 times as many *thirds*, which will be 9 *thirds*; or, expressed thus, $\frac{9}{3}$.

Ans. $\frac{9}{3}$.

3. How many *fourths* in 3?

Ans. $\frac{12}{4}$.

4. How many *fifths* in 4?

Ans. —.

5. How many *sixths* in 5?

Ans. —.

6. How many *tenths* in 6?

Ans. —.

7. How many *ninths* in 5?

Ans. —.

8. Change 3 whole ones into *fifths*.

Ans. $\frac{15}{5}$.

9. Change 5 whole ones into *sevenths*.

Ans. —.

10. Change 4 units into *ninths*.

Ans. —.

11. How many *thirds* in $4\frac{2}{3}$?

Analysis. In 1 unit there are 3 *thirds*, and in 4 units there are 4 times as many *thirds*, which make 12 *thirds*, and 2 more *thirds* make 14 *thirds*; or, expressed thus, $\frac{14}{3}$.

Ans. $\frac{14}{3}$.

12. How many *fourths* in $5\frac{3}{4}$?

Analysis. In 1 unit there are 4 *fourths*, and in 5 units there are 5 times as many *fourths*, which make 20 *fourths*, and 3 more *fourths* make 23 *fourths*; or, written thus, $\frac{23}{4}$.

Ans. $\frac{23}{4}$.

13. How many *halves* in $7\frac{1}{2}$?

Ans. $\frac{15}{2}$.

14. How many *thirds* in $5\frac{1}{3}$?

Ans. —

15. How many fourths in $4\frac{1}{4}$? *Ans.* —.

16. How many fifths in $3\frac{3}{5}$? *Ans.* —.

17. How many sevenths in $4\frac{3}{7}$? *Ans.* —.

18. How many sixths in $5\frac{2}{3}$? *Ans.* —.

19. How many ninths in $3\frac{4}{9}$? *Ans.* —.

20. How many tenths in $3\frac{5}{10}$? *Ans.* —.

21. If $\frac{1}{3}$ of a number is 9, what is the number?

Analysis. If one third of the number is 9, then three thirds of the number, or the entire number, will be 3 times 9, which will be 27. *Ans.* 27.

22. If $\frac{1}{4}$ of a pound of candy cost 8 cents, what would be the cost of a whole pound?

Analysis. If one fourth of a pound cost 8 cents, then four fourths, or a whole pound, would cost 4 times 8 cents, which will be 32 cents. *Ans.* 32 cents.

23. Mary has a piece of ribbon, and $\frac{1}{3}$ of it measures 6 inches; how long is the whole piece? *Ans.* 18 inches.

24. If half an ounce of mace cost 10 cents, what would a whole ounce cost? *Ans.* —.

25. A man paid 9 dollars for $\frac{1}{4}$ of an acre of land; what was the price of a whole acre? *Ans.* —.

26. One sixth of a man's life is 12 years; how old is he? *Ans.* —.

27. Jane has a basket of roses, and $\frac{1}{6}$ of the number of roses is 6; how many has she? *Ans.* —.

28. One ninth of the number of acres in a field is 15; how many acres in the field? *Ans.* —.

29. William has a bag of chestnuts, and $\frac{1}{8}$ of the number is 13; how many chestnuts has he? *Ans.* —.

30. If $\frac{1}{10}$ of a firkin of butter contain 8 pounds, how many pounds in the whole firkin? *Ans.* —.

31. A man paid 18 dollars for one fifth of a bale of cotton; what was a whole bale worth?

32. If a pound of figs cost 28 cents, how much would $\frac{1}{4}$ of a pound cost?

Analysis. If 4 *fourths*, or a whole pound, cost 28 cents, then $\frac{1}{4}$ of a pound would cost *one fourth* of 28 cents, which will be 7 cents.

Ans. 7 cents.

33. If a box contains 48 oranges, how many oranges in $\frac{1}{6}$ of the box?

Analysis. If 6 *sixths* of the box, or the whole box, contain 48 oranges, then $\frac{1}{6}$ of the box will contain *one sixth* of 48 oranges, which will be 8 oranges.

Ans. 8 oranges.

34. If a yard of muslin cost 35 cents, what would be the cost of $\frac{1}{5}$ of a yard?

Ans. —.

35. If a bushel of wheat weigh 60 pounds, what would be the weight of $\frac{1}{10}$ of a bushel?

Ans. —.

36. Thomas has a fishing-line which measures 32 feet; how long is $\frac{1}{8}$ of the line?

Ans. —.

37. If a lot of land is worth 216 dollars, what is the value of $\frac{1}{3}$ of the lot?

Ans. —.

38. There are 60 minutes in an hour; how many minutes in $\frac{1}{2}$ of an hour?

Ans. —.

39. If a man travels 252 miles in a week, how far could he travel in $\frac{1}{4}$ of a week?

Ans. —.

40. If $\frac{1}{5}$ of a number is 8, how much would $\frac{3}{5}$ of it be?

Analysis. If *one fifth* of the number is 8, then *three fifths* of the number would be 3 times 8, which will be 24.

Ans. 24.

How many *times* more is $\frac{3}{5}$ than $\frac{1}{5}$? *Ans.* 3 times.

41. If $\frac{1}{7}$ of an acre of land costs 20 dollars, what would be the cost of $\frac{6}{7}$ of an acre?

Analysis. If *one seventh* of an acre costs 20 dollars, then *six sevenths* would cost 6 times as much, which will be 120 dollars.

Ans. 120 dollars.

How many *times* more is $\frac{6}{7}$ than $\frac{1}{7}$? *Ans.* —

42. If $\frac{1}{3}$ of a number is 7, how much is $\frac{2}{3}$ of it?

Ans. —

43. If $\frac{1}{3}$ of a farm contains 48 acres, how many acres would $\frac{2}{3}$ of the farm contain?

Ans. —.

44. If $\frac{1}{10}$ of a grist-mill be worth 475 dollars, how much would $\frac{7}{10}$ of it be worth?

Ans. —.

45. If $\frac{1}{3}$ of a factory be worth 7253 dollars, what would be the value of $\frac{2}{3}$ at the same rate?

Ans. —.

46. If $\frac{1}{2}$ of a pound of pepper cost 20 cents, what would $\frac{1}{4}$ of a pound cost?

Analysis. If 4 fifths of a pound cost 20 cents, then *one fifth* of a pound would cost *one fourth* of 20 cents, which will be 5 cents.

Ans. 5 cents.

47. If $\frac{7}{8}$ of a yard of muslin cost 56 cents, what would be the cost of $\frac{1}{8}$ of a yard?

Analysis. If 7 *eighths* of a yard cost 56 cents, then *one eighth* would cost *one seventh* of 56 cents, which will be 8 cents.

Ans. 8 cents.

48. If $\frac{3}{4}$ of a pound of butter sell for 36 cents, how much would $\frac{1}{4}$ of a pound sell for?

Ans. 12 cents.

49. If $\frac{3}{4}$ of a clothes-line measures 45 feet, what is the length of $\frac{1}{4}$ of the line?

Ans. —.

50. In $\frac{7}{10}$ of a dollar there are 70 cents; how many cents in $\frac{1}{10}$ of a dollar?

Ans. —.

51. In $\frac{5}{8}$ of an hour there are 50 minutes; how many minutes in $\frac{1}{8}$ of an hour?

Ans. —.

52. If $\frac{7}{8}$ of a piece of silk contain 35 yards, how many yards in $\frac{1}{8}$ of the piece?

Ans. —.

53. If $\frac{3}{12}$ of a lot of land sell for 72 dollars, how much would $\frac{1}{12}$ of the lot sell for at the same rate?

Ans. —.

54. If $\frac{9}{11}$ of a piece of calico measure 45 yards, how many yards in $\frac{1}{11}$ of the piece?

Ans. —

55. If $\frac{3}{18}$ of a piece of broadcloth is worth 42 dollars, what would be the value of $\frac{1}{18}$ of the piece? *Ans. —.*

56. If $\frac{7}{15}$ of a ton of iron cost 35 dollars, what would be the cost of $\frac{1}{15}$ of a ton? *Ans. —.*

57. If $\frac{3}{4}$ of a pound of coffee cost 15 cents, what would a whole pound cost?

Analysis. If *three* sevenths of a pound cost 15 cents, then *one* seventh of a pound would cost *one third* of 15 cents, which will be 5 cents; and if *one* seventh costs 5 cents, then *seven sevenths*, or a whole pound, would cost 7 times as many cents, which will be 35 cents. *Ans. 35 cents.*

58. If $\frac{5}{9}$ of a man's age is 35 years, how old is he?

Analysis. If *five* ninths of his age is 35 years, then *one* ninth of his age would be *one fifth* of 35 years, which will be 7 years; and if *one* ninth of his age is 7 years, then *nine* ninths, or his whole age, would be 9 times as many years which will be 63 years. *Ans. 63 years.*

59. If $\frac{3}{4}$ of a number is 48, what is the number?

Ans. 64.

60. If $\frac{4}{9}$ of a bushel of potatoes be worth 72 cents, what would be the value of a bushel? *Ans. 90 cents.*

61. If $\frac{7}{8}$ of a man's age is 42 years, what is his age?

Ans. 48 years.

62. If $\frac{7}{10}$ of a certain number is 105, what is the number?

Ans. 150.

63. If $\frac{5}{12}$ of a farm contain 115 acres, how many acres in the whole farm? *Ans. 276 acres.*

64. A gentleman sold three eighths of a lot of land for 972 dollars; what was the value of the whole lot?

Ans. 2592 dollars.

65. Three fifteenths of a factory was valued at 4275 dollars; what was the value of the whole factory?

Ans. 21375 dollars.

66. Four ninths of a gentleman's estate was valued at \$708 dollars. How much was his estate worth?

Ans. 12843 dollars.

67. In $\frac{1}{3}$, how many *ninths* are there?

Analysis. In 1 unit there are 9 *ninths*, and in $\frac{1}{3}$ of a unit there will be *one third* as many *ninths*, which will be 3 *ninths*, or expressed thus, $\frac{3}{3}$. Therefore, $\frac{1}{3} = \frac{3}{3}$.

68. In $\frac{1}{4}$, how many *sixteenths*?

Analysis. In 1 unit there are 16 *sixteenths*, and in $\frac{1}{4}$ of a unit there will be *one fourth* as many *sixteenths*, which will be 4 *sixteenths*, or, written thus, $\frac{4}{16}$. Therefore, $\frac{1}{4} = \frac{4}{16}$.

69. In $\frac{1}{8}$, how many *twelfths*? *Ans.* $\frac{4}{12}$.

70. In $\frac{1}{5}$, how many *fifteenths*? *Ans.* —.

71. In $\frac{1}{4}$, how many *twentieths*? *Ans.* —.

72. In $\frac{1}{8}$, how many *thirtieths*? *Ans.* —.

73. In $\frac{1}{7}$, how many *thirty-fifths*? *Ans.* —.

74. In $\frac{1}{4}$, how many *twenty-fourths*? *Ans.* —.

75. In $\frac{3}{8}$, how many *tenths*?

Analysis. 1 unit is equal to 10 *tenths*; then $\frac{1}{5}$ of a unit will be equal to *one fifth* as many *tenths*, which will be 2 *tenths*, and $\frac{3}{5}$ will be equal to 3 *times* as many *tenths*, which will be 6 *tenths*, or, written thus, $\frac{6}{10}$.

Ans. $\frac{6}{10}$.

76. Change $\frac{5}{8}$ to *thirtieths*.

Analysis. 1 unit is equal to 30 *thirtieths*, and $\frac{1}{6}$ of a unit will be equal to *one sixth* as many *thirtieths*, which will be 5 *thirtieths*, and $\frac{5}{6}$ will be 5 *times* as many *thirtieths*, which will be 25 *thirtieths*, or, written thus, $\frac{25}{30}$.

Ans. $\frac{25}{30}$.

77. Change $\frac{4}{5}$ to *fifteenths*. *Ans.* $\frac{12}{15}$.

78. Change $\frac{3}{4}$ to *fourteenths*. *Ans.* —.

79. Change $\frac{5}{7}$ to *thirty-fifths*. *Ans.* $\frac{25}{35}$.

80. Change $\frac{3}{4}$ to *twentieths*. *Ans.* —.

81. Change $\frac{7}{8}$ to *forty-fifths*. *Ans.* $\frac{35}{40}$.

82. Change $\frac{9}{10}$ to *forty-fifths*. *Ans.* —.

83. Change $\frac{7}{12}$ to thirty-sixths.

Ans. $\frac{11}{12}$.

84. If 5 apples be equally divided among 7 boys, what part of an apple would be the share of each boy?

Analysis. If only 1 apple were divided among 7 boys, each boy would receive $\frac{1}{7}$ of an apple, and if *five* apples were divided, each boy would receive 5 *times* $\frac{1}{7}$ of an apple, which would be $\frac{5}{7}$ of an apple.

Ans. $\frac{5}{7}$ of an apple.

85. If 6 cantaloupes be equally divided among 10 men, what part of a cantaloupe would be the share of each?

Analysis. If only 1 cantaloupe were divided among 10 men, each man would receive $\frac{1}{10}$ of a cantaloupe, and if 6 cantaloupes were divided, each man would receive 6 *times* $\frac{1}{10}$ of a cantaloupe, which would be $\frac{6}{10}$ of a cantaloupe.

Ans. $\frac{6}{10}$ of a cantaloupe.

86. If 4 oranges be equally divided among 6 girls, how much of an orange would each girl receive?

Ans. $\frac{2}{3}$ of an orange.

87. William divided 4 cakes equally among 5 of his playmates; what part of a cake did each one get?

Ans. $\frac{4}{5}$ of a cake.

88. A gentleman distributed 5 dollars equally among 6 poor persons; what was the share of each?

Ans. $\frac{5}{6}$ of a dollar.

89. Nine apples were equally divided among 5 boys; what was the share of each boy?

Ans. $\frac{9}{5}$ of an apple.

90. A society distributed 6 cords of wood among 4 poor families; how much did each family get?

Ans. $\frac{3}{2}$ of a cord.

91. In $\frac{1}{4}$ of an orange, how many *whole* oranges and *parts* of an orange?

Analysis. Since 4 *fourths* make 1 *whole* orange, then in 11 *fourths* there will be as many whole oranges as 4 is contained times in 11, which will be 2 times and 3 *fourths* over. Hence, there will be 2 oranges and $\frac{3}{4}$ of an orange.

92. In $\frac{14}{3}$, how many *units* and *parts* of a unit?

Analysis. Since 3 *thirds* make 1 unit, then in 14 *thirds* there will be as many units as 3 is contained *times* in 14, which will be 4 *times* and 2 *thirds* over. Ans. $4\frac{2}{3}$.

93. In $\frac{11}{2}$, how many units and parts of a unit? Ans. $5\frac{1}{2}$.

94. In $\frac{9}{2}$, how many units, etc.? Ans. $3\frac{1}{2}$.

95. In $\frac{12}{8}$, how many units, etc.? Ans. —.

96. In $\frac{20}{8}$, how many units, etc.? Ans. —

97. In $\frac{24}{9}$ of a dollar, how many whole dollars, etc.? Ans. —.

98. In $\frac{22}{6}$ of a yard, how many yards, etc.? Ans. —.

99. William had $\frac{7}{8}$ of a dollar; his father gave him $\frac{6}{8}$, and his mother $\frac{5}{8}$; how much did he then have?

Analysis. 7 *eighths* and 6 *eighths* are 13 *eighths*, and 5 *eighths* are 18 *eighths*; or, written thus, $\frac{13}{8} = 2\frac{5}{8}$.

Ans. $2\frac{5}{8}$ dollars.

100. Thomas had $\frac{5}{8}$ of an orange; his cousin gave him $\frac{4}{8}$ of another, and his sister gave him $\frac{3}{8}$. How many oranges did he then have? Ans. 2 oranges.

101. James had $\frac{9}{10}$ of a dollar; his mother gave him $\frac{6}{10}$; he made $\frac{7}{10}$ by selling papers, and $\frac{3}{10}$ by running on errands. How much did he then have? Ans. $2\frac{7}{10}$ dollars.

102. Robert had $\frac{8}{9}$ of a melon; he bought $\frac{7}{9}$ of another melon, his brother gave him $\frac{6}{9}$, and his cousin $\frac{5}{9}$. How many melons did he then have? Ans. $3\frac{1}{9}$ melons.

103. Richard had $\frac{1}{2}$ of an orange, and his brother gave him $\frac{1}{4}$ of an orange. What part of an orange did he then have?

Analysis. One *half* is equal to 2 *fourths*. Then, 2 *fourths* and 1 *fourth* make 3 *fourths*, or thus, $\frac{3}{4}$.

Ans. $\frac{3}{4}$ of an orange.

104. Jane had $\frac{1}{4}$ of a pound of candy, and she bought $\frac{1}{8}$ of another pound. What part of a pound did she then have?

Analysis. One *fourth* is equal to 3 *twelfths*, and $\frac{1}{8}$ is equal

to 4 *twelfths*. Then, 3 *twelfths* and 4 *twelfths* make 7 *twelfths*; or, expressed thus, $\frac{7}{12}$. *Ans.* $\frac{7}{12}$ of a pound.

105. Susan had $\frac{1}{2}$ of a cantaloupe, and her classmate gave her $\frac{1}{3}$ of another. What part of a cantaloupe did she have?

Ans. $\frac{5}{6}$ of a cantaloupe.

106. A man bought $\frac{1}{4}$ of a ton of coal at one time and $\frac{1}{8}$ a another time; how much did he buy? *Ans.* $\frac{3}{8}$ of a ton.

107. Grace had $\frac{1}{3}$ of a pineapple, and her brother gave her $\frac{1}{6}$ of another; how much had she? *Ans.* $\frac{1}{2}$ of a pineapple.

108. George had $\frac{1}{2}$ of a dollar, and earned $\frac{1}{4}$ of a dollar more; how much did he have? *Ans.* $\frac{3}{4}$ of a dollar.

109. Richard had a melon, and gave $\frac{1}{5}$ of it to his little brother; how much did he keep for himself?

Analysis. The *whole* melon is equal to 5 *fifths*, and as he gave away 1 *fifth*, he had 4 *fifths* left; or, written thus, $\frac{4}{5}$.

Ans. $\frac{4}{5}$ of a melon.

110. Katie had a dollar, and spent $\frac{3}{4}$ of it for a doll; how much of a dollar had she left? *Ans.* —.

111. Henry had an orange, and gave away $\frac{3}{4}$ of it; how much did he keep for himself? *Ans.* —.

112. A merchant bought a firkin of butter, and sold $\frac{3}{4}$ of it; what part remained unsold? *Ans.* —.

113. Two men jointly owned a factory. One man owned $\frac{7}{8}$; how much did the other own? *Ans.* —.

114. If a man can do a job of work in 7 days, what part of it could he do in 5 days?

Analysis. If he can do the whole job in 7 days, then in 1 day he can do *one seventh* of it, and in 5 days he could do 5 times as much, which will be 5 *sevenths*; or, written thus, $\frac{5}{7}$.

Ans. $\frac{5}{7}$ of the job.

115. If 9 men can reap a field of wheat in a day, what part of the field could 7 men reap in the same time?

Ans. —.

116. If a barrel of flour last a family 10 weeks, what part of a barrel would last them 8 weeks? *Ans.* —.

117. If 12 yards of muslin make a dress for a lady, what part of a dress would 9 yards make? *Ans.* —.

118. If 11 persons divide a sum of money equally, what part of it would be the share of 5 persons? *Ans.* —.

119. If 15 cars can carry a regiment of soldiers, what part of the regiment could be carried in 10 cars? *Ans.* —.

120. If a yard of cloth cost $\frac{3}{4}$ of a dollar, how much would 5 yards cost?

Analysis. If 1 yard cost 3 *fourths* of a dollar, then 5 yards would cost 5 *times* 3 *fourths*, which will be 15 *fourths*; or, expressed thus, $1\frac{5}{4} = 3\frac{3}{4}$. *Ans.* $3\frac{3}{4}$ dollars.

121. If an apple cost $\frac{2}{10}$ of a cent, how much would be the cost of 7 apples? *Ans.* $6\frac{4}{10}$ cents.

122. If a pineapple cost $\frac{2}{3}$ of a dollar, how much would 9 pineapples cost? *Ans.* $3\frac{2}{3}$ dollars.

123. If a bushel of peas cost $\frac{7}{8}$ of a dollar, what would be the cost of 5 bushels? *Ans.* —.

124. A merchant bought 8 bags of apples, each bag holding $1\frac{1}{2}$ of a bushel; how many bushels did he buy? *Ans.* —.

125. From a bolt of ribbon a merchant cut 9 strips, each strip measuring $1\frac{5}{8}$ of a yard. How many yards did he cut off? *Ans.* $8\frac{7}{8}$ yards.

126. A market-man sold 11 baskets of strawberries, each basket holding $\frac{4}{5}$ of a gallon; how many gallons did he sell? *Ans.* —.

127. At $\frac{2}{3}$ of a dollar per gallon, what would be the cost of 9 gallons of oil? *Ans.* —.

128. At $\frac{7}{8}$ of a dollar per bushel, how much would 10 bushels of corn cost? *Ans.* —.

129. What would be the cost of $\frac{1}{2}$ peaches at $1\frac{2}{3}$ of a cent apiece? *Ans.* —.

130. If 2 pounds of raisins cost $\frac{1}{2}$ of a dollar, what will be the cost of one pound?

Analysis. If 2 pounds cost 4 *eighths* of a dollar, then 1 pound will cost *one half* of 4 *eighths*, which will be 2 *eighths*, or $\frac{1}{4}$.

Ans. $\frac{1}{4}$ of a dollar.

131. If 4 yards of muslin cost $\frac{1}{2}$ of a dollar, how much will 1 yard cost?

Analysis. If 4 yards cost 12 *sixteenths* of a dollar, then 1 yard will cost *one fourth* of 12 *sixteenths*, which will be 3 *sixteenths*, or $\frac{1}{8}$.

Ans. $\frac{1}{8}$ of a dollar.

132. If 3 bags hold $\frac{2}{3}$ of a barrel of apples, how many apples will one bag hold?

Ans. —.

133. If 5 oranges cost $\frac{1}{3}$ of a dollar, what will be the cost of one orange?

Ans. —.

134. If 7 bananas sell for $\frac{2}{3}$ of a dollar, what would be the price of one banana?

Ans. —.

135. How much is $\frac{1}{2}$ of $\frac{1}{3}$?

Analysis. $\frac{1}{3}$ is equal to 2 *sixths*, and *one half* of 2 *sixths* is *one sixth*, or thus, $\frac{1}{6}$. Hence, $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$.

Ans. $\frac{1}{6}$.

136. How much is $\frac{1}{3}$ of $\frac{1}{4}$?

Analysis. $\frac{1}{4}$ is equal to 3 *twelfths*, and $\frac{1}{3}$ of 3 *twelfths* is *one twelfth*, or thus, $\frac{1}{12}$. Hence, $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$.

Ans. $\frac{1}{12}$.

137. How much is $\frac{2}{3}$ of $\frac{3}{4}$?

Analysis. $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$; then $\frac{2}{3}$ of $\frac{1}{4}$ is *twice* as much, which will be $\frac{2}{12}$, and $\frac{2}{3}$ of $\frac{3}{4}$ will be 3 times $\frac{2}{12}$, which will be $\frac{6}{12}$.

Ans. $\frac{6}{12}$.

138. How much is $\frac{1}{2}$ of $\frac{1}{4}$?

Ans. —.

139. How much is $\frac{1}{2}$ of $\frac{3}{4}$?

Ans. —.

140. How much is $\frac{1}{3}$ of $\frac{1}{3}$?

Ans. —.

141. How much is $\frac{1}{3}$ of $\frac{2}{3}$?

Ans. —.

142. How much is $\frac{2}{3}$ of $\frac{1}{3}$?

Ans. —.

143. How much is $\frac{2}{3}$ of $\frac{2}{3}$?

Ans. —.

144. How much is $\frac{2}{3}$ of $\frac{3}{4}$?

Ans. —.

145. How much is $\frac{3}{4}$ of $\frac{5}{7}$?

Ans. —.

146. At $\frac{3}{4}$ of a dollar per peck, how many pecks of apples could be bought for 4 dollars?

Analysis. One dollar is equal to 3 *thirds* of a dollar, and 4 dollars are equal to 12 *thirds*. Then, if 2 *thirds* of a dollar buy one peck, 12 *thirds* will buy as many pecks as 2 *thirds* are contained times in 12 *thirds*, which will be 6 times.

Ans. 6 pecks.

147. If a sack hold $\frac{7}{8}$ of a bushel of peas, how many sacks would be required to hold 7 bushels?

Analysis. One bushel is equal to 8 *eighths* of a bushel, and 7 bushels will be equal to 7 times 8 *eighths*, which will be 56 *eighths*. Then, if 7 *eighths* fill one sack, 56 *eighths* will fill as many sacks as 7 *eighths* are contained times in 56 *eighths*, which will be 8 times.

Ans. 8 sacks.

148. At $\frac{3}{4}$ of a dollar per bushel, how many bushels of potatoes could be bought for 6 dollars?

Ans. 8 bushels.

149. A merchant cut 10 yards of ribbon into pieces, each $\frac{5}{8}$ of a yard in length. How many pieces were there?

Ans. 12 pieces.

150. At $\frac{3}{4}$ of a cent apiece, how many peaches could you buy for 12 cents?

Ans. 20 peaches.

151. If you pay $\frac{3}{4}$ of a dollar for a yard of flannel, how many yards could you buy for $5\frac{1}{4}$ dollars?

Ans. 7 yards.

152. If $\frac{5}{8}$ of a yard of linen make an apron, how many aprons could be made out of $7\frac{1}{2}$ yards?

Ans. 12 aprons.

ART. 104. If we take the fraction $\frac{2}{7}$ and multiply the numerator by 3, we shall have $\frac{6}{7}$. Now, 6 *sevenths* is evidently *three* times as much as 2 *sevenths*. Hence we derive the following

PRINCIPLE I. *Multiplying the numerator of a fraction by any number MULTIPLIES the value of the fraction by that number.*

Again. If we divide the numerator of $\frac{2}{3}$ by 3, we have $\frac{2}{9}$. Now, 2 *sevenths* is evidently one third of 6 *sevenths*. Hence we derive the following

PRINCIPLE II. *Dividing the numerator of a fraction by any number DIVIDES the value of the fraction by that number.*

Again. If we multiply the denominator of $\frac{2}{3}$ by 3, we have $\frac{2}{9}$. But 2 *ninths* is only one third as much as 2 *thirds*, because it requires 3 *ninths* to make 1 *third*. Hence,

PRINCIPLE III. *Multiplying the denominator of a fraction by any number DIVIDES the value of the fraction by that number.*

Again. If we divide the denominator of $\frac{2}{3}$ by 3, we have $\frac{2}{9}$. But 2 *thirds* is 3 times as much as 2 *ninths*, since it requires 3 *ninths* to make 1 *third*. Hence,

PRINCIPLE IV. *Dividing the denominator of a fraction by any number MULTIPLIES the value of the fraction by that number.*

ART. 105. According to the Principles above stated, it is evident that MULTIPLYING the *numerator* has the same effect as DIVIDING the *denominator*; and DIVIDING the *numerator* has the same effect as MULTIPLYING the *denominator*.

ART. 106. The fraction $\frac{2}{1}$ is evidently equal to 2. Now, if we multiply both numerator and denominator of $\frac{2}{1}$ by 3, we shall have $\frac{6}{3}$, which is equal to 2.

Again, if we divide both numerator and denominator of $\frac{2}{1}$ by 2, we shall have $\frac{1}{1}$, which is also equal to 2. Therefore, the *value* of the fraction remains unchanged. Hence,

PRINCIPLE V. *If the numerator and the denominator be both multiplied or both divided by the same number, the value of the fraction will not be altered.*

Observation I. When we multiply the *numerator* of a fraction by any number, we increase the *number* of parts that are used; and when we multiply the *denominator* of the

fraction we *decrease* the *size* of the parts. Hence, as we *increase* the *number* of parts precisely as much as we *decrease* the *size* of the parts, the *value* of the fraction is not altered.

Observation II. When we divide the *numerator* we *decrease* the *number* of parts that are used; when we divide the *denominator* we *increase* the *size* of the parts. Hence, as we *decrease* the *number* of parts precisely as much as we *increase* the *size* of the parts, the *value* of the fraction remains unaltered.

REDUCTION OF FRACTIONS.

ART. 107. REDUCTION of fractions consists in changing their *form* without altering their *value*.

CASE I.

ART. 108. *To reduce a fraction to its lowest terms.*

A fraction is in its *lowest* terms when no number greater than 1 will divide its numerator and denominator without a remainder.

Ex. 1. Reduce $\frac{1}{3}\frac{2}{5}$ to its lowest terms.

Analysis. Since dividing both numerator and denominator of a fraction by the same number does not alter its value (Art. 106), we divide both terms of $\frac{1}{3}\frac{2}{5}$ by 2, and get $\frac{1}{3}\frac{1}{5}$; dividing $\frac{1}{3}$ by 3, we obtain $\frac{1}{9}$, whose terms are the lowest to which the given fraction can be reduced.

OPERATION.

$$2)\frac{1}{3}\frac{2}{5} = \frac{1}{3}\frac{1}{5}$$

$$3)\frac{1}{3}\frac{1}{5} = \frac{1}{9}\frac{1}{5}$$

Or we may simplify the operation thus: If we divide both terms of the fraction by 6, their *greatest common divisor*, the fraction will at once be reduced to its lowest terms.

OPERATION

$$6)\frac{1}{3}\frac{2}{5} = \frac{1}{9}\frac{1}{5}$$

From the foregoing explanations we obtain the following

RULE.

Divide both numerator and denominator by any number that will divide them without a remainder, and continue the oper-

ation until there is no number greater than 1 that will divide them exactly.

Or,

Divide them at once by their greatest common divisor.

EXAMPLES.

Reduce the following fractions to their lowest terms :

2. $\frac{12}{16}$.	Ans. $\frac{3}{4}$.	8. $\frac{96}{108}$.	Ans. $\frac{8}{9}$.	14. $\frac{32}{104}$.	Ans. $\frac{4}{13}$.
3. $\frac{18}{24}$.	Ans. $\frac{3}{4}$.	9. $\frac{54}{63}$.	Ans. $\frac{6}{7}$.	15. $\frac{630}{810}$.	Ans. $\frac{7}{9}$.
4. $\frac{24}{36}$.	Ans. $\frac{2}{3}$.	10. $\frac{80}{96}$.	Ans. $\frac{5}{6}$.	16. $\frac{336}{864}$.	Ans. $\frac{7}{27}$.
5. $\frac{36}{60}$.	Ans. $\frac{3}{5}$.	11. $\frac{120}{128}$.	Ans. $\frac{15}{16}$.	17. $\frac{484}{800}$.	Ans. $\frac{11}{20}$.
6. $\frac{48}{64}$.	Ans. $\frac{3}{4}$.	12. $\frac{84}{252}$.	Ans. $\frac{1}{3}$.	18. $\frac{315}{990}$.	Ans. $\frac{7}{22}$.
7. $\frac{60}{84}$.	Ans. $\frac{5}{7}$.	13. $\frac{155}{441}$.	Ans. $\frac{5}{11}$.	19. $\frac{2016}{2268}$.	Ans. $\frac{8}{9}$.

CASE II.

ART. 109. *To reduce an improper fraction to a whole or mixed number.*

Ex. 1. Reduce $\frac{20}{6}$ to a mixed number.

Analysis. The operation here required is to find out how many *whole ones* and *parts* of a whole one there are in 20 *sixths*. Since 6 *sixths* make 1 *whole one*, 20 *sixths* will make as many *whole ones* as 6 is contained times in 20, which will be 3 times and 2 *sixths*. Ans. $3\frac{2}{6}$.

From the foregoing analysis we derive the following

RULE.

Divide the numerator by the denominator

EXAMPLES.

Reduce the following fractions to whole or mixed numbers :

2. $\frac{19}{6}$.	Ans. $6\frac{1}{3}$.	6. $\frac{89}{6}$.	Ans. $11\frac{1}{6}$.	10. $\frac{112}{16}$.	Ans. 7
3. $\frac{29}{6}$.	Ans. $5\frac{1}{3}$.	7. $\frac{105}{8}$.	Ans. $11\frac{3}{8}$.	11. $\frac{126}{12}$.	Ans. $10\frac{1}{2}$.
4. $\frac{35}{6}$.	Ans. $5\frac{5}{6}$.	8. $\frac{252}{16}$.	Ans. 14.	12. $\frac{132}{18}$.	Ans. $7\frac{1}{3}$.
5. $\frac{53}{8}$.	Ans. $6\frac{5}{8}$.	9. $\frac{96}{16}$.	Ans. $6\frac{1}{2}$.	13. $\frac{132}{17}$.	Ans. $8\frac{2}{17}$.

14. $4\frac{25}{8}$. Ans. $28\frac{1}{8}$. | 17. $3\frac{10}{7}$. Ans. $12\frac{1}{7}$. | 20. $7\frac{29}{8}$. Ans. $28\frac{1}{8}$.
 15. $5\frac{9}{7}$. Ans. $18\frac{1}{2}$. | 18. $1\frac{7}{8}$. Ans. $13\frac{5}{8}$. | 21. $20\frac{28}{8}$. Ans. $45\frac{1}{4}$.
 16. $2\frac{2}{5}$. Ans. $5\frac{1}{5}$. | 19. $3\frac{9}{8}$. Ans. $14\frac{1}{4}$. | 22. $22\frac{7}{8}$. Ans. $163\frac{1}{8}$.

CASE III.

ART. 110. To reduce a whole number to a fraction having a given denominator.

Ex. 1. Reduce 5 to a fraction whose denominator shall be 7.

Analysis. The operation here required is to change 5 whole ones into sevenths. In 1 unit there are 7 sevenths, and in 5 units there are 5 times as many sevenths, which will be 35 sevenths.

Ans. $5\frac{35}{7}$.

From the *Analysis* we deduce the following

RULE.

Multiply the whole number by the required denominator, and place the product over the denominator.

EXAMPLES.

2. Reduce 4 to a fraction whose denominator shall be 6.
Ans. $2\frac{4}{6}$.
3. Reduce 12 to sixths.
Ans. $7\frac{2}{3}$.
4. Change 9 to fifteenths.
Ans. —.
5. Reduce 10 to ninths.
Ans. —.
6. Fifteen is equal to how many tenths?
Ans. $1\frac{5}{10}$.
7. Express 7 as a fraction having 8 for its denominator.
Ans. —.
8. Reduce 6 to fourteenths.
Ans. —.
9. Change 8 to twelfths.
Ans. —.
10. Sixteen is equal to how many eighths?
Ans. —.
11. Reduce 25 to fifths.
Ans. —.

CASE IV.

ART. 111. *To reduce a mixed number to an improper fraction.*

Ex. 1. Reduce $9\frac{3}{7}$ to an improper fraction.

Analysis. The operation here required is to find how many *sevenths* there are in 9 *whole ones* and 3 *sevenths*. In 1 unit there are 7 *sevenths*; and in 9 units there are 9 times as many *sevenths*, which make 63 *sevenths*, and 3 more *sevenths* make 66 *sevenths*, or, written thus, $\frac{66}{7}$. Ans. $\frac{66}{7}$.

From the *Analysis* we get the following

RULE.

Multiply the whole number by the denominator, add the numerator to the product, and place the sum over the denominator.

EXAMPLES.

Reduce the following to improper fractions :

2. $4\frac{1}{2}$. Ans. $\frac{14}{2}$.	9. $12\frac{2}{3}$. Ans. $\frac{62}{3}$.	16. $19\frac{3}{4}$. Ans. $\frac{155}{4}$.
3. $7\frac{2}{3}$. Ans. $\frac{22}{3}$.	10. $13\frac{2}{3}$. Ans. $\frac{41}{3}$.	17. $22\frac{1}{4}$. Ans. $\frac{202}{4}$.
4. $8\frac{1}{2}$. Ans. $\frac{17}{2}$.	11. $15\frac{2}{3}$. Ans. $\frac{47}{3}$.	18. $25\frac{1}{4}$. Ans. $\frac{101}{4}$.
5. $9\frac{1}{2}$. Ans. $\frac{19}{2}$.	12. $20\frac{1}{4}$. Ans. $\frac{81}{4}$.	19. $31\frac{3}{10}$. Ans. $\frac{313}{10}$.
6. $6\frac{1}{2}$. Ans. $\frac{13}{2}$.	13. $25\frac{1}{2}$. Ans. $\frac{51}{2}$.	20. $35\frac{1}{2}$. Ans. $\frac{71}{2}$.
7. $8\frac{1}{2}$. Ans. $\frac{17}{2}$.	14. $18\frac{1}{2}$. Ans. $\frac{37}{2}$.	21. $42\frac{1}{2}$. Ans. $\frac{85}{2}$.
8. $10\frac{1}{2}$. Ans. $\frac{21}{2}$.	15. $16\frac{1}{2}$. Ans. $\frac{33}{2}$.	22. $55\frac{1}{11}$. Ans. $\frac{606}{11}$.

CASE V.

ART. 112. *To reduce a fraction to higher terms.*

Ex. 1. Reduce $\frac{4}{5}$ to a fraction whose denominator shall be 15.

Analysis. The operation here required is to change *fifths* into *fifteenths*. 1 unit is equal to 15 *fifteenths*; then $\frac{1}{5}$ of a unit will be 3 *fifteenths*, and 4 *fifths* will be equal to 4 times as many *fifteenths*, which will be 12 *fifteenths*, or $\frac{12}{15}$.

Ans. $\frac{12}{15}$.

From the *Analysis* we get the following

RULE.

Divide the required denominator by the denominator of the given fraction, multiply the quotient by the numerator, and place the product over the required denominator.

EXAMPLES.

2 Change $\frac{5}{8}$ to a fraction whose denominator shall be 18.

Ans. $\frac{15}{18}$.

3. Reduce $\frac{3}{4}$ to a fraction whose denominator shall be 16.

Ans. $\frac{12}{16}$.

4. Reduce $\frac{5}{7}$ to thirty-fifths.

Ans. $\frac{25}{35}$.

5. Change $\frac{3}{5}$ to fortieths.

Ans. $\frac{24}{40}$.

6. Reduce $\frac{4}{5}$ to thirty-sixths.

Ans. $\frac{16}{36}$.

7. Change $\frac{2}{3}$ and $\frac{3}{4}$ to twelfths.

Ans. $\frac{8}{12}, \frac{9}{12}$.

8. Reduce $\frac{3}{5}$ and $\frac{1}{4}$ to twentieths.

Ans. $\frac{12}{20}, \frac{5}{20}$.

9. Change $\frac{2}{3}, \frac{3}{4}$ and $\frac{5}{6}$ to twenty-fourths.

Ans. $\frac{16}{24}, \frac{18}{24}, \frac{20}{24}$.

10. Reduce $\frac{5}{6}, \frac{3}{5}$ and $\frac{7}{12}$ to thirty-sixths.

Ans. $\frac{30}{36}, \frac{12}{36}, \frac{21}{36}$.

11. In $3\frac{2}{3}$ how many tenths?

Ans. $\frac{20}{3}$.

12. In $4\frac{2}{3}$ how many fifteenths?

Ans. $\frac{10}{3}$.

CASE VI.

ART. 113. *To reduce a compound fraction to a simple one*

Ex. 1. Reduce $\frac{1}{3}$ of $\frac{1}{4}$ to a simple fraction.

Analysis. $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$. How do we know this? Because, if we cut an apple into 4 equal parts, each part will be $\frac{1}{4}$ of the whole apple. Now, if we again cut each of these *fourths* into *three* equal parts, we see that the whole apple will be divided into *twelve* equal parts, making each part $\frac{1}{12}$ of the whole apple; therefore $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$.

Ans. $\frac{1}{12}$.

2. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

Analysis. $\frac{1}{4}$ of $\frac{1}{5}$ is $\frac{1}{20}$; then $\frac{3}{4}$ of $\frac{1}{5}$ will be 3 times as much, which will be $\frac{3}{20}$; and $\frac{3}{4}$ of $\frac{4}{5}$ will be 4 times as much, which will be $\frac{12}{20}$.

$$\text{Ans. } \frac{12}{20} = \frac{3}{5}.$$

By observing the *Analysis* we see the reason for the following

RULE.

Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

NOTE.—If there are *mixed numbers*, they should first be reduced to *improper fractions*.

EXAMPLES.

3. Reduce $\frac{3}{5}$ of $\frac{2}{3}$ to a simple fraction. $\text{Ans. } \frac{2}{5} = \frac{2}{5}.$

4. Change $\frac{2}{3}$ of $\frac{5}{7}$ to a simple fraction. $\text{Ans. } \frac{10}{21} = \frac{10}{21}.$

5. Reduce $\frac{5}{6}$ of $\frac{3}{4}$ to a simple fraction. $\text{Ans. } \frac{15}{24} = \frac{5}{8}.$

6. Change $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction. $\text{Ans. } \frac{12}{20} = \frac{3}{5}.$

7. Reduce $\frac{7}{8}$ of $\frac{3}{4}$ to a simple fraction. $\text{Ans. } \frac{21}{32} = \frac{21}{32}.$

8. What simple fraction is equal to $\frac{3}{5}$ of $\frac{2}{3}$? $\text{Ans. } \frac{2}{5}.$

9. What simple fraction is equal to $\frac{3}{4}$ of $\frac{7}{11}$? $\text{Ans. } \frac{21}{44}.$

10. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{2}{5}$ to a simple fraction. $\text{Ans. } \frac{2}{15}.$

11. Change $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{2}{3}$ to a simple fraction. $\text{Ans. } \frac{5}{14}.$

12. Reduce $\frac{5}{6}$ of $3\frac{1}{2}$ to a simple fraction. $\text{Ans. } \frac{25}{4}.$

13. Reduce $\frac{5}{6}$ of $2\frac{1}{2}$ to an improper fraction. $\text{Ans. } \frac{25}{6}.$

14. A man owning $\frac{3}{4}$ of a steamboat sold $\frac{2}{3}$ of his share; what part of the boat did he sell? $\text{Ans. } \frac{1}{2}$ of the boat.

15. A gentleman owning $\frac{3}{4}$ of a factory gave $\frac{2}{3}$ of what he owned to his son; what part of the whole factory was the son's share? $\text{Ans. } \frac{1}{2}$ of the factory.

ART. 114. The operation of reducing compound fractions to simple ones may be shortened in *practice* by *canceling* those terms that are found in both the numerator and denominator.

Ex. 1. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{9}{11}$ to a simple fraction.

$$\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{9}{11} = \frac{3}{11}.$$

Analysis. By canceling the *fours* and *fives* and *nines*, both in the numerators and in the denominators, we get the fraction $\frac{3}{11}$. In canceling the *fours*, we simply *divide* both numerator and denominator by 4, which does not alter the *value* of the fraction (Art. 106). In canceling the *fives* we divide both terms of the fraction by 5; and in canceling the *nines* we divide both terms by 9.

Hence, we deduce the following

RULE.

Cancel all the factors COMMON to the numerators and the denominators, then multiply the remaining numerators together for a new numerator, and the remaining denominators for a new denominator.

EXAMPLES.

2. Reduce $\frac{4}{5}$ of $\frac{7}{8}$ of $\frac{1}{6}$ of $\frac{1}{3}$ to a simple fraction.

Ans. $\frac{1}{18}$

3. Reduce $\frac{4}{7}$ of $\frac{5}{12}$ of $\frac{3}{4}$ of $\frac{7}{10}$ to a simple fraction.

Ans. $\frac{1}{8}$

4. Change $\frac{3}{5}$ of $\frac{6}{11}$ of $\frac{5}{8}$ of $\frac{1}{4}$ to a simple fraction.

Ans. $\frac{3}{44}$

5. Reduce $\frac{7}{8}$ of $\frac{6}{21}$ of $\frac{5}{18}$ of $\frac{9}{15}$ to a simple fraction.

Ans. $\frac{1}{21}$

6. Change $\frac{3}{4}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{6}{10}$ to a simple fraction.

Ans. $\frac{1}{4}$

7. Reduce $\frac{5}{7}$ of $\frac{1}{3}$ of $\frac{5}{11}$ of $2\frac{3}{4}$ to a simple fraction.

Ans. $\frac{5}{88}$

8. Reduce $\frac{7}{11}$ of $\frac{3}{4}$ of $\frac{1}{7}$ of $2\frac{3}{4}$ to a simple fraction.

Ans. $\frac{1}{11}$

CASE VII.

ART 115. *To reduce two or more fractions to a common denominator.*

NOTE.—By *common denominator* is meant the *same denominator*.

Ex. 1. Reduce $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{1}{5}$ to a common denominator.

Analysis. We will take 60 as the common denominator, because 60 is the product of the three denominators of the given fractions, and therefore is a *common multiple* of the denominators; the question then is, simply, to reduce $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{1}{5}$ to *sixtieths*. We say, 1 unit is equal to 60 *sixtieths*; then, $\frac{1}{3}$ of a unit will be equal to 20 *sixtieths*, and $\frac{2}{3}$ will be equal to *twice* as many *sixtieths*, which will be 40 *sixtieths*, or $\frac{40}{60}$. Again, since 1 unit is equal to 60 *sixtieths*, $\frac{1}{4}$ of a unit will be equal to 15 *sixtieths*, and $\frac{3}{4}$ will be 45 *sixtieths*, or $\frac{45}{60}$. Lastly, since 1 unit is equal to 60 *sixtieths*, $\frac{1}{5}$ of a unit will be 12 *sixtieths*, and $\frac{1}{5}$ will be 12 *sixtieths*, or $\frac{12}{60}$. Hence, we see that $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{1}{5}$ when reduced become respectively $\frac{40}{60}$, $\frac{45}{60}$ and $\frac{12}{60}$.

Ans. $\frac{40}{60}$, $\frac{45}{60}$, $\frac{12}{60}$.

Or, we may explain the process thus :

$$\frac{2}{3} + \frac{3}{4} + \frac{1}{5} = \frac{40}{60} + \frac{45}{60} + \frac{12}{60}.$$

If we multiply *both* terms of $\frac{2}{3}$ by 4 and 5 (the denominators of the other two fractions), we shall have $\frac{40}{60}$. Again, if we multiply *both* terms of $\frac{3}{4}$ by 3 and 5 (the denominators of the other two fractions), we shall have $\frac{45}{60}$. Lastly, if we multiply *both* terms of $\frac{1}{5}$ by 3 and 4 (the denominators of the other two fractions), we shall have $\frac{12}{60}$. The operation therefore consists in multiplying the denominators of the fractions together for a *common denominator*, and in multiplying each numerator into all the denominators except its own for a new numerator; in other words, it consists in multiplying *both terms* of each fraction by the same number, which does not alter the values of the fractions

Hence the reason is obvious for the following

RULE.

Multiply both terms of each fraction by all the denominators except its own.

NOTE.—Mixed numbers must first be reduced to improper fractions, and all compound fractions to simple ones.

EXAMPLES.

Reduce the following fractions to others having a *common denominator* :

- | | |
|---|---|
| 2. Reduce $\frac{2}{3}$ and $\frac{7}{4}$. | Ans. $\frac{27}{12}$, $\frac{25}{12}$. |
| 3. Reduce $\frac{4}{7}$ and $\frac{9}{10}$. | Ans. $\frac{40}{70}$, $\frac{63}{70}$. |
| 4. Reduce $\frac{3}{5}$ and $\frac{7}{11}$. | Ans. $\frac{33}{55}$, $\frac{35}{55}$. |
| 5. Reduce $\frac{3}{8}$, $\frac{2}{3}$ and $\frac{5}{4}$. | Ans. $\frac{45}{96}$, $\frac{54}{96}$, $\frac{90}{96}$. |
| 6. Reduce $\frac{5}{7}$, $\frac{2}{3}$ and $\frac{4}{5}$. | Ans. $\frac{75}{105}$, $\frac{70}{105}$, $\frac{84}{105}$. |
| 7. Reduce $\frac{4}{5}$, $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{3}{4}$. | Ans. $\frac{320}{480}$, $\frac{240}{480}$, $\frac{300}{480}$, $\frac{270}{480}$. |
| 8. Reduce $\frac{2}{3}$, $\frac{1}{5}$, $\frac{2}{7}$ and $\frac{1}{4}$. | Ans. $\frac{252}{420}$, $\frac{84}{420}$, $\frac{120}{420}$, $\frac{105}{420}$. |
| 9. Reduce $\frac{5}{6}$, $\frac{2}{7}$, $\frac{3}{8}$ and $\frac{1}{2}$. | Ans. $\frac{315}{840}$, $\frac{192}{840}$, $\frac{252}{840}$, $\frac{192}{840}$. |
| 10. Reduce $\frac{5}{6}$, $\frac{1}{3}$, $\frac{5}{8}$ and $2\frac{3}{4}$. | Ans. $\frac{350}{840}$, $\frac{140}{840}$, $\frac{480}{840}$, $\frac{1584}{840}$. |
| 11. Reduce $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$ and $\frac{1}{2}$ of $\frac{3}{4}$. | Ans. $\frac{210}{840}$, $\frac{252}{840}$, $\frac{270}{840}$, $\frac{210}{840}$. |

CASE VIII.

ART. 116. *To reduce two or more fractions to others having the LEAST COMMON denominator.*

Ex. 1. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{8}$ to their *least common denominator*.

Analysis. A little examination of the denominators of these fractions will show us that the *least* number in which they can all be contained is 24. Hence, 24 is their *least common multiple*, and therefore will be their *least common denominator*. The question then is to reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{8}$ to *twenty-fourths*. Proceeding according to the *Analysis* in CASE V., we have

$\frac{1}{24}$, $\frac{1}{12}$, $\frac{3}{24}$ and $\frac{1}{4}$.

Hence, in practice we employ the following

RULE.

Find the least common multiple of all the denominators for the least common denominator. Divide this least common denominator by the denominator of each fraction, and multiply the quotient by its numerator; the several products will be the numerators required.

NOTE 1.—In applying this Rule, each fraction should be reduced to its *lowest* terms before commencing the operation.

NOTE 2.—The object of reducing fractions to a common denominator is to prepare them for the operations of Addition and Subtraction.

EXAMPLES.

Reduce the following to equivalent fractions having the *least* common denominator :

- | | |
|--|--|
| 2. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{1}{2}$. | Ans. $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$, $\frac{6}{12}$ |
| 3. Reduce $\frac{1}{3}$, $\frac{4}{6}$, $\frac{5}{9}$, $\frac{7}{18}$. | Ans. $\frac{6}{18}$, $\frac{12}{18}$, $\frac{10}{18}$, $\frac{7}{18}$. |
| 4. Reduce $\frac{3}{6}$, $\frac{5}{10}$, $\frac{6}{8}$, $\frac{4}{15}$. | Ans. $\frac{18}{60}$, $\frac{15}{60}$, $\frac{25}{60}$, $\frac{8}{60}$. |
| 5. Reduce $\frac{4}{7}$, $\frac{5}{14}$, $\frac{3}{21}$, $\frac{5}{6}$. | Ans. $\frac{24}{42}$, $\frac{15}{42}$, $\frac{10}{42}$, $\frac{35}{42}$. |
| 6. Reduce $\frac{2}{3}$, $\frac{1}{2}$, $\frac{5}{6}$, $\frac{3}{4}$. | Ans. $\frac{8}{12}$, $\frac{6}{12}$, $\frac{10}{12}$, $\frac{9}{12}$. |
| 7. Reduce $\frac{4}{5}$, $\frac{3}{10}$, $\frac{5}{12}$, $\frac{5}{20}$. | Ans. $\frac{48}{60}$, $\frac{18}{60}$, $\frac{25}{60}$, $\frac{15}{60}$. |
| 8. Reduce $\frac{2}{7}$, $\frac{3}{8}$, $\frac{9}{10}$, $\frac{9}{14}$. | Ans. $\frac{60}{70}$, $\frac{42}{70}$, $\frac{63}{70}$, $\frac{45}{70}$. |
| 9. Reduce $\frac{5}{6}$, $\frac{4}{7}$, $\frac{11}{12}$, $\frac{2}{3}$ of $\frac{5}{7}$. | Ans. $\frac{70}{84}$, $\frac{48}{84}$, $\frac{77}{84}$, $\frac{40}{84}$. |
| 10. Reduce $\frac{1}{15}$, $\frac{8}{21}$, $\frac{3}{35}$, $2\frac{2}{3}$ | Ans. $\frac{98}{105}$, $\frac{40}{105}$, $\frac{9}{105}$, $\frac{273}{105}$. |

ADDITION OF FRACTIONS.

CASE I.

ART. 117. *When the denominators are alike.*

Ex. 1. Jane paid $\frac{1}{10}$ of a dollar for a slate, $\frac{2}{10}$ of a dollar for a reader, and $\frac{4}{10}$ of a dollar for an arithmetic; how much did she spend for all?

Analysis. Since the parts of the fractions are all *tenths* of a dollar, they may be added just as we add *whole* dollars; thus 2 *tenths* and 3 *tenths* are 5 *tenths*, and 4 *tenths* are 9 *tenths*, or expressed thus, $\frac{9}{10}$. *Ans.* $\frac{9}{10}$ of a dollar.

Hence we employ the following

RULE.

Add the numerators, and place their sum over the common denominator.

EXAMPLES.

2. What is the sum of $\frac{5}{8}$, $\frac{7}{8}$ and $\frac{3}{8}$? *Ans.* $1\frac{1}{2}$.
3. What is the sum of $\frac{5}{9}$, $\frac{7}{9}$ and $\frac{3}{9}$? *Ans.* $2\frac{2}{3}$.
4. Add together $\frac{5}{8}$, $\frac{4}{8}$, $\frac{2}{8}$ and $\frac{1}{8}$. *Ans.* $2\frac{1}{2}$.
5. Find the sum of $\frac{5}{12}$, $\frac{9}{12}$, $\frac{7}{12}$ and $\frac{3}{12}$. *Ans.* $2\frac{1}{2}$.
6. Add together $\frac{7}{15}$, $\frac{8}{15}$, $\frac{6}{15}$ and $\frac{9}{15}$. *Ans.* 2.
7. William paid $\frac{5}{8}$ of a dollar for a book, $\frac{3}{8}$ of a dollar for a slate, $\frac{7}{8}$ of a dollar for a cap, and then had $\frac{5}{8}$ of a dollar left; how much did he have at first? *Ans.* $2\frac{1}{2}$ dollars.

CASE II.

ART. 118. *When the denominators are unlike.*

Ex. 1. What is the sum of $\frac{3}{8}$, $\frac{3}{4}$ and $\frac{5}{8}$?

Analysis. We cannot add the fractions in their present shape, but we must first reduce them to a common denominator. We see that 12 is the *least* common multiple of the denominators. Reducing them all to *twelfths*, we have $\frac{3}{8} = \frac{9}{12}$; $\frac{3}{4} = \frac{9}{12}$; and $\frac{5}{8} = \frac{10}{12}$. Then $\frac{9}{12}$ and $\frac{9}{12}$ are $\frac{17}{12}$, and $\frac{17}{12}$ are $2\frac{5}{12}$, $= 2\frac{3}{4}$ or $2\frac{1}{2}$. *Ans.* $2\frac{1}{2}$.

Hence, we derive the following

RULE.

Reduce the fractions to a common denominator; add the numerators and place the sum over the common denominator.

NOTE.—Compound fractions must be reduced to simple fractions before reducing them to a common denominator.

EXAMPLES.

2. What is the sum of $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{1}{2}$? *Ans.* $1\frac{1}{4}$.
3. Find the sum of $\frac{3}{5}$, $\frac{5}{8}$ and $\frac{7}{10}$. *Ans.* $2\frac{2}{5}$.
4. What is the sum of $\frac{1}{4}$, $\frac{3}{5}$ and $\frac{1}{2}$? *Ans.* $1\frac{7}{20}$.
5. Add together $\frac{5}{8}$, $\frac{7}{8}$, $\frac{9}{12}$ and $\frac{1}{3}$. *Ans.* $2\frac{1}{4}$.
6. What is the sum of $\frac{5}{7}$, $\frac{3}{10}$, $\frac{4}{5}$ and $\frac{3}{14}$? *Ans.* $2\frac{1}{5}$.
7. Find the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$. *Ans.* $1\frac{1}{4}$.
8. What is the sum of $\frac{5}{8}$, $\frac{7}{12}$, $\frac{5}{18}$ and $\frac{7}{24}$? *Ans.* $1\frac{1}{2}$.
9. Add together $\frac{5}{7}$, $\frac{2}{3}$, $\frac{5}{8}$ and $\frac{5}{21}$. *Ans.* $2\frac{1}{4}$.
10. Add together $\frac{3}{5}$, $\frac{1}{3}$ of $\frac{1}{4}$ and $\frac{9}{10}$. *Ans.* $1\frac{1}{2}$.
11. Find the sum of $\frac{3}{5}$ of $\frac{2}{3}$, $\frac{7}{10}$ and $\frac{5}{8}$. *Ans.* $1\frac{1}{5}$.

When there are mixed numbers we may add the fractional parts, and then unite their sum to that of the whole numbers.

12. What is the sum of $9\frac{1}{5}$ and $5\frac{3}{5}$?

Analysis. We first reduce $\frac{1}{5}$ and $\frac{3}{5}$ to a common denominator; $\frac{1}{5} = \frac{2}{10}$, and $\frac{3}{5} = \frac{6}{10}$. Adding $\frac{2}{10}$ and $\frac{6}{10}$ we have $\frac{8}{10}$; then adding the whole numbers, 9 and 5, we have 14. Hence, the entire sum is $14\frac{8}{10}$. *Ans.* $14\frac{4}{5}$.

OPERATION.

$$9\frac{1}{5} = 9\frac{2}{10}$$

$$5\frac{3}{5} = 5\frac{6}{10}$$

$$\text{Ans. } 14\frac{8}{10}$$

13. What is the sum of $15\frac{1}{2}$ and $27\frac{1}{3}$? *Ans.* $42\frac{5}{6}$.

14. Find the sum of $25\frac{3}{4}$ and $32\frac{3}{4}$. *Ans.* $58\frac{5}{2}$.

15. A farmer sold a quantity of wheat for $67\frac{3}{8}$ dollars, a load of fodder for $22\frac{1}{4}$ dollars and a quantity of barley for $37\frac{1}{2}$ dollars. What was the amount of his sales?

Ans. $127\frac{1}{4}$ dollars.

16. A merchant sold 4 pieces of alpaca. The first contained $36\frac{3}{4}$ yards, the second, $42\frac{1}{2}$ yards, the third, $45\frac{7}{8}$ yards, and the fourth, $35\frac{3}{4}$ yards; how many yards did he sell?

Ans. $160\frac{1}{4}$ yards.

17. A grocer sold 4 firkins of butter. The first weighed

48 $\frac{3}{4}$ pounds, the second, 62 $\frac{1}{4}$ pounds, the third, 56 $\frac{3}{4}$ pounds, and the fourth, 73 $\frac{1}{4}$ pounds; what was the entire weight of the butter?

Ans. 241 $\frac{1}{8}$ pounds.

18. A gentleman bought a quarter of beef for 9 $\frac{7}{8}$ dollars, a side of mutton for 2 $\frac{1}{4}$ dollars, a bunch of fish for 2 $\frac{3}{4}$ dollars and a turkey for 2 $\frac{3}{10}$ dollars; what was the amount of his purchases?

Ans. 17 $\frac{11}{20}$ dollars.

19. A farmer bought 4 tracts of land. The first contained 217 $\frac{7}{10}$ acres, the second, 328 $\frac{5}{8}$ acres, the third, 425 $\frac{1}{8}$ acres, and the fourth, 487 $\frac{1}{8}$ acres; how many acres in the 4 tracts?

Ans. 1459 $\frac{1}{8}$ acres.

20. A lady went to a store and bought a pair of gaiters for 4 $\frac{3}{4}$ dollars, a bonnet for 7 $\frac{1}{2}$ dollars, a pair of gloves for 2 $\frac{1}{8}$ dollars, a lace collar for 3 $\frac{1}{8}$ dollars and a parasol for 5 $\frac{3}{8}$ dollars; what was the amount of her bill?

Ans. 23 $\frac{87}{120}$ dollars.

21. A grocer has 4 barrels of syrup. The first contains 42 $\frac{4}{5}$ gallons, the second 45 $\frac{3}{5}$ gallons, the third, 52 $\frac{4}{5}$ gallons, and the fourth, 48 $\frac{7}{10}$ gallons; how many gallons in the 4 barrels?

Ans. 189 $\frac{1}{5}$ gallons.

SUBTRACTION OF FRACTIONS.

CASE I.

ART. 119. *When the denominators are alike.*

Ex. 1. If $\frac{3}{8}$ be taken from $\frac{8}{8}$, what will remain?

Analysis. Since the parts of the fractions are all *ninths*, they may be taken from each other, just as we take whole numbers from each other; hence, 3 *ninths* from 8 *ninths* leave 5 *ninths*.

Ans. $\frac{5}{8}$.

For this case we employ the following

RULE.

Take the less numerator from the greater, and place the difference over the common denominator.

EXAMPLES.

2. If $\frac{4}{7}$ be taken from $\frac{9}{7}$, what will remain? *Ans.* $\frac{5}{7}$
 3. Subtract $\frac{7}{15}$ from $\frac{12}{15}$. *Ans.* $\frac{5}{15} = \frac{1}{3}$
 4. If $\frac{3}{10}$ be taken from $\frac{9}{10}$, how much will be left? *Ans.* $\frac{6}{10}$
 5. Take $\frac{7}{18}$ from $\frac{10}{18}$. *Ans.* $\frac{3}{18}$
 6. Subtract $\frac{9}{17}$ from $\frac{15}{17}$. *Ans.* $\frac{6}{17}$
 7. From $\frac{5}{10}$ take $\frac{2}{10}$. *Ans.* $\frac{3}{10}$
 8. From $\frac{9}{12}$ take $\frac{5}{12}$. *Ans.* $\frac{4}{12}$

CASE II.

ART. 120. *When the denominators are unlike.*

Ex. 1. From $\frac{4}{3}$ take $\frac{2}{5}$.

Analysis. We cannot subtract these fractions in their present shape, for 2 thirds taken from 4 fifths will leave neither 2 thirds nor 2 fifths, but by reducing them to a common denominator (Art. 115) we have $\frac{4}{3} = \frac{12}{15}$, and $\frac{2}{5} = \frac{4}{15}$, and now, subtracting $\frac{4}{15}$ from $\frac{12}{15}$, we have $\frac{8}{15}$ as the remainder. *Ans.* $\frac{8}{15}$.

From the foregoing explanation we deduce the

RULE.

Reduce the fractions to a common denominator; take the less numerator from the greater, and place the difference over the common denominator.

EXAMPLES.

2. From $\frac{3}{4}$ take $\frac{1}{4}$. *Ans.* $\frac{2}{4}$
 3. From $\frac{1}{3}$ take $\frac{2}{6}$. *Ans.* $\frac{1}{6}$
 4. Take $\frac{1}{2}$ from $\frac{5}{6}$. *Ans.* $\frac{1}{6}$
 5. Subtract $\frac{3}{15}$ from $\frac{8}{15}$. *Ans.* $\frac{5}{15}$
 6. Subtract $\frac{3}{8}$ from $\frac{5}{8}$. *Ans.* $\frac{2}{8}$
 7. From $\frac{7}{8}$ take $\frac{3}{8}$. *Ans.* $\frac{4}{8}$
 8. From $\frac{3}{4}$ of $\frac{3}{4}$ take $\frac{1}{16}$. *Ans.* $\frac{7}{16}$

9. From $\frac{3}{4}$ of $\frac{5}{8}$ take $\frac{1}{2}$ of $\frac{1}{4}$. Ans. $\frac{1}{8}$.
 10. Subtract $\frac{1}{4}$ from $\frac{1}{8}$. Ans. $\frac{1}{8}$.
 11. Take $\frac{5}{8}$ from $\frac{1}{2}$. Ans. $\frac{1}{8}$.
 12. From $\frac{7}{8}$ subtract $\frac{3}{8}$. Ans. $\frac{1}{2}$.
 13. From $8\frac{5}{8}$ subtract $5\frac{1}{4}$.

Analysis. We first reduce the fractions to a common denominator, then take the *difference* of the fractions, and add it to the *difference* of the whole numbers. Thus, $\frac{3}{8}$ from $\frac{1}{2}$ leaves $\frac{1}{8}$; and then 5 from 8 leaves 3, making the remainder $3\frac{1}{8}$.

OPERATION.

$$8\frac{5}{8} = 8\frac{1}{2}$$

$$5\frac{1}{4} = 5\frac{2}{8}$$

$$\text{Ans. } 3\frac{1}{8}$$

$$\text{Ans. } 3\frac{1}{8}$$

14. From $9\frac{3}{4}$ subtract $4\frac{3}{8}$.

$$\text{Ans. } 5\frac{3}{8}$$

15. Subtract $12\frac{1}{2}$ from $17\frac{1}{2}$.

$$\text{Ans. } 5\frac{1}{2}$$

16. Take $25\frac{3}{4}$ from $34\frac{1}{2}$.

$$\text{Ans. } 9\frac{1}{4}$$

17. A grocer had a barrel of molasses containing $42\frac{1}{2}$ gallons, from which he sold $17\frac{1}{2}$ gallons; how much was left?

$$\text{Ans. } 25\frac{1}{2} \text{ gallons.}$$

18. A merchant had $37\frac{3}{4}$ yards of silk, and sold off $23\frac{1}{2}$ yards; how many yards remained?

$$\text{Ans. } 14\frac{1}{2} \text{ yards.}$$

19. From a cask of wine containing $42\frac{3}{4}$ gallons, $24\frac{1}{2}$ gallons were drawn; how much still remained?

$$\text{Ans. } 18\frac{1}{4} \text{ gallons.}$$

20. A man started on a journey of $84\frac{1}{2}$ miles, and after traveling $52\frac{3}{4}$ miles, how much farther had he to travel?

$$\text{Ans. } 32\frac{1}{4} \text{ miles.}$$

21. A man deposited in bank $72\frac{3}{4}$ dollars; after drawing out $45\frac{1}{2}$ dollars, how much had he still in bank?

$$\text{Ans. } 27\frac{1}{2} \text{ dollars.}$$

22. From a piece of cloth containing $35\frac{3}{4}$ yards, a merchant sold $23\frac{1}{2}$ yards; how much remained?

$$\text{Ans. } 12\frac{1}{2} \text{ yards.}$$

23. From 6 subtract $\frac{7}{8}$.

Analysis. In solving this example we borrow 1 unit from the 6 units, and call the unit borrowed $\frac{8}{8}$. Then we say: $\frac{7}{8}$

from $\frac{3}{8}$ will leave $\frac{1}{8}$; and as we borrowed 1 from the 6, we left only 5; hence the answer is $5\frac{1}{8}$.

Ans. $5\frac{1}{8}$.

24. From 12 take $\frac{5}{8}$.

Ans. $11\frac{3}{8}$.

25. Subtract $\frac{7}{10}$ from 15.

Ans. $14\frac{3}{10}$.

26. From 26 subtract $12\frac{3}{4}$.

Ans. $13\frac{1}{4}$.

27. Subtract $18\frac{7}{8}$ from 32.

Ans. $13\frac{1}{8}$.

28. From 35 take $15\frac{5}{8}$.

Ans. $19\frac{3}{8}$.

29. Take $27\frac{3}{12}$ from 45.

Ans. $17\frac{1}{2}$.

30. From $9\frac{1}{2}$ take $5\frac{3}{8}$.

Analysis. We first reduce the fractional parts to a common denominator; we then have $9\frac{2}{4}$ and $5\frac{3}{4}$. Now, as we cannot take $\frac{3}{4}$ from $\frac{2}{4}$, we borrow 1 unit from the 9 units, and call the unit borrowed $\frac{4}{4}$; we then add the $\frac{4}{4}$ to the $\frac{2}{4}$, and we have $\frac{6}{4}$; then subtracting the $\frac{3}{4}$ from the $\frac{6}{4}$, we have left $\frac{3}{4}$; and as we borrowed 1 unit from the 9 units, and therefore left but 8 units, we take the 5 from 8, which leaves

OPERATION

$$9\frac{2}{4} = 9\frac{2}{4}$$

$$5\frac{3}{4} = 5\frac{3}{4}$$

$$\text{Ans. } 3\frac{3}{4}$$

3. Hence the result is $3\frac{5}{8}$.

Ans. $3\frac{5}{8}$.

31. From $15\frac{1}{2}$ take $9\frac{3}{4}$.

Ans. $5\frac{1}{2}$.

32. Subtract $17\frac{5}{8}$ from $25\frac{1}{2}$.

Ans. $7\frac{7}{8}$.

33. Take $24\frac{5}{8}$ from $43\frac{1}{8}$.

Ans. $18\frac{3}{4}$.

34. Subtract $325\frac{7}{8}$ from $442\frac{3}{4}$.

Ans. $116\frac{3}{8}$.

35. From a cheese which weighed $48\frac{1}{2}$ pounds a grocer cut $23\frac{3}{4}$ pounds; how much was left?

Ans. $24\frac{9}{8}$ pounds.

36. Henry had $25\frac{1}{2}$ dollars, and bought a coat for $16\frac{3}{4}$ dollars; how much had he left?

Ans. $8\frac{5}{8}$ dollars.

37. Jane went to a store with $81\frac{1}{2}$ cents, and bought a handkerchief for $62\frac{1}{2}$ cents; how much money had she left?

Ans. $18\frac{3}{4}$ cents.

38. From a jar of honey weighing $28\frac{1}{2}$ pounds, a grocer sold $12\frac{3}{4}$ pounds; how much remained?

Ans. $15\frac{3}{4}$ pounds.

39. From a rod $32\frac{1}{4}$ feet long, $18\frac{3}{4}$ feet were cut off; how much was left?

Ans. $13\frac{7}{4}$ feet.

MULTIPLICATION OF FRACTIONS.

CASE I.

ART. 121. *To multiply a fraction by a whole number.*

Ex. 1. What would be the cost of 4 bushels of corn at $\frac{7}{8}$ of a dollar per bushel?

Analysis. If 1 bushel cost $\frac{7}{8}$ of a dollar, 4 bushels would evidently cost 4 *times* as much, which would be $\frac{28}{8}$ of a dollar, or $3\frac{4}{8}$ dollars = $3\frac{1}{2}$ dollars. *Ans.* $3\frac{1}{2}$ dollars.

Here we *multiplied* the *numerator* of the fraction by 4, but if we had *divided* the *denominator*, we would have obtained $\frac{7}{2}$ of a dollar = $3\frac{1}{2}$ dollars, the same as before.

From this illustration we derive a

RULE.

Multiply the numerator by the whole number, or divide the denominator when it can be done without a remainder.

EXAMPLES.

2. Multiply $\frac{5}{7}$ by 6. *Ans.* $4\frac{2}{7}$.

3. Multiply $1\frac{9}{12}$ by 3. *Ans.* $2\frac{1}{2}$.

4. Multiply $1\frac{2}{3}$ by 7. *Ans.* $4\frac{1}{3}$.

5. Multiply $1\frac{1}{8}$ by 6. *Ans.* $3\frac{3}{4}$.

6. Multiply $1\frac{2}{4}$ by 15. *Ans.* $9\frac{3}{2}$.

7. Multiply $2\frac{5}{6}$ by 9. *Ans.* $6\frac{1}{2}$.

8. Multiply $1\frac{3}{7}$ by 10. *Ans.* $7\frac{1}{7}$.

9. Multiply $2\frac{2}{3}$ by 5. *Ans.* $4\frac{2}{3}$.

10. If a bushel of wheat cost $1\frac{1}{8}$ of a dollar, what would be the cost of 10 bushels? *Ans.* $9\frac{5}{4}$ dollars.

11. What would be the cost of 9 yards of muslin at $\frac{4}{5}$ of a dollar per yard? *Ans.* $7\frac{1}{5}$ dollars.

12. If a dozen eggs sell for $\frac{2}{3}$ of a dollar, what must you pay for 11 dozen? *Ans.* $4\frac{1}{3}$ dollars.

13. If a sack hold $1\frac{1}{8}$ of a bushel of potatoes, how many bushels could you put in 11 sacks? *Ans.* $9\frac{1}{8}$ bushels.

Observation. Whenever the multiplier happens to be exactly equal to the denominator of a fraction, the product will be equal to the *numerator* of the fraction; or, in other words,

A fraction is multiplied by a number equal to its denominator, simply by canceling the denominator; thus, $\frac{5}{8} \times 8 = 5$; $\frac{7}{8} \times 8 = 7$; $\frac{9}{9} \times 9 = 3$.

14. A man bought 16 sacks of potatoes, each sack holding $1\frac{1}{8}$ of a bushel; how many bushels did he buy?

Ans. 15 bushels.

15. A lady bought 10 yards of muslin at $\frac{7}{10}$ of a dollar per yard; how much did she pay for the muslin?

Ans. 7 dollars.

16. Multiply $3\frac{3}{4}$ by 5.

Analysis. We first multiply the fractional part, $\frac{3}{4}$ by 5, which gives $1\frac{5}{4}$ or $3\frac{1}{4}$. We then multiply the whole number, 3 by 5, which gives 15; adding these two products, we have $18\frac{1}{4}$, as the entire product.

Ans. $18\frac{1}{4}$.

OPERATION.

$$\begin{array}{r} 3\frac{3}{4} \\ 5 \\ \hline 15 \\ 1\frac{5}{4} \\ \hline 16\frac{1}{4} \end{array}$$

In *practice* we do not always write down the product obtained by multiplying the fractional part by the whole number, but we carry it in the *mind*, and add it to the other product. Thus, we would say above: 5 times $\frac{3}{4}$ are $1\frac{5}{4}$, or $3\frac{1}{4}$; 5 times 3 are 15, and $3\frac{1}{4}$ to carry are $18\frac{1}{4}$.

Ans. $18\frac{1}{4}$.

17. Multiply $5\frac{3}{4}$ by 7.

Ans. $18\frac{1}{4}$.

18. Multiply $6\frac{5}{8}$ by 8.

Ans. $40\frac{1}{4}$.

19. Multiply $8\frac{5}{8}$ by 6.

Ans. $54\frac{3}{4}$.

20. Multiply $12\frac{3}{4}$ by 5.

Ans. $51\frac{1}{2}$.

21. Multiply $25\frac{3}{8}$ by 7.

Ans. $62\frac{1}{2}$.

22. How much would 7 barrels of apples cost at $6\frac{3}{4}$ dollars per barrel?

Ans. $177\frac{5}{8}$.

Ans. $47\frac{3}{4}$ dollars.

23. If $7\frac{1}{4}$ yards of cassimere make a suit of clothes, how many yards would be required for 9 suits?

Ans. $65\frac{1}{4}$ yards.

24. What would be the cost of 8 pounds of crackers at $16\frac{1}{2}$ cents per pound?

Ans. $133\frac{1}{2}$ cents.

25. What would 9 yards of calico cost at $15\frac{3}{4}$ cents per yard?

Ans. $141\frac{3}{4}$ cents.

26. A rod is $16\frac{1}{2}$ feet; how many feet in 7 rods?

Ans. $115\frac{1}{2}$ feet.

27. At $12\frac{3}{4}$ dollars per barrel, how much must you pay for 13 barrels of oranges?

Ans. $163\frac{3}{4}$ dollars.

28. How much must you pay for 15 cords of wood at $7\frac{3}{4}$ dollars per cord?

Ans. 115 dollars.

CASE II.

ART. 122. *To multiply a whole number by a fraction.*

Ex. 1. If a yard of calico cost 18 cents, what would be the cost of $\frac{5}{6}$ of a yard?

Analysis. If 1 yard cost 18 cents, then $\frac{1}{6}$ of a yard would cost *one sixth* of 18 cents, which is 3 cents; and $\frac{5}{6}$ of a yard would cost 5 times as much as $\frac{1}{6}$, which would be 15 cents.

1st OPERATION.

$$\begin{array}{r} 6 \overline{)18} \\ \underline{3} \\ 5 \end{array}$$

Ans. $\overline{15}$ cents.

To get the cost of $\frac{1}{6}$ of a yard, we divided the 18 cents by 6, the *denominator* of the fraction; and to get the cost of $\frac{5}{6}$, we multiplied the quotient by 5, the *numerator* of the fraction. We might have multiplied first by 5, and then divided the product by 6, and the result would have

2d OPERATION

$$\begin{array}{r} 18 \\ 5 \\ \hline 6 \overline{)90} \end{array}$$

Ans. 15 cents

been the same. This latter plan is the one usually practiced. Hence we derive the

RULE.

Multiply the whole number by the numerator, and divide the product by the denominator.

EXAMPLES.

2. Multiply 27 by $\frac{5}{9}$. *Ans.* 15.
3. Multiply 35 by $\frac{4}{7}$. *Ans.* 20.
4. Multiply 72 by $\frac{5}{8}$. *Ans.* 45.
5. Multiply 42 by $\frac{7}{9}$. *Ans.* $32\frac{2}{3}$.
6. Multiply 56 by $\frac{4}{7}$. *Ans.* $44\frac{4}{7}$.
7. Multiply 75 by $\frac{7}{12}$. *Ans.* $43\frac{5}{12}$.
8. A merchant had a piece of calico measuring 36 yards, and sold $\frac{3}{4}$ of it; how much was left in the piece?
Ans. 9 yards.
9. William had 48 chestnuts and Henry had $\frac{7}{8}$ as many; how many had Henry?
Ans. 42 chestnuts.
10. If a yard of muslin cost 40 cents, what would be the cost of $\frac{3}{4}$ of a yard?
Ans. 32 cents.
11. A yard is 36 inches; how long is $\frac{7}{12}$ of a yard?
Ans. 21 inches.
12. Forty rods make a furlong; how many rods in $\frac{5}{8}$ of a furlong?
Ans. $28\frac{1}{2}$ rods.
13. If a bushel of corn weigh 56 pounds, what would be the weight of $\frac{3}{4}$ of a bushel?
Ans. $33\frac{3}{4}$ pounds.
14. If an acre of land produce 40 bushels of wheat, how much would $\frac{5}{8}$ of an acre produce?
Ans. $34\frac{1}{2}$ bushels.
15. A farmer had 84 sheep and sold $\frac{7}{12}$ of them; how many sheep had he left?
Ans. 35 sheep.

CASE III.

ART. 123. *To multiply a fraction by a fraction.*

Ex. 1. Multiply $\frac{2}{3}$ by $\frac{3}{4}$.

Analysis. $\frac{1}{4}$ of $\frac{1}{3}$ is $\frac{1}{12}$ (Art. 113); then $\frac{2}{3}$ of $\frac{1}{4}$ will be

twice as much, which is $\frac{2}{12}$; and $\frac{2}{3}$ of $\frac{2}{3}$ will be 3 times as much, which is $\frac{6}{12}$. Ans. $\frac{6}{12}$.

From the *Analysis* we deduce the

RULE.

Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

NOTE.—Mixed numbers should be reduced to improper fractions before multiplying.

NOTE TO TEACHERS.—In solving examples like the one just given, it is an excellent plan to make the pupil *write* each step of the operation on the slate or blackboard.

This will impress the *Analysis* more clearly on his mind.

The example above given might be written thus :

$$\begin{aligned}\frac{1}{3} \text{ of } \frac{1}{4} &= \frac{1}{12}, \\ \frac{2}{3} \text{ of } \frac{1}{4} &= \frac{2}{12}, \\ \frac{2}{3} \text{ of } \frac{2}{3} &= \frac{6}{12} = \frac{1}{2}.\end{aligned}$$

EXAMPLES.

2. Multiply $\frac{4}{5}$ by $\frac{3}{7}$. Ans. $\frac{12}{35}$.

3. Multiply $\frac{3}{8}$ by $\frac{2}{3}$. Ans. $\frac{2}{8}$.

4. Multiply $\frac{5}{6}$ by $\frac{4}{5}$. Ans. $\frac{4}{3}$.

5. Multiply $\frac{7}{13}$ by $\frac{3}{4}$. Ans. $\frac{21}{52}$.

6. Multiply $\frac{23}{42}$ by $\frac{7}{10}$. Ans. $\frac{23}{60}$.

7. Multiply $\frac{12}{15}$ by $\frac{9}{11}$. Ans. $\frac{36}{55}$.

8. Henry had $\frac{2}{3}$ of a melon, and gave his brother $\frac{3}{4}$ of it. what part of the whole melon did he give away?

Ans. $\frac{3}{10}$ of a melon.

9. A gentleman owned $\frac{7}{10}$ of a factory, and sold $\frac{3}{4}$ of what he owned ; what part of the whole factory did he sell?

Ans. $\frac{21}{40}$ of factory.

10. Thomas had $\frac{7}{8}$ of a pound of candy, and gave away $\frac{2}{3}$ of it ; what part of a pound did he give away?

Ans. $\frac{7}{12}$ of a pound.

11. What is the product of $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{2}{5}$? *Ans.* $\frac{1}{80}$.
 12. Multiply $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ together. *Ans.* $\frac{5}{12}$.
 13. Multiply $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{9}{10}$ together. *Ans.* $\frac{3}{2}$.
 14. William had $\frac{3}{4}$ of a cantaloupe, and gave his sister $\frac{2}{3}$ of $\frac{3}{4}$ of what he had; what part of a cantaloupe did he give away?
Ans. $\frac{3}{10}$ of a cantaloupe.

15. A gentleman owning $\frac{5}{7}$ of a saw-mill sold $\frac{2}{3}$ of $\frac{5}{7}$ of what he owned; what part of the whole mill did he sell?

Ans. $\frac{2}{7}$ of mill.

16. What would be the cost of $\frac{2}{3}$ of $\frac{3}{4}$ of a pound of citron at $\frac{5}{8}$ of a dollar per pound? *Ans.* $\frac{1}{4}$ of a dollar.

17. Multiply $2\frac{2}{3}$ by $3\frac{1}{2}$.

OPERATION.

Analysis. We first reduce the mixed numbers to improper fractions: $2\frac{2}{3} = \frac{8}{3}$, and $3\frac{1}{2} = \frac{7}{2}$. Then multiplying $\frac{8}{3}$ by $\frac{7}{2}$, we have $\frac{128}{15} = 8\frac{8}{15}$. *Ans.* $8\frac{8}{15}$.

$$\begin{array}{r} 2\frac{2}{3} = \frac{8}{3} \\ 3\frac{1}{2} = \frac{7}{2} \\ \hline \frac{128}{15} = 8\frac{8}{15} \end{array}$$

18. Multiply $3\frac{1}{2}$ by $3\frac{1}{2}$. *Ans.* $12\frac{1}{4}$.

19. Multiply $5\frac{1}{2}$ by $2\frac{3}{4}$. *Ans.* $15\frac{1}{8}$.

20. Multiply $3\frac{1}{4}$ by $2\frac{1}{2}$. *Ans.* $8\frac{1}{8}$.

21. What would be the cost of $3\frac{1}{4}$ yards of cassimere at $3\frac{1}{2}$ dollars per yard? *Ans.* $10\frac{5}{8}$ dollars.

22. If a pound of soap cost $12\frac{1}{2}$ cents, how much would $9\frac{1}{2}$ pounds cost? *Ans.* $118\frac{3}{4}$ cents.

23. At $2\frac{2}{3}$ dollars per yard, what would be the cost of $4\frac{1}{2}$ yards of silk? *Ans.* 12 dollars.

24. If a man walk $3\frac{5}{8}$ miles an hour, how far could he walk in $2\frac{3}{4}$ hours? *Ans.* $9\frac{3}{8}$ miles.

25. How many cords of wood could a man cut in $5\frac{1}{4}$ days, by cutting $2\frac{3}{4}$ cords in one day? *Ans.* $13\frac{3}{4}$ cords.

26. What would be the cost of $18\frac{3}{4}$ acres of land at $18\frac{1}{4}$ dollars per acre? *Ans.* $351\frac{9}{16}$ dollars.

27. A lady bought $16\frac{3}{4}$ yards of calico at $16\frac{3}{4}$ cents per yard; how much did the calico cost? *Ans.* $277\frac{7}{8}$ cents.

28. A merchant bought 5 pieces of cassimere, each piece containing $25\frac{1}{2}$ yards, at $2\frac{1}{2}$ dollars per yard; what was the cost of the 5 pieces? *Ans.* $343\frac{1}{2}$ dollars

DIVISION OF FRACTIONS.

CASE I.

ART. 124. *To divide a fraction by a whole number.*

Ex. 1. If 3 pounds of prunes cost $\frac{3}{10}$ of a dollar, what would be the cost of 1 pound? ☛

Analysis. If 3 pounds cost 9 *tenths* of a dollar, 1 pound would cost *one third* as much, which will be 3 *tenths* of a dollar, or thus, $\frac{3}{10}$.

Ans. $\frac{3}{10}$ of a dollar.

In this example we *divided* the *numerator*, 9, by 3, and obtained $\frac{3}{10}$ as the result. If we had *multiplied* the *denominator*, 10, by 3, we would have obtained $\frac{9}{30}$, which, being reduced to its lowest terms, gives $\frac{3}{10}$, the same as before.

2. If 5 yards of cloth cost $\frac{1}{5}$ of a dollar, what would be the cost of 1 yard?

Analysis. If 5 yards cost 15 *sixteenths* of a dollar, it is plain that 1 yard would cost *one fifth* as much, which will be 3 *sixteenths* of a dollar; or $\frac{3}{16}$.

Ans. $\frac{3}{16}$ of a dollar.

Here we *divided* the *numerator*, 15, by 5, and obtained $\frac{3}{16}$ as the result. Had we *multiplied* the *denominator*, 16, by 5, we should have had $\frac{15}{80}$, which, reduced to its lowest terms, gives $\frac{3}{16}$, the same as before.

Hence we derive the following

RULE.

Divide the numerator, or multiply the denominator of the fraction, by the whole number.

NOTE.—Mixed numbers must be reduced to improper fractions, and compound fractions to simple ones.

EXAMPLES.

3. Divide $1\frac{2}{3}$ by 3.*Ans.* $\frac{4}{9} = \frac{2}{3}$.4. Divide $\frac{2}{3}$ by 4.*Ans.* $\frac{2}{12} = \frac{1}{6}$.5. Divide $1\frac{5}{8}$ by 5.*Ans.* $\frac{1}{8}$.6. Divide $\frac{4}{10}$ by 6.*Ans.* $\frac{1}{15}$.7. Divide $2\frac{1}{3}$ by 7.*Ans.* $\frac{2}{21}$.8. Divide $\frac{2}{3}$ by 9.*Ans.* $\frac{2}{27}$.9. Divide $2\frac{2}{3}$ by 8.*Ans.* $\frac{2}{12} = \frac{1}{6}$.10. Divide $1\frac{5}{6}$ by 10.*Ans.* $\frac{2}{60} = \frac{1}{30}$.

11. If $\frac{2}{10}$ of a factory belong equally to 6 owners, what part of the factory does each own? *Ans.* $\frac{2}{60}$ of the factory.

12. Henry divided $\frac{2}{3}$ of a melon among 5 of his school-mates; what part of a melon did each get?

Ans. $\frac{2}{15}$ of a melon.

13. A father at his death left $\frac{2}{3}$ of a paper-mill to his 3 sons and 3 daughters; what part of the mill was the share of each?

Ans. $\frac{1}{12}$ of the mill.

14. A gentleman distributed $\frac{2}{3}$ of a dollar equally among 4 beggars; what part of a dollar did each get?

Ans. $\frac{1}{6}$ of a dollar.

15. William had $\frac{7}{8}$ of a pine-apple and divided it among his 3 sisters and his 2 brothers; what part did each have?

Ans. $\frac{7}{40}$ of a pine-apple.16. Divide $3\frac{2}{3}$ by 5.

Analysis. We first reduce $3\frac{2}{3}$ to an improper fraction, making $\frac{16}{3}$; then $\frac{16}{3}$ divided by 5, gives $\frac{16}{15}$.

Ans. $\frac{16}{15}$.17. Divide $6\frac{1}{4}$ by 5.*Ans.* $1\frac{1}{4}$.18. Divide $7\frac{2}{3}$ by 6.*Ans.* $1\frac{4}{9}$.19. Divide $7\frac{1}{2}$ by 10.*Ans.* $\frac{5}{4}$.20. Divide $10\frac{2}{3}$ by 8.*Ans.* $1\frac{1}{4}$.21. Divide $15\frac{2}{3}$ by 7.*Ans.* $2\frac{1}{7}$.22. Divide $12\frac{2}{3}$ by 6.*Ans.* $2\frac{1}{3}$.

23. If 8 yards of flannel cost $5\frac{1}{2}$ dollars, what would be the cost of one yard? *Ans.* $\frac{2}{3}$ of a dollar.

24. If a man can cut $15\frac{3}{4}$ cords of wood in 7 days, how many cords could he cut in one day? *Ans.* $2\frac{1}{4}$ cords.

25. If 9 yards of velvet cost $34\frac{1}{2}$ dollars, what is the price per yard? *Ans.* $3\frac{3}{4}$ dollars.

26. If a man can walk $38\frac{1}{2}$ miles in 10 hours, how far could he walk in one hour? *Ans.* $3\frac{5}{8}$ miles.

27. If 7 tons of coal cost $60\frac{2}{3}$ dollars, what is the price per ton? *Ans.* $8\frac{2}{3}$ dollars.

CASE II.

ART. 125. *To divide a whole number by a fraction.*

Ex. 1. At the rate of $\frac{1}{5}$ of a dollar per yard, how many yards of linen could be bought for 5 dollars?

Analysis. If $\frac{1}{5}$ of a dollar will buy 1 yard, 5 dollars would buy as many yards as $\frac{1}{5}$ is contained times in 5. Now, $\frac{1}{5}$ is contained in 1 unit 5 times; then in 5 units it will be contained 5 times as often, which will be 25 times. Therefore, 25 yards could be bought for 5 dollars. *Ans.* 25 yards.

2. At $\frac{2}{3}$ of a dollar per gallon, how many gallons of strawberries could be bought for 8 dollars?

Analysis. If $\frac{2}{3}$ of a dollar will buy 1 gallon, 8 dollars would buy as many gallons as $\frac{2}{3}$ is contained times in 8. Now, $\frac{1}{3}$ is contained in 1 unit, 3 times; then in 8 units it will be contained 8 times as often, which will be 24 times; and 4 thirds will be contained only one fourth as many times, which will be 6 times. Therefore, 6 gallons could be bought for 8 dollars. *Ans.* 6 gallons.

By observing the *Analysis* we see that the operation consists in multiplying the whole number by the *denominator* of the fraction, and dividing the product by the *numerator*.

Hence we derive the following

RULE.

Multiply the whole number by the denominator of the fraction, and divide the product by the numerator.

EXAMPLES.

3. Divide 10 by $\frac{3}{4}$ *Ans.* $13\frac{1}{3}$.

4. Divide 9 by $\frac{3}{8}$. *Ans.* 15.

5. Divide 12 by $\frac{5}{7}$. *Ans.* $16\frac{4}{5}$.

6. Divide 16 by $\frac{7}{8}$. *Ans.* $18\frac{2}{7}$.

7. Divide 18 by $\frac{5}{8}$. *Ans.* $32\frac{2}{5}$.

8. Divide 24 by $\frac{5}{8}$. *Ans.* $28\frac{4}{5}$.

9. At $\frac{3}{4}$ of a dollar per bushel, how many bushels of corn could be bought for 12 dollars? *Ans.* 18 bushels.

10. If a sack hold $\frac{7}{8}$ of a bushel of apples, how many sacks would be required to hold 21 bushels? *Ans.* 24 sacks.

11. If a yard of muslin cost $\frac{7}{10}$ of a dollar, how many yards could be bought for 14 dollars? *Ans.* 20 yards.

12. At $\frac{3}{8}$ of a dollar per pound, how many pounds of mace could be bought for 8 dollars? *Ans.* $13\frac{1}{3}$ pounds.

13. If a family consume $\frac{5}{8}$ of a barrel of flour per month how long would 9 barrels last them? *Ans.* $10\frac{2}{5}$ months

14. Divide 18 by $2\frac{1}{4}$.

Analysis. We first reduce $2\frac{1}{4}$ to an improper fraction, making $\frac{9}{4}$, and we say: $\frac{1}{4}$ is contained in 1 unit 4 times; then, in 18 units it will be contained 18 times as often, which will be 72 times, and 9 fourths will be contained only one ninth as many times, which will be 8 times. *Ans.* 3.

15. Divide 24 by $2\frac{3}{4}$. *Ans.* 9.

16. Divide 32 by $3\frac{3}{8}$. *Ans.* $8\frac{8}{15}$.

17. Divide 20 by $2\frac{5}{8}$. *Ans.* $7\frac{1}{2}$.

18. Divide 39 by $2\frac{3}{8}$. *Ans.* 15.

19. At $2\frac{3}{4}$ dollars per yard, how many yards of cassimere could be bought for 33 dollars? *Ans.* 12 yards.

20. How many sacks, each containing $2\frac{1}{2}$ yards, could be made out of 28 yards of cloth? *Ans.* 12 sacks.

21. If a man can cut $3\frac{1}{2}$ cords of wood in a day, how many days would be required to cut 25 cords? *Ans.* $7\frac{1}{2}$ days.

22. At $3\frac{1}{2}$ cents apiece, how many lemons could you buy for 75 cents? *Ans.* 24 lemons.

CASE III.

ART. 126. *To divide a fraction by a fraction.*

Ex. 1. Divide $\frac{3}{4}$ by $\frac{4}{5}$.

Analysis. 1 unit is contained in $\frac{3}{4}$ *three fourths* of a time; then, *one fifth* of 1 unit will be contained in $\frac{3}{4}$ *five times* as often, which will be $\frac{15}{4}$ times, and 4 *fifths* of a unit will be contained only *one fourth* as many times, which will be $\frac{15}{8}$ of a time. *Ans.* $\frac{15}{8}$.

2. Divide $\frac{3}{7}$ by $\frac{5}{9}$.

Analysis. One unit is contained in $\frac{3}{7}$ *three sevenths* of a time; then, *one ninth* of a unit will be contained *nine times* as often, which will be $\frac{27}{7}$ times, and 5 *ninths* will be contained only *one fifth* as many times, which will be $\frac{27}{5}$ times. *Ans.* $\frac{27}{5}$.

In both these examples it will be observed that the operation consists in multiplying the dividend by the divisor *inverted*; thus, in the first example, multiplying $\frac{3}{4}$ by $\frac{5}{4}$ we obtain $\frac{15}{8}$; and in the second, multiplying $\frac{3}{7}$ by $\frac{9}{5}$ we get $\frac{27}{5}$, the same as by the analysis. From these illustrations we derive a

RULE.

Invert the divisor, and multiply the numerators together for a numerator, and the denominators together for a denominator.

NOTE.—Mixed numbers must be reduced to improper fractions, and compound fractions to simple ones, before inverting or multiplying.

EXAMPLES.

3. Divide $\frac{7}{8}$ by $\frac{3}{8}$. *Ans.* $1\frac{1}{3}$.
4. Divide $\frac{5}{8}$ by $\frac{3}{8}$. *Ans.* $1\frac{1}{3}$.
5. Divide $\frac{3}{4}$ by $\frac{7}{8}$. *Ans.* $\frac{6}{7}$.
6. Divide $\frac{5}{8}$ by $\frac{3}{8}$. *Ans.* $\frac{5}{3}$.
7. Divide $\frac{7}{12}$ by $\frac{5}{8}$. *Ans.* $\frac{2}{3}\frac{1}{2}$.
8. Divide $3\frac{3}{8}$ by $\frac{5}{8}$. *Ans.* $6\frac{3}{5}$.
9. Divide $4\frac{1}{6}$ by $\frac{9}{16}$. *Ans.* $4\frac{2}{9}$.
10. Divide $3\frac{3}{8}$ by $1\frac{1}{4}$. *Ans.* $2\frac{1}{4}$.
11. Divide $5\frac{3}{4}$ by $2\frac{3}{8}$. *Ans.* $2\frac{1}{8}$.
12. Divide $\frac{3}{4}$ of $\frac{3}{4}$ by $\frac{5}{8}$. *Ans.* $\frac{3}{5}$.
13. Divide $\frac{3}{8}$ of $\frac{1}{2}$ by $\frac{5}{8}$. *Ans.* $\frac{1}{2}\frac{3}{5}$.
14. Divide $\frac{5}{7}$ by $\frac{1}{3}$ of $\frac{2}{3}$. *Ans.* $5\frac{5}{14}$.
15. Divide $\frac{1}{10}$ by $\frac{2}{3}$ of $\frac{3}{4}$. *Ans.* $2\frac{9}{20}$.
16. Divide $9\frac{1}{8}$ by $2\frac{1}{8}$. *Ans.* 4.
17. Divide $5\frac{1}{4}$ by $1\frac{3}{4}$. *Ans.* 3.
18. Divide $32\frac{3}{8}$ by $3\frac{1}{8}$. *Ans.* $10\frac{5}{4}$.
19. At $4\frac{3}{4}$ dollars per yard, how many yards of broadcloth could you buy for $74\frac{3}{4}$ dollars? *Ans.* 16 yards.
20. If a bushel of apples cost $2\frac{1}{4}$ dollars, how many bushels could be bought for $60\frac{3}{4}$ dollars? *Ans.* 27 bushels.
21. A gentleman purchased a farm for $3412\frac{1}{2}$ dollars, paying $12\frac{1}{2}$ dollars per acre; of how many acres did the farm consist? *Ans.* 273 acres.
22. How many yards of calico, at $\frac{1}{8}$ of a dollar per yard, could be purchased for $\frac{1}{8}$ of a dollar? *Ans.* $7\frac{1}{2}$ yards.

ART. 127. Examples like the following frequently occur, which can be more satisfactorily solved by *Analysis* than by any particular system of Rules.

Ex. 1. If $\frac{5}{8}$ of a yard of muslin cost 30 cents, how much would a whole yard cost?

Analysis. If 5 *ninths* of a yard cost 35 cents, one *ninth* of

a yard would cost *one fifth* of 35 cents, which will be 7 cents; and if *one ninth* cost 7 cents, 9 *ninths*, or a whole yard, would cost 9 times 7 cents, which will be 63 cents. *Ans.* 63 cents.

NOTE.—We would again suggest to the teacher to make his pupils *write* each step on the slate or blackboard, in a series of “Equations.”

The example just analyzed would stand thus :

OPERATION.

$$\frac{5}{7} = 35 \text{ cents.}$$

$$\frac{1}{9} = 7 \quad “$$

$$\frac{2}{3} = 63 \quad “$$

2. If $\frac{7}{8}$ of a number is 42, what is the number?
Ans. 48.
3. If $\frac{4}{5}$ of a pound of rasins cost 20 cents, what would a pound be worth?
Ans. 25 cents.
4. If $\frac{5}{8}$ of a man's age be 40 years, what is his age?
Ans. 56 years.
5. If $\frac{3}{10}$ of a barrel of flour last a family 15 days, how long would a barrel last?
Ans. 50 days.
6. A carpenter received 42 dollars for $\frac{5}{8}$ of a month's work how much would he have received for the whole month?
Ans. 49 dollars.
7. In a certain school 42 pupils study arithmetic, and they are $\frac{7}{10}$ of the whole school; how many pupils are there?
Ans. 60 pupils.
8. A farmer has 70 sheep in one pasture, and they are $\frac{5}{12}$ of his entire flock; how many sheep has he? *Ans.* 168 sheep.
9. Four ninths of the distance from Augusta to Atlanta is 76 miles; what is the distance between these cities?
Ans. 171 miles.
10. Four sevenths of a man's life, amounting to 48 years, was spent in Virginia; at what age did he die?
Ans. 84 years.

11. Nine tenths of a grist-mill was valued at 8775 dollars; what was the value of the whole mill? *Ans.* 9750 dollars.

12. If $\frac{7}{11}$ of an acre of land sell for 126 dollars, for how much would an acre sell? *Ans.* 198 dollars.

13. A drover at Cincinnati sold 224 hogs, which were $\frac{7}{18}$ of his whole number; how many hogs had he at first? *Ans.* 480 hogs.

14. A gentleman owning $\frac{3}{8}$ of a factory sold $\frac{4}{9}$ of what he owned for 34976 dollars; what was the value of the whole factory? *Ans.* 65580 dollars.

15. If $\frac{3}{8}$ of $\frac{3}{8}$ of $\frac{5}{7}$ of a certain number be 1446, what is the number? *Ans.* 5061.

16. At an election 1344 persons voted for the successful candidate, and he received $\frac{6}{11}$ of all the votes; how many persons voted? *Ans.* 2464 persons.

17. If $\frac{3}{4}$ of a pound of tea be worth 72 cents, what would be the value of $\frac{1}{12}$ of a pound?

Analysis. If 3 *fourths* of a pound be worth 72 cents, one *fourth* of a pound would be worth *one third* of 72 cents, which is 24 cents; and if one *fourth* be worth 24 cents, 4 *fourths*, or a whole pound, would cost 4 *times* 24 cents, which will be 96 cents; then, one *twelfth* of a pound would be worth *one twelfth* of 96 cents, which will be 8 cents, and 7 *twelfths* will be worth 7 *times* 8 cents, which will be 56 cents. *Ans.* 56 cents.

NOTE.—This example might be written on the board thus :

OPERATION.

$$\frac{3}{4} = 72 \text{ cents.}$$

$$\frac{1}{4} = 24 \text{ "}$$

$$\frac{4}{4} = 1 = 96 \text{ "}$$

$$\frac{1}{12} = 8 \text{ "}$$

$$\frac{7}{12} = 56 \text{ cents. } \textit{Ans.}$$

18. If $\frac{3}{4}$ of a yard of cloth cost 108 cents, what would be the cost of $\frac{5}{8}$ of a yard? *Ans.* 105 cents.

19. If $\frac{2}{3}$ of a number be 80, how much would $\frac{1}{4}$ of it be?

Ans. 126.

20. If $\frac{1}{4}$ of a bushel of oats cost 75 cents, what would be the cost of $\frac{3}{4}$ of a bushel?

Ans. 62 $\frac{1}{2}$ cents.

21. *Seven ninths* of 540 is $\frac{2}{3}$ of what number?

Ans. 560.

22. If the rent of a house for $\frac{5}{6}$ of a year be 330 dollars, how much would it be for $\frac{2}{11}$ of a year?

Ans. 324 dollars.

23. *Three fifths* of 560 is $\frac{1}{7}$ of what number?

Ans. 432

ART. 128. *To reduce complex fractions to simple ones.*

NOTE.—Complex fractions present the subject of division of fractions under all its different forms, and therefore admit of the same analysis.

Ex. 1. Reduce $\frac{2\frac{2}{3}}{3\frac{2}{3}}$ to a simple fraction.

Analysis. $\frac{2\frac{2}{3}}{3\frac{2}{3}} = 2\frac{2}{3} \div 3\frac{2}{3} = \frac{8}{3} \div \frac{11}{3} = \frac{8}{3} \times \frac{3}{11} = \frac{8}{11} = \frac{29}{11}.$

Hence, to reduce a complex fraction to a simple one,

Divide the numerator by the denominator.

EXAMPLES.

2. Reduce $\frac{3\frac{1}{2}}{5\frac{1}{2}}$ to a simple fraction. *Ans.* $\frac{7}{11}.$

3. Reduce $\frac{\frac{5}{7}}{2\frac{2}{3}}$ to a simple fraction. *Ans.* $\frac{15}{28}.$

4. Reduce $\frac{\frac{7}{8}}{3\frac{2}{3}}$ to a simple fraction. *Ans.* $\frac{7}{36}.$

5. Reduce $2\frac{1}{2} \times \frac{4}{5} \div 5\frac{1}{2} \times 6\frac{3}{4}.$ *Ans.* $\frac{1}{18}.$

6. Reduce $2\frac{2}{3} \times \frac{5}{8} \times 1\frac{1}{2} \div \frac{3}{4} \times 8\frac{1}{2}.$ *Ans.* $\frac{4}{9}.$

7. Reduce $3\frac{1}{2} \times 2\frac{2}{5} \div 2\frac{1}{10} \times 2\frac{2}{3}.$ *Ans.* $1\frac{1}{2}.$

8. Reduce $\frac{7}{9} \times \frac{8\frac{1}{2}}{10} \div \frac{5}{8} \times \frac{12\frac{1}{2}}{18\frac{1}{2}}.$ *Ans.* $1\frac{5}{8}.$

9. Reduce $\frac{10\frac{1}{2}}{12\frac{1}{2}} \times 9 \div \frac{5\frac{1}{2}}{7} \times 8\frac{3}{5}.$ *Ans.* $1\frac{1}{2}.$

10. Simplify $\frac{3\frac{1}{2}}{5\frac{2}{3}} \times \frac{4\frac{2}{3}}{4\frac{2}{3}} \times \frac{5\frac{1}{2}}{14} \times \frac{6}{2\frac{1}{2}}$. *Ans.* $\frac{1}{2}\frac{5}{8}$.
11. Simplify $\frac{5}{8} \times \frac{4\frac{2}{3}}{2\frac{1}{4}} \div \frac{4}{9} \times \frac{5\frac{1}{2}}{8}$. *Ans.* $4\frac{1}{8}$.
12. Simplify $4\frac{2}{3} \times \frac{3\frac{1}{2}}{7} \div \frac{4\frac{1}{2}}{6\frac{1}{4}} \times 1\frac{2}{3}$. *Ans.* $2\frac{1}{12}$.
13. Simplify $\frac{2\frac{1}{2} \times 4\frac{2}{3}}{6\frac{1}{2} \times 1\frac{2}{5}}$. *Ans.* $2\frac{3}{8}\frac{2}{3}$.
14. Simplify $\frac{\frac{2}{3} \times \frac{8}{9} \div 4 \times \frac{1}{6}}{3\frac{1}{2} \times \frac{2}{3} \div \frac{1}{4} \times 7\frac{1}{2}}$. *Ans.* $\frac{8}{9}$.
15. Simplify $\frac{5}{16} \times \frac{5\frac{1}{2}}{6\frac{1}{4}} \div \frac{5}{9} \times \frac{9\frac{3}{4}}{12\frac{1}{2}}$. *Ans.* $\frac{1}{2}\frac{5}{8}$.
16. Reduce $\frac{\frac{9}{11} \times \frac{3}{16}}{\frac{1}{9} \times \frac{5}{16}}$. *Ans.* $1\frac{3}{16}$.
17. Reduce $\frac{\frac{1}{3} \times \frac{8}{9} \times 2\frac{1}{2}}{\frac{8}{9} \times \frac{1}{17} \times 5\frac{1}{2}}$. *Ans.* $\frac{2}{17}$.
18. Reduce $\frac{\frac{7}{16} \times \frac{2}{3} \times 5\frac{1}{2}}{\frac{7}{11} \times \frac{2}{3} \times 3\frac{1}{2}}$. *Ans.* $\frac{2}{11}$.
19. Simplify $\frac{8}{7} \times 2\frac{2}{3} + \frac{2}{3} \times 6\frac{3}{4}$. *Ans.* $\frac{8}{7}\frac{2}{3}$.
20. Simplify $\frac{6\frac{9}{10}}{9\frac{1}{8}} \times \frac{3\frac{1}{2}}{5\frac{1}{4}} \times \frac{2\frac{1}{2}}{2\frac{3}{8}}$. *Ans.* $\frac{2}{3}\frac{7}{8}$.
21. Simplify $3\frac{2}{3} \times 1\frac{2}{3} \times 2\frac{1}{4} + 2\frac{1}{2} \times 1\frac{2}{3}$. *Ans.* $3\frac{2}{3}$.
22. Reduce $\frac{3\frac{2}{3} \text{ of } 4\frac{2}{3}}{7\frac{1}{2} + 1\frac{1}{4}}$. *Ans.* $2\frac{1}{4}$.
23. Reduce $\frac{4\frac{4}{5} + 2\frac{5}{8}}{3\frac{2}{3} \text{ of } 2\frac{3}{4}}$. *Ans.* $\frac{2}{3}$.
24. Reduce $\frac{\frac{7}{10} \text{ of } 3\frac{2}{3} \text{ of } 2\frac{1}{2}}{\frac{2}{10} \text{ of } 2\frac{2}{3} \text{ of } 4\frac{2}{3}}$. *Ans.* $1\frac{1}{4}\frac{2}{3}$.
25. Reduce $\frac{6\frac{2}{7} \div 3\frac{3}{4}}{2\frac{2}{3} + 3\frac{3}{4}}$. *Ans.* $\frac{5}{17}$.
26. Reduce $(2\frac{2}{3} + 3\frac{2}{3}) \div (3\frac{2}{3} + 4\frac{1}{2})$. *Ans.* $\frac{2}{3}\frac{2}{3}$.
27. Reduce $(\frac{1}{3} + \frac{5}{6}) \times (2\frac{1}{2} + 3\frac{3}{4})$. *Ans.* $8\frac{3}{4}$.
28. Reduce $(\frac{1}{3} + \frac{1}{4} + \frac{1}{6}) \div (\frac{1}{2} + \frac{1}{3} + \frac{1}{15})$. *Ans.* $1\frac{1}{15}$.

PROMISCUOUS EXAMPLES.

1. Reduce $1\frac{1}{8}$ to its lowest terms. Ans. $\frac{9}{8}$
2. Reduce $12\frac{5}{8}$ to an improper fraction. Ans. $1\frac{1}{8}$
3. Reduce $\frac{768}{1152}$ to its lowest terms. Ans. $\frac{2}{3}$
4. Reduce $2\frac{5}{14}$ to a mixed number. Ans. $18\frac{5}{14}$
5. Change $17\frac{7}{11}$ to an improper fraction. Ans. $1\frac{8}{11}$
6. Reduce 18 to *ninths*. Ans. —
7. Reduce 12 to *sixteenths*. Ans. —
8. Reduce $\frac{5}{7}$ of $\frac{3}{10}$ to a simple fraction. Ans. $\frac{1}{14}$
9. Reduce $\frac{3}{4}$, $\frac{2}{5}$, $\frac{9}{10}$ and $\frac{5}{6}$ to a common denominator. Ans. $\frac{45}{60}$, $\frac{24}{60}$, $\frac{54}{60}$, $\frac{50}{60}$
10. What is the sum of $\frac{1}{8}$, $\frac{3}{4}$ and $\frac{5}{8}$? Ans. $2\frac{1}{8}$
11. A merchant has 3 pieces of cloth, the first measuring $32\frac{1}{2}$ yards, the second $44\frac{1}{4}$ yards, and the third $35\frac{5}{8}$ yards; how many yards in the three pieces? Ans. $112\frac{3}{8}$ yards.
12. From $\frac{5}{7}$ take $\frac{2}{3}$. Ans. $\frac{1}{21}$
13. From $5\frac{3}{4}$ subtract $3\frac{5}{8}$. Ans. $1\frac{1}{2}$
14. From a piece of cloth containing $35\frac{1}{2}$ yards, $18\frac{3}{4}$ yards were cut off; how many yards were left? Ans. $16\frac{7}{2}$ yards.
15. Thomas has 231 walnuts, and Richard has $\frac{7}{11}$ as many; how many has Richard? Ans. 147 walnuts.
16. If a yard of broadcloth cost $3\frac{5}{8}$ dollars, what would be the cost of 15 yards? Ans. $54\frac{3}{8}$ dollars.
17. What would be the cost of $5\frac{3}{4}$ yards of cassimere at $3\frac{1}{2}$ dollars per yard? Ans. $18\frac{5}{2}$ dollars.
18. If $\frac{2}{3}$ of a certain number be 135, what is the number? Ans. 195.
19. A gentleman being asked how much money he had in his pocket, replied that $\frac{3}{4}$ of $\frac{4}{5}$ of the amount made 72 dollars; how much had he? Ans. 135 dollars.
20. If $\frac{7}{18}$ of a number be 126, what is $\frac{5}{9}$ of the number? Ans. 130.

21. At $\frac{1}{2}$ of a dollar per bushel, what would be the cost of $9\frac{1}{2}$ bushels of wheat? *Ans.* $8\frac{1}{2}$ dollars.

22. If $4\frac{1}{2}$ yards of calico cost 91 cents, how much is that per yard? *Ans.* 21 cents.

23. A gentleman paid 36 dollars for barley at the rate of $\frac{1}{2}$ of a dollar per bushel; how many bushels did he buy? *Ans.* 40 bushels.

24. Reduce $\frac{5}{\frac{1}{2}}$ to a simple fraction. *Ans.* $2\frac{5}{2}$.

25. Reduce $\frac{4\frac{3}{4}}{7\frac{1}{2}}$ to a simple fraction. *Ans.* $\frac{3}{4}$.

26. How many coats, each containing $4\frac{1}{2}$ yards, could be made out of $59\frac{1}{2}$ yards of cloth? *Ans.* 14 coats.

DECIMAL FRACTIONS.

ART. 129. In Common Fractions the unit is divided into *halves, thirds, fourths, fifths*, and so on; but there is a class of fractions in which the subdivisions of the unit follow a *regular* order, increasing and decreasing in the same manner as whole numbers.

ART. 130. If a unit be divided into *ten* equal parts, each part is *one tenth*, or expressed thus, $\frac{1}{10}$; 3 of the parts are 3 *tenths*, or thus, $\frac{3}{10}$; 7 parts are 7 *tenths*, or thus, $\frac{7}{10}$. Now, if a *tenth* be divided into *ten* equal parts, one of the parts will be *one hundredth*, or expressed thus, $\frac{1}{100}$; 5 parts will be 5 *hundredths*, or $\frac{5}{100}$. In like manner, if a *hundredth* be divided into *ten* equal parts, *one* of the parts will be *one thousandth*, or thus, $\frac{1}{1000}$; 3 parts will be 3 *thousandths*, or $\frac{3}{1000}$, etc.

ART. 131. Such fractions as these, in which the unit is divided into *tenths, hundredths, thousandths, ten-thousandths*, etc., are called *Decimal Fractions*. They are called *decimal*

from the Latin word *decem*, signifying *ten*, because they increase and decrease in a *tenfold* ratio. In *whole numbers* we proceed from units *up*, through *tens*, *hundreds*, *thousands*, and so on; in *decimals* we go from units *down*, through *tenths*, *hundredths*, *thousandths*, etc.

ART. 132. A DECIMAL FRACTION is a fraction whose *denominator* is always 10, 100, 1000, or some number obtained by multiplying 10 into itself.

The *denominator* of a decimal fraction is not usually written. Decimal fractions are commonly called *decimals*.

ART. 133. A DECIMAL FRACTION is usually expressed by writing the *numerator* with a point or period (.) before it.

Thus, $\frac{1}{10}$	is written	.1
$\frac{3}{10}$	"	.3
$\frac{5}{10}$	"	.5
$\frac{1}{100}$	"	.01
$\frac{4}{100}$	"	.04
$\frac{25}{100}$	"	.25
$\frac{1}{1000}$	"	.001
$\frac{25}{1000}$	"	.025
$\frac{475}{1000}$	"	.475
$\frac{1}{10000}$	"	.0001
$\frac{8}{10000}$	"	.0008
$\frac{85}{10000}$	"	.0035
$\frac{524}{10000}$	"	.0524
$\frac{7825}{10000}$	"	.7825

The point placed before decimals is called the *decimal point*; it is sometimes called *separatrix*, because it separates the fractional parts from the whole numbers.

ART. 134. The *first* place on the right of the decimal point is that of *tenths*; the *second* place is that of *hundredths*; the *third* is that of *thousandths*; the *fourth* is that of *ten-thou-*

In the same manner let the student read the following decimals :

.7	.005	.60735	.3872
.54	.4256	.07243	.506
.86	.0875	.00487	.29
.09	.0048	.00028	.035
.325	.0009	.00007	.4276
.093	.0528	.070256	.35794
.467	.48976	.05308	.0008

ART. 137. An expression made up of a whole number and a decimal is called a *mixed* number. Thus, 7.45 is a *mixed* number, because it consists of the whole number 7 and the decimal .45 ; it is read, 7 and 45 *hundredths*.

Let the student *write* the fractional parts of the following *mixed* numbers as decimals :

$8\frac{3}{10}$.	Ans. 8.3.	$253\frac{78}{100}$.	Ans. —.	$83\frac{798}{10000}$.	Ans. —.
$5\frac{63}{100}$.	Ans. 5.63.	$28\frac{456}{1000}$.	Ans. —.	$417\frac{38}{100000}$.	Ans. —.
$34\frac{5}{10}$.	Ans. —.	$18\frac{23}{1000}$.	Ans. —.	$72\frac{63925}{1000000}$.	Ans. —.
$78\frac{54}{100}$.	Ans. —.	$48\frac{6}{1000}$.	Ans. —.	$15\frac{528}{1000000}$.	Ans. —.
$56\frac{7}{100}$.	Ans. 56.07.	$36\frac{792}{1000}$.	Ans. —.	$23\frac{35}{1000000}$.	Ans. —.
$62\frac{4}{100}$.	Ans. —.	$98\frac{5216}{10000}$.	Ans. —.	$95\frac{26}{1000}$.	Ans. —.

ART. 138. Write the following as decimals :

8 Tenths.	Ans. .8.	9243 Ten-thousandths	Ans.
47 Hundredths.	Ans. .47.	856 Ten-thousandths.	Ans.
9 Hundredths.	Ans. .09.	45 Ten-thousandths	Ans.
6 Tenths.	Ans. —.	7 Ten-thousandths.	Ans.
752 Thousandths.	Ans. —.	69843 Hundred-thousandths	Ans.
89 Thousandths.	Ans. —.	387 Hundred-thousandths.	Ans.
7 Thousandths.	Ans. —.	783 Millionths.	Ans.
364 Thousandths.	Ans. —.	37 Thousandths.	Ans.

ART. 139 Read the following decimals :

.8	.052	.6548	9.07
.59	.007	.0923	8.3752
.03	5.37	9.72	26.54975
.425	8.763	6.387	39.003

ART. 140. A cipher placed on the *right* of a decimal does not alter the value; thus, .5 is the same as .50, or .500; for we may write the first fraction as $\frac{5}{10}$, the second as $\frac{50}{100}$, and the third as $\frac{500}{1000}$; and by reducing each of these fractions to its lowest terms, we get $\frac{1}{2}$ as the value of each.

Any number of ciphers, therefore, may be placed at the right of a decimal, without altering its value.

This principle very much facilitates the operation of *addition* and *subtraction* of decimals. All that is necessary to reduce *decimals* to a common denominator is simply to annex ciphers to the several fractions until they shall all have the *same* number of decimal *places*, in which case their denominators would be the same. Thus, suppose we were required to add 7.8, 5.64 and 9.375. As the last fraction has *three* decimal places, we give *three* places to the other two, by annexing *two* ciphers to the first, and *one* cipher to the second. The numbers will then stand 7.800, 5.640 and 9.375, being all reduced to *thousandths*, and now ready for addition.

ART. 141. A cipher placed on the *left* of a decimal *diminishes* its value *ten times*; thus,

9	is equal to the common fraction	$\frac{9}{10}$,
.09	"	" $\frac{9}{100}$,
.009	"	" $\frac{9}{1000}$,
.0009	"	" $\frac{9}{10000}$, etc.

Hence, we see that the *value* of any decimal figure depends upon its distance from the decimal point.

We also learn from the same that a decimal is divided by

10 by removing the decimal point one place to the **LEFT**; or divided by 100 by removing the decimal point two places to the *left*; by 1000, by removing the decimal point three places to the *left*; and so on.

It also naturally follows that by removing the decimal point one place to the *right*, we *multiply* the decimal by 10; by removing it two places to the *right*, we *multiply* the decimal by 100, and so on.

DECIMAL TABLE.

1 unit	is equal to 10 tenths,
1 tenth	" 10 hundredths,
1 hundredth	" 10 thousandths,
1 thousandth	" 10 ten-thousandths, etc.

ART. 142. Since decimals increase and decrease in a *tenfold* ratio, just as whole numbers, they may be added, subtracted, multiplied or divided in the same way as whole numbers, the only difference being in the management of the decimal point.

ADDITION OF DECIMALS.

ART. 143. Ex. 1. What is the sum of 6.58, 8.369 and 9.785?

Analysis. We write the numbers, placing *tenths* under *tenths*, *hundredths* under *hundredths*, etc. We begin with the *lowest* denomination, which, in this example, is *thousandths*, and adding the numbers in this column we get 14 *thousandths* = 1 *hundredth* and 4 *thousandths*. We write the 4 *thousandths* in the *thousandths*' place, and carry the 1 *hundredth* to the *hundredths*' column. Adding the *hundredths*, with the 1 we carry, we get 23 *hundredths* = 2 *tenths* and 3 *hundredths*; we write the 3 *hundredths* in the *hundredths*'

OPERATION.

6.58
8.369
9.785

Ans. 24.734

column, and carry the 2 *tenths* to the *tenths'* column. Then adding the *tenths*, carrying the 2 *tenths*, we get 17 *tenths* = 1 *unit* and 7 *tenths*; we write the 7 *tenths* in the column of *tenths*, and carry the 1 *unit* to the *units'* column. We then add the *units*, carrying the 1 *unit*, and get 24 *units* = 2 *tens* and 4 *units*, each of which we write in their proper columns.

Ans. 24.734

From the foregoing explanation we get the

RULE.

Write the numbers so that tenths may stand under tenths, hundredths under hundredths, and so on. Begin with the lowest denomination and add as in whole numbers, placing the decimal point directly under the decimal point above.

EXAMPLES.

2. What is the sum of 7.45, 8.63, 5.25 and 7.42 ?

Ans. 28.75.

3. Add together 18.37, 16.7, 14.375 and 23.504.

Ans. 72.949.

4. Add together 6.428, 23.73, 18.456 and 15.907.

Ans. 64.521.

5. Find the sum of 23.564, 18.4217, 6.25 and 13.5208.

Ans. 61.7565.

6. Add together 28.256, 427.48, 45.309 and 32.5346

Ans. 533.5796.

- 7 Add 15.3275, 9.46817, 25.407 and 217.28527.

Ans. 267.48794

- 8 Find the sum of 8 and 6 tenths, 36 and 59 hundredths, 84 and 65 hundredths and 24 and 369 thousandths.

Ans. 104.209.

9. What is the sum of 23 and 9 tenths, 17 and 58 hundredths, 35 and 608 thousandths, and 324 and 8 hundredths ?

Ans. 401.168.

10. A grocer has four barrels of syrup measuring respectively, 28.7 gallons, 32.625 gallons, 34.75 gallons and 36.45 gallons; how many gallons in the four barrels?

Ans. 132.525 gallons.

11. A merchant bought 4 firkins of butter, weighing as follows, viz., 62.75 pounds, 53.8 pounds, 48.675 pounds and 52.7 pounds; how many pounds in the 4 firkins?

Ans. 217.925 pounds.

12. John bought a knife for 2.25 dollars, a coat for 8.5 dollars, a vest for 3.625 dollars and a cap for 3.875 dollars; how many dollars would pay for his purchases?

Ans. 18.25 dollars.

SUBTRACTION OF DECIMALS.

ART. 144. Ex. 1. From 54.67 take 32.328.

Analysis. The decimal places in the *minuend* being fewer than those in the *subtrahend*, we suppose a cipher to be annexed to the *minuend*, which makes the number of decimal places the same

OPERATION.

54.67	Minuend.
32.328	Subtrahend.
<u>22.342</u>	Remainder.

in each, thereby reducing them to the same denominator; we then subtract precisely as in whole numbers, being careful to place the decimal point in the remainder directly under the decimal point above.

Hence we have the following

RULE.

Write the less number under the greater, placing tenths under tenths, hundredths under hundredths, etc. Begin with the lowest denomination and subtract as in whole numbers, placing the decimal point in the remainder directly under the decimal point above.

EXAMPLES.

2. From 75.47 take 39.23. *Ans* 36.24.
3. From 425.358 take 272.183. *Ans.* 153.175.
4. Subtract 58.875 from 96.25. *Ans.* 37.375.
5. Take 235.78 from 527.24. *Ans.* 291.46.
6. From 425.824 take 226.75. *Ans.* 199.074.
7. Subtract 47.508 from 125.26. *Ans.* 77.752
8. From 43 and 4 tenths take 25 and 67 hundredths.
Ans. 17.73.
9. Take 53 and 647 thousandths from 75 and 4 tenths.
Ans. 21.753.
10. From a cask of wine containing 43.5 gallons a merchant sold 28.75 gallons; how much was left?
Ans. 14.75 gallons.
11. James had 23.25 dollars; after paying for a coat worth 13.875 dollars, how much money had he left?
Ans. 9.375 dollars.
12. A farmer owning 654 acres of land, at one time sold off 225.75 acres, and at another time, 182.5 acres; how many acres remained?
Ans. 245.75 acres.
13. A gentleman having 756 dollars deposited in a bank, drew out 325.25 dollars at one time, and 248.625 dollars at another; how much remained in bank?
Ans. 182.125 dollars

MULTIPLICATION OF DECIMALS.

ART. 145. Ex. 1. Multiply .9 by .5.

Analysis. .9 may be written thus, $\frac{9}{10}$; and .5 may be written $\frac{5}{10}$. From what we have learned in *Common Fractions*, we know that $\frac{9}{10}$ multiplied by $\frac{5}{10}$ gives $\frac{45}{100}$ as the product; or, expressing it decimally,
45.

OPERATION.

.9 Multiplicand
.5 Multiplier.
—
45 Product

Ans. 45.

Here we see that *tenths* multiplied by *tenths* will give *hundredths*; but to express *hundredths* requires *two* decimal places, and therefore we cut off *two* places for decimals in this example.

2. What is the product of .75 multiplied by .7?

Analysis. .75 may be written $\frac{75}{100}$; and .7 may be written $\frac{7}{10}$; and $\frac{75}{100}$ multiplied by $\frac{7}{10}$ gives $\frac{525}{1000}$; or, expressed decimally, .525. *Ans.* .525.

OPERATION.

.75 Multiplicand.

.7 Multiplier.

.525 Product.

Here we see that *hundredths* multiplied by *tenths* will give *thousandths*, but to express *thousandths* requires *three* decimal places; and this alone is the reason why in this example we cut off *three* places for decimals.

3. What is the product of .45 multiplied by .09?

Analysis. .45 may be written $\frac{45}{100}$; and .09 may be written $\frac{9}{100}$; but $\frac{45}{100}$ multiplied by $\frac{9}{100}$ gives $\frac{405}{10000}$, which expressed decimally, is .0405.

OPERATION.

.45 Multiplicand.

.09 Multiplier.

.0405 Product.

Here we see that *hundredths* multiplied by *hundredths* will give *ten-thousandths*; but to express *ten-thousandths* requires *four* decimal places, and therefore in this example it was necessary to prefix* a cipher, in order that the product might be *ten-thousandths*.

These examples make it clear that there will *always* be as many decimal places in the product as there are in the multiplicand and multiplier counted together. Hence the

RULE.

Multiply as in whole numbers, and in the product point off as many figures from the RIGHT for decimals as there are

* Prefix means to place before; i. e., on the left.

decimal places in both the multiplicand and the multiplier prefixing ciphers when necessary to supply the deficiency.

EXAMPLES.

- | | |
|--|------------------------------|
| 4. Multiply 4.5 by 5.7. | <i>Ans.</i> 25.65. |
| 5. Multiply 8.47 by 4.6. | <i>Ans.</i> 38.962. |
| 6. Multiply 4.75 by 5.35. | <i>Ans.</i> 25.4125. |
| 7. Multiply 15.225 by 6.7. | <i>Ans.</i> 102.0075. |
| 8. Multiply 8.625 by 4.25. | <i>Ans.</i> 36.65625. |
| 9. Multiply 324.7 by .75. | <i>Ans.</i> 243.525. |
| 10. Multiply .357 by .27. | <i>Ans.</i> .09639. |
| 11. Multiply .48 by .009. | <i>Ans.</i> .00432. |
| 12. Multiply 35.7 by 3.25. | <i>Ans.</i> 116.025. |
| 13. Multiply .007 by .09. | <i>Ans.</i> .00063. |
| 14. Multiply 16.5 by 16.5. | <i>Ans.</i> 272.25. |
| 15. What would be the cost of 17.25 yards of velvet at 4.5 dollars per yard? | <i>Ans.</i> 77.625 dollars. |
| 16. A cubic foot of water weighs 62.5 pounds; what would be the weight of 9.75 cubic feet? | <i>Ans.</i> 609.375 pounds. |
| 17. If an acre of land cost 7.875 dollars, what would be the cost of 9.5 acres? | <i>Ans.</i> 74.8125 dollars. |
| 18. Multiply 45 thousandths by 17 hundredths. | <i>Ans.</i> .00765. |
| 19. Multiply 7.854 by 100. | <i>Ans.</i> 785.4. |
| 20. Multiply .0576 by 1000. | <i>Ans.</i> 57.6 |

DIVISION OF DECIMALS.

ART. 146. Ex. 1. Divide 15.75 by 4.5.

Analysis. Here the dividend is *hundredths* and the divisor is *tenths*. Now, if *hundredths* be divided by *units*, the quotient will evidently be *hundredths*, and if *hundredths* be divided by *tenths*, the quotient will be 10 times as great

OPERATION.

$$\begin{array}{r}
 4.5 \overline{) 15.75} (3.5 \\
 \underline{13.5} \\
 2.25 \\
 \underline{2.25} \\
 0
 \end{array}$$

as when divided by units; *i. e.*, it must be tenths, but to express *tenths* requires *one* decimal place; hence in this example we cut off *one* decimal figure. *Ans.* 3.5.

2. Divide 87.75 by 3.25.

Analysis. Here there are *two* decimal places in the dividend and *two* in the divisor, and the quotient is a *whole* number. This ought to be so, for it is plain that 3 *hundredths* will be contained in 9 *hundredths* just as often as 3 *units* are contained in 9 *units*; therefore, whenever the decimal places in the dividend and the divisor are equal, the quotient will *always* be a whole number.

OPERATION.

$$\begin{array}{r} 3.25)87.75(27 \\ \underline{65.0} \\ 22.75 \\ \underline{22.75} \end{array}$$

Ans. 27.

3. Divide 56.4 by .24.

Analysis. In this example, as there were *two* decimal places in the divisor and only *one* in the dividend, we made the decimal places equal in each by annexing a cipher to the dividend, which did not alter the value of the decimal part (Art. 140). The quotient, therefore, is a whole number, just as was explained in Example 2. *Ans.* 235.

OPERATION.

$$\begin{array}{r} .24)56.40(235 \\ \underline{48} \\ 84 \\ \underline{72} \\ 120 \\ \underline{120} \end{array}$$

4. Divide 18.144 by 324.

Analysis. Here the dividend is *thousandths* and the divisor is *units*. *Thousandths* divided by *units* will evidently give *thousandths*; but to express *thousandths* requires *three* decimal places, and since we get but *two* figures in the quotient, a cipher must be *prefixed* to make up the *three* decimals required. *Ans.* .056.

OPERATION.

$$\begin{array}{r} 324)18.144(.056 \\ \underline{16.20} \\ 1.944 \\ \underline{1.944} \end{array}$$

The four examples just given embrace all the cases that can occur in the division of decimals. The explanations *above given* are summed up in the following

RULE.

I Divide as in whole numbers, and point off from the right of the quotient as many figures for decimals as the decimal places in the dividend exceed those in the divisor. If there are not enough figures to point off, supply the deficiency by prefixing ciphers.

II. When there are more decimal places in the DIVISOR than in the DIVIDEND, make the places equal by annexing ciphers to the dividend before dividing. The quotient in such cases will be a whole number.

III. After all the figures in the dividend have been brought down, if there be a remainder, ciphers may be annexed, and the division continued as far as may be desired. The ciphers so annexed must be regarded as decimals belonging to the dividend.

EXAMPLES.

- | | |
|--|---------------------------|
| 5. Divide 26.075 by 3.5. | <i>Ans.</i> 7.45. |
| 6. Divide 26.88 by 5.6. | <i>Ans.</i> 4.8. |
| 7. Divide 256.5 by 4 5. | <i>Ans.</i> 57. |
| 8. Divide 182.75 by 4.25. | <i>Ans.</i> 43. |
| 9. Divide 883.2 by 3.45. | <i>Ans.</i> 256. |
| 10. Divide 29.945 by 565. | <i>Ans.</i> .053. |
| 11. Divide 33.995 by 6.5. | <i>Ans.</i> 5.23. |
| 12. Divide 270 by .72. | <i>Ans.</i> 375. |
| 13. Divide 2.6368 by 412. | <i>Ans.</i> .0064. |
| 14. Divide 4.745 by .65. | <i>Ans.</i> 7.3. |
| 15. Divide 119.4 by 6.94. | <i>Ans.</i> 17.2046 +. |
| 16. Divide 540.5 by 19.78. | <i>Ans.</i> 27.3255 +. |
| 17. A man traveled 471.15 miles in 13.5 days; what was his average travel per day? | <i>Ans.</i> 34.9 miles. |
| 18. A gentleman paid 86.25 dollars for 23 days' board at a hotel; how much did he pay per day? | <i>Ans.</i> 3.75 dollars. |

19. A merchant invested 74.25 dollars in cassimere, paying 2.25 dollars per yard; how many yards did he buy?

Ans. 33 yards.

20. A farmer sold 45 bushels of oats for 33.75 dollars; what was the price per bushel?

Ans. .75 dollars.

21. How many sacks would be required to hold 396 bushels of wheat, each sack holding 2.75 bushels?

Ans. 144 sacks.

22. A grocer bought 327 oranges for 21.255 dollars; how much were they apiece?

Ans. .065 dollars

REDUCTION OF DECIMALS.

CASE I.

ART. 147. *To reduce a common fraction to a decimal.*

Ex. 1. Reduce $\frac{3}{5}$ to a decimal.

Analysis. We annex *one* cipher to the numerator, thereby changing the 3 *units* into 30 *tenths*, and 30 *tenths* divided by 5 gives 6 *tenths* for a quotient, which, expressed decimally, is .6.

OPERATION.

$$\begin{array}{r} 5 \overline{)3.0} \\ \underline{.6} \end{array}$$

Ans. .6.

2. Reduce $\frac{3}{4}$ to a decimal.

Analysis. In this example we annex *two* ciphers, thereby changing the numerator into 300 *hundredths*, and 300 *hundredths* divided by 4 gives 75 *hundredths* for a quotient, which, expressed decimally, is .75.

OPERATION

$$\begin{array}{r} 4 \overline{)3.00} \\ \underline{.75} \end{array}$$

Ans. .75.

From these illustrations we derive the

RULE.

Annex ciphers to the numerator and divide by the denominator, cutting off as many places for decimals as there are ciphers annexed. Prefix ciphers to the quotient when necessary.

EXAMPLES.

Reduce the following fractions to decimals :

3. $\frac{1}{4}$.	Ans. .25.	11. $\frac{3}{4}$ of $\frac{5}{8}$.	Ans. .625.
4. $\frac{5}{8}$.	Ans. .625.	12. $\frac{2}{3}$.	Ans. .66666 +.
5. $\frac{7}{16}$.	Ans. .4375.	13. $\frac{5}{8}$.	Ans. .55555 +.
6. $\frac{2}{5}$.	Ans. .4.	14. $\frac{7}{11}$.	Ans. .636363 +.
7. $\frac{11}{16}$.	Ans. .85.	15. $12\frac{3}{4}$.	Ans. 12.375
8. $1\frac{3}{8}$.	Ans. .8125.	16. $25\frac{1}{2}$.	Ans. 25.75
9. $\frac{3}{32}$.	Ans. .09375.	17. $18\frac{1}{2}$.	Ans. 18.5.
10. $1\frac{7}{16}$.	Ans. .056.	18. $35\frac{9}{16}$.	Ans. 35.5625.

CASE II.

ART. 148. *To reduce a decimal to a common fraction.*

Ex. 1. Reduce .6 to a common fraction.

Analysis. .6 may be written thus, $\frac{6}{10}$; and reducing it to lower terms by dividing both the numerator and the denominator by 2, we have $\frac{3}{5}$ as the result. Ans. $\frac{3}{5}$.

2. Reduce .75 to its corresponding common fraction.

Analysis. .75 may be written $\frac{75}{100}$, and reducing it to its lowest terms we have $\frac{3}{4}$. Ans. $\frac{3}{4}$.

From the foregoing illustrations we derive a

RULE.

Omit the decimal point; write the given decimal over its proper denominator, and reduce this fraction to its lowest terms.

EXAMPLES.

Reduce the following decimals to common fractions :

3. .55.	Ans. $\frac{11}{20}$.	9. .025.	Ans. $\frac{1}{40}$.
4. .125.	Ans. $\frac{1}{8}$.	10. .0375.	Ans. $\frac{3}{80}$.
5. .875.	Ans. $\frac{7}{8}$.	11. .8125.	Ans. $1\frac{13}{16}$.
6. .25.	Ans. $\frac{1}{4}$.	12. .15625.	Ans. $\frac{5}{32}$.
7. .4375.	Ans. $\frac{7}{16}$.	13. .075.	Ans. $\frac{3}{40}$.
8. .225.	Ans. $\frac{9}{40}$.	14. .9375.	Ans. $1\frac{3}{8}$.

MISCELLANEOUS EXAMPLES.

1. A merchant has 4 pieces of calico, measuring respectively as follows, viz.: 33.75 yards, 42.5 yards, 35.625 yards and 44.25 yards; how many yards in the 4 pieces?

Ans. 156.125 yards.

2. A farmer sold a load of wheat for 56.375 dollars, a lot of pork for 93.75 dollars, a quantity of corn for 123.5 dollars and a ton of hay for 23.625 dollars; how much did he receive for all the articles?

Ans. 297.25 dollars.

3. A merchant has 4 casks of wine. The first contains 42.625 gallons, the second, 38.75 gallons, the third, 44.5 gallons and the fourth, 40.65 gallons; how many gallons in the 4 casks?

Ans. 166.525 gallons.

4. What is the sum of 75 and 64 *thousandths*, 43 and 9 *tenths* and 256 and 7 *hundredths*?

Ans. 375.034.

5. A farmer owned 573.8 acres of land, and sold off 328.625 acres; how much remained?

Ans. 245.175 acres.

6. What is the difference between 9 *tenths* and 9 *hundredths*?

Ans. .81.

7. Subtract 8 *thousandths* from 8 *tenths*.

Ans. .792.

8. A man had 636.25 dollars, and spent 327.375 dollars; how many dollars had he left?

Ans. 308.875 dollars.

9. If a man can travel 32.73 miles in a day, how far could he travel in 17.25 days?

Ans. 564.5925 miles.

10. If a yard of alpaca cost .875 of a dollar, what would be the cost of .75 of a yard?

Ans. .65625 dollars.

11. In a square rod there are 30.25 square yards; how many square yards in 22.625 rods?

Ans. 684.40625 rods.

12. What is the product of 35 and 7 *tenths* multiplied by 42 and 65 *thousandths*?

Ans. 1501.7205.

13. How many sacks would be required to hold 93.15 bushels of barley, each sack holding 3.45 bushels?

Ans. 27 sacks.

14. If 45 acres of land produce 1173.825 bushels of peas, what is the yield per acre? *Ans.* 26.085 bushels.

15. At .625 of a dollar per yard, how many yards of linen could be bought for 22.5 dollars? *Ans.* 36 yards.

16. A grocer bought 525 bananas for 28.875 dollars; what was the cost of the bananas apiece? *Ans.* .055 of a dollar.

17. What decimal fraction is equivalent to $\frac{1}{8}$? *Ans.* .6875.

18. What decimal fraction is equivalent to $\frac{3}{32}$? *Ans.* .09375

19. Reduce .8125 to a common fraction. *Ans.* $\frac{13}{16}$.

20. Reduce .0375 to a common fraction. *Ans.* $\frac{3}{80}$.

FEDERAL MONEY.

ARTICLE 149. The CURRENCY of a nation means its money. The national currency of the United States is called **FEDERAL MONEY**.

ART. 150. The denominations of **FEDERAL MONEY** are *eagles, dollars, dimes, cents* and *mills*. The dollar is the unit.

ART. 151. The character \$, written before any number, indicates Federal money. Thus, \$27 is read 27 dollars.

TABLE OF FEDERAL MONEY

10 mills (m.)	make	1 cent, ct.
10 cents	"	1 dime, d.
10 dimes	"	1 dollar, \$.
10 dollars	"	1 eagle, E.

ART. 152. From the above table it is seen that dimes are *tenths* of a dollar, cents are *hundredths*, and mills are *thousandths*. The dime is always written on the right of the

dollar, and separated from it by a (.) point; thus, \$9.7 represents 9 dollars and 7 dimes.

ART. 153. In the practical business of keeping accounts dollars and cents are the only denominations employed. Eagles are read as dollars, dimes are read as cents, and mills as fractions of a cent. Thus, 3 eagles and 4 dollars would be read, 34 dollars; and 6 dimes, 2 cents and 5 mills would be read, 62½ cents.

ART. 154. The first two places on the right of the decimal point being always assigned to cents, when the number of cents is less than 10 a cipher must be written in the place of dimes. Thus, 5 dollars and 6 cents would be written, \$5.06.

To express dollars and mills when there are no cents, *two* ciphers must be placed before the mills. Thus, 8 dollars and 5 mills would be written, \$8.005.

EXAMPLES.

1. Read \$43.25.

Analysis. This *may* be read 43 dollars, 2 dimes and 5 cents, or 43 dollars and 25 cents. This latter mode is the *practical* one.

2. Read \$27.06.

Analysis. This *may* be read 27 dollars, 0 dimes and 6 cents: in *practice* it is read 27 dollars and 6 cents.

3. Read \$56.375.

Analysis. This *may* be read 56 dollars, 3 dimes, 7 cents and 5 mills, or 56 dollars, 37 cents and 5 mills; *usually* it is read 56 dollars and 37½ cents.

Let the pupil *read* the following:

4. \$15.36	8. \$32.07	12. \$54.375	16. \$48.23
5. \$23.74	9. \$45.80	13. \$243.405	17. \$65.038
6. \$32.85	10. \$83.05	14. \$324.095	18. \$47.60
7. \$56.39	11. \$92.30	15. \$75.006	19. \$53.025

Let the pupil *write* the following in figures :

1. Twenty-five dollars and thirty-six cents. *Ans.* \$25.36.
2. Nine dollars and seventy-five cents. *Ans.* —.
3. Twelve dollars and forty-six cents. *Ans.* —.
4. Forty dollars and thirty cents. *Ans.* —.
5. Thirty-nine dollars and seven cents. *Ans.* \$39.07.
6. Forty-five dollars and nine cents. *Ans.* —.
7. Fifty-three dollars and seven cents. *Ans.* —.
8. Nine dollars, six cents and five mills. *Ans.* \$9.065.
9. Twelve dollars, nine cents and three mills. *Ans.* —.
10. Fifteen dollars, twelve cents and two mills. *Ans.* —.
11. Forty-three dollars and five mills. *Ans.* \$43.005.
12. Sixty-two dollars and nine mills. *Ans.* —.
13. Fifty-nine cents and four mills. *Ans.* \$0.594.
14. Seventy-five cents and six mills. *Ans.* —.
15. Nine cents and five mills. *Ans.* —.

REDUCTION OF FEDERAL MONEY.

ART. 155. REDUCTION consists in changing numbers from one *form* to another without changing their *value*.

Ex. 1. Reduce 5 dollars to dimes.

Analysis. In 1 dollar there are 10 dimes, and in 5 dollars there are 5 times as many dimes, which will be 50 dimes.

Ans. 50 dimes.

2. Reduce 35 dollars to cents.

Analysis. In 1 dollar there are 100 cents, and in 35 dollars there are 35 times as many cents, which will be 3500 cents.

Ans. 3500 cents.

3. Reduce 42 dollars to mills.

Analysis. In 1 dollar there are 1000 mills, and in 42 dollars there are 42 times as many mills = 42000 mills.

Ans. 42000 mills.

4. Reduce \$6.75 to cents.

Analysis. In 1 dollar there are 100 cents, and in 6 dollars there are 6 times as many cents = 600 cents, and the 75 cents to be added make 675 cents. *Ans.* 675 cents.

The foregoing examples clearly illustrate the following

RULE.

I. To reduce dollars to dimes, multiply by 10, or annex ONE ciphers.

II. To reduce dollars to cents, multiply by 100, or annex TWO ciphers.

III. To reduce dollars to mills, multiply by 1000, or annex THREE ciphers.

IV. To reduce dollars and cents to cents, remove the \$-sign and the decimal point.

EXAMPLES.

- | | |
|-------------------------------------|----------------|
| 5. Reduce \$37 to cents. | <i>Ans.</i> —. |
| 6. Change \$56 to cents. | <i>Ans.</i> —. |
| 7. In \$365 how many mills? | <i>Ans.</i> —. |
| 8. Reduce \$18 to dimes. | <i>Ans.</i> —. |
| 9. Change \$27 to dimes. | <i>Ans.</i> —. |
| 10. Reduce \$83 to cents. | <i>Ans.</i> —. |
| 11. Change \$315 to mills. | <i>Ans.</i> —. |
| 12. Reduce 45 cents to mills. | <i>Ans.</i> —. |
| 13. Change \$15.48 to cents. | <i>Ans.</i> —. |
| 14. Reduce \$28.38 to cents. | <i>Ans.</i> —. |
| 15. In \$53.09 how many cents? | <i>Ans.</i> —. |
| 16. In 4285 cents how many dollars? | |

Analysis. Since 100 cents make \$1, then 1 cent is $\frac{1}{100}$ of \$1, and 4285 cents will be 4285 times as much = $\frac{4285}{100}$ of \$1, or \$42 $\frac{85}{100}$; or \$42.85. *Ans.* \$42.85.

17. In 25346 mills how many dollars?

Analysis. Since 1000 mills make \$1, then 1 mill is $\frac{1}{1000}$ of \$1, and 25346 mills will be 25346 times as much = $\frac{25346}{1000}$ of \$1, or \$25. $\frac{346}{1000}$; or \$25.346. *Ans.* \$25.346.

From these illustrations we derive the

RULE.

I. To reduce cents to dollars, divide by 100, or remove the decimal point TWO places from the right.

II. To reduce mills to dollars, divide by 1000, or remove the point THREE places from the right.

18. Reduce 3984 cents to dollars. *Ans.* \$39.84.

19. In 56427 mills, how many dollars? *Ans.* \$56.427.

20. Change 46258 cents to dollars. *Ans.* —.

21. Reduce 23598 mills to dollars. *Ans.* —.

22. In 73927 cents how many dollars? *Ans.* —.

23. Reduce 956243 mills to dollars. *Ans.* —.

ADDITION OF FEDERAL MONEY.

ART. 156. Ex. 1. What is the sum of 23 dollars and 35 cents, 37 dollars, 9 cents and 6 mills, 32 dollars and 56 cents, and 8 dollars and 5 cents? *OPERATION.*

23 dollars and 35 cents,	\$23.35
37 dollars, 9 cents and 6 mills,	37.096
32 dollars and 56 cents,	32.56
8 dollars and 5 cents?	8.05
	\$101.056

Analysis. We write dollars under dollars, cents under cents and mills under mills. We then commence at the right and add exactly as in Addition of Decimals. *Ans.* \$101.056.

2. What is the sum of 25 dollars and 64 cents, 36 dollars and 73 cents, 45 dollars and 37 cents and 9 dollars and 59 cents? *Ans.* \$117.33.

3 Add together 54 dollars and 67 cents, 34 dollars and 87 cents, 231 dollars and 52 cents and 7 dollars and 16 cents.

Ans. \$328.22.

4. A gentleman bought a barrel of potatoes for \$7.75, a box of soap for \$4.37, a cask of syrup for \$26.33 and a sack of salt for \$3.25; what was the amount of his purchases?

Ans. \$41.70

5. What sum should be paid for a barrel of flour at \$12.25, a firkin of butter at \$23.12½, a barrel of apples at \$6.37½ and a bag of coffee at \$16.50?

Ans. \$58.25.

6. A lady paid \$9 for a bonnet, \$18.37½ for a shawl, \$32.75 for a silk dress and \$22.50 for a set of furs; what was the amount of her purchases?

Ans. \$82.62½.

7. A merchant invested \$375.95 in broadcloths, \$278.93½ in cassimeres, \$185.65 in alpacas and \$213.37½ in calicoes; what sum did he invest?

Ans. \$1053.91½.

8. James bought a knife for 75 cents, a slate for 37½ cents, a pencil for 6¼ cents, an arithmetic for 93¾ cents and a copy-book for 12½; how much did he pay out?

Ans. \$2.25.

9. A farmer sold wheat for \$213.50, corn for \$183.81½, bacon for \$176.75, and barley for \$56.18¾; what was the amount of his sales?

Ans. \$630.25.

SUBTRACTION OF FEDERAL MONEY.

ART. 157. Ex. 1. What is the difference between \$87.35 and \$45.82?

OPERATION

Analysis. We write dollars under dollars, cents under cents, and mills under mills, and begin at the right and subtract precisely as in Subtraction of Decimals.

\$87.35
45.82

\$41.53

Ans. \$41.53.

2. Subtract \$65.45 from \$92.83.

Ans. \$27.38.

3. From \$423.28 take \$212.65.

Ans. \$210.63.

4. Take \$326.58 from \$682.93.

Ans. \$356.35.

5. A man bought a house for \$4282.25, and sold it for \$4826.15; what was his profit on the sale?

Ans. \$543.90.

6. A gentleman bought a watch for \$162.75, and sold it for \$197.25; how much did he make on the sale? *Ans.* \$34.50.

7. A speculator bought cotton for \$762.85, and sold it for \$728.32; how much did he lose? *Ans.* \$34.53

8. A man owed \$746, and paid \$428.45; how much had he still to pay? OPERATION

Analysis. Since there are no cents with the \$746, we fill out the vacant places of cents with ciphers, and then subtract as in Decimals.

$$\begin{array}{r} \$746\ 00 \\ \underline{428\ 45} \\ \$317.55 \end{array}$$

Ans. \$317.55.

9. A gentleman had \$875 deposited in a bank. At one time he drew out \$325.75, and at another time \$258.68; how much still remained in bank? *Ans.* \$290.57.

10. A lady went to a store with a 50-dollar greenback. After paying \$12.25 for a bonnet, \$14.50 for a mantilla, \$2.12½ for a veil, \$2.25 for a pair of gloves, and \$3.75 for a pair of gaiters, how much change ought the merchant to return to her? *Ans.* \$15.12½.

MULTIPLICATION OF FEDERAL MONEY.

ART. 158. Ex. 1. If a box of candles cost \$8.25, what would be the cost of 9.5 boxes? OPERATION.

Analysis. If 1 box cost \$8.25, then 9.5 boxes would cost 9.5 times as much = \$8.25
 $\times 9.5 = \$78.375$. We multiply and point off exactly as in Multiplication of Decimals.

$$\begin{array}{r} \$8.25 \\ 9.5 \\ \hline 4125 \\ 7425 \\ \hline \$78.375 \end{array}$$

Ans. \$78.375.

2. If a yard of cassimere cost \$3.25, what would be the cost of 9 yards? *Ans.* \$29.25.

3. What would be the cost of 7 merino shawls at \$6.75 apiece? *Ans.* \$47.25.

4. A merchant bought 15 sacks of salt at \$2.15 per sack; what was the cost of the salt? *Ans.* \$32.25.

5. A lady bought 18 yards of silk at \$2.85 per yard; what did she pay for the silk? *Ans.* \$51.30.

6. A gentleman boarded 17 days at a hotel, paying \$3.25 per day; what was the amount of his bill? *Ans.* \$55.25.

7. A grocer bought 23 barrels of crushed sugar at \$25 45 per barrel; what was the cost of the sugar? *Ans.* \$585.35.

8. A farmer bought of a grocer 55 pounds of coffee at 27 cents per pound; 2 barrels of flour at \$9.87½ per barrel; 75 pounds of sugar at 12½ cents per pound, and 4 sacks of salt at \$2.18½ per sack; what was the amount of the farmer's bill?
Ans. \$52.72½.

9. A man bought 423.5 acres of land. For 264 acres he paid \$8.62½ per acre, and for the remainder he paid \$9.25 per acre; what was the cost of the whole? *Ans.* \$3752.37½.

10. A farmer bought of a merchant 186 pounds of sugar at 16½ cents per pound, and the merchant took in part payment 24 bushels of peas worth 93¼ cents per bushel; how much did the farmer still owe the merchant? *Ans.* \$8.19.

11. A cotton-broker bought a bale of cotton weighing 512 pounds at 21½ cents per pound, and sold it at 19¼ cents per pound; how much did he lose by the operation?
Ans. \$11.52.

DIVISION OF FEDERAL MONEY.

ART. 159. Ex. 1. If 21 yards of velvet cost \$99.75, what would be the cost of one yard?

Analysis. If 21 yards cost \$99.75, then 1 yard would cost $\frac{1}{21}$ of \$99.75; and to get $\frac{1}{21}$ of \$99.75, we divide it by 21, which gives \$4.75. Hence, in all cases, we divide and point off exactly as in Division of Decimals.

OPERATION.

$$\begin{array}{r} 21) \$99.75 (\$4.75 \\ \underline{84} \\ 157 \\ \underline{147} \\ 105 \\ \underline{105} \\ 00 \end{array}$$

Ans. \$4.75

2. If 9 barrels of oranges cost \$137.25, what would be the cost of one barrel? *Ans.* \$15.25.

3. A cotton-broker bought 8 bales of cotton for \$788.48; what was the price per bale? *Ans.* \$98.56.

4. A merchant paid \$152.01 for a piece of broadcloth containing 27 yards; what was the price per yard? *Ans.* \$5.63.

5. If 15 barrels of flour sell for \$186.75, what is the price per barrel? *Ans.* \$12.45.

6. If 8.5 bushels of wheat cost \$9.775, what is the cost per bushel? *Ans.* \$1.15.

7. A merchant bought of a farmer 28 bushels of barley at \$1.31 $\frac{1}{2}$ per bushel. The farmer took sugar in payment at 15 cents per pound; how many pounds did the farmer receive? *Ans.* 245 pounds.

8. A bale of cotton weighing 512 pounds was sold for \$94.72; what was the price per pound? *Ans.* 18 $\frac{1}{2}$ cents.

9. If 32 tons of hay cost \$792, what is the cost of one ton?

OPERATION.

Analysis. If 32 tons of hay cost \$792, then 1 ton will cost $\frac{1}{32}$ of \$792; i. e., we divide by 32. As there is a remainder after dividing the *dollars*, we annex two ciphers, thereby reducing it to cents.

$$\begin{array}{r}
 32 \overline{) \$792.00} \quad (\$24.75 \\
 \underline{64} \\
 152 \\
 \underline{128} \\
 240 \\
 \underline{224} \\
 160 \\
 \underline{160} \\
 0
 \end{array}$$

Ans. \$24.75.

10. A merchant sold 36 yards of velvet for \$225; what was the price per yard? *Ans.* \$6.25.

11. A farmer sold 64 acres of wild land for \$592; how much did he receive per acre? *Ans.* \$9.25.

12. A grocer sold 42 cans of peaches for \$15.75; what was the price per can? *Ans.* \$0.375.

13. A provision-dealer paid \$21 for 24 ducks; how much did he pay apiece for them? *Ans.* \$0.875.

COMPOUND QUANTITIES.

ARTICLE 160. COMPOUND QUANTITIES are quantities which consist of different denominations. Thus, 5 pounds 3 shillings 2 pence is a *compound* quantity; so, 4 days 5 hours 10 minutes is a *compound* quantity.

ART. 161. COMPOUND QUANTITIES are used in expressing WEIGHTS, MEASURES and MONEY.

ART. 162. ENGLISH, or STERLING MONEY.

TABLE.

4 farthings (qr. or far.)	make 1 penny, d.
12 pence	" 1 shilling, s.
20 shillings	" 1 pound, £.

The above table represents the currency of Great Britain.

Ex. 1. In £3 5s. 6d. 2far., how many farthings?

Analysis. In £1 there are 20 shillings, and in £3 there are 3 times as many shillings = 60 shillings; and the 5 shillings to be added make 65 shillings. Again, in 1 shilling there are 12 pence, and in 65 shillings there are 65 times as many pence = 780 pence; and 6 pence make 786 pence. Lastly, in 1 penny there are 4 farthings, and in 786 pence there are 786 times as many farthings = 3144 farthings; and 2 farthings make 3146 farthings.

OPERATION.

£	s.	d.	far.
3	5	6	2
20			
65	shillings.		
12			
136			
65			
786	pence.		
4			
3146	farthings.		

Ans. 3146far.

The operation which we have performed in this example is called REDUCTION. We have merely changed the *form* of the compound quantity without altering its *value*, for 3146

farthings is *exactly* the same in value as £3 5s. 6d. 2far.
Hence,

ART. 163. REDUCTION is the process of changing quantities from one form to another without altering their VALUE.

EXAMPLES.

2. In £5 8s. 7d., how many pence? *Ans.* 1303d.

3. Reduce £7 4s. 5d. 3far. to farthings. *Ans.* 6935far.

4. In 10s. 8d. 2far., how many farthings? *Ans.* 514far

5. Change £9 3s. 6d. 3far. to farthings. *Ans.* 8811far.

6. In £15 7s. 9d. 2far., how many farthings?
Ans. 14774far.

7. Reduce £5 8s. 9d. to farthings. *Ans.* 5220far.

In all the foregoing examples we have changed quantities from a *higher* denomination to a *lower*. This operation is called REDUCTION DESCENDING.

8. In 2087 farthings, how many pounds, shillings, etc.

Analysis. Since 4 farthings make 1 penny, then in 2087 farthings there are $\frac{1}{4}$ as many pence as farthings = 521 pence, and 3 farthings over. Again, since 12 pence make 1 shilling, in 521 pence there are $\frac{1}{12}$ as many shillings as pence = 43 shillings, and 5 pence over. Lastly, since 20 shillings make 1 pound, then in 43 shillings there are $\frac{1}{20}$ as many pounds as there are shillings = £2, and 3 shillings over. Hence, 2087 farthings are the *same in value* as £2 3s. 5d 3far

OPERATION.

$$\begin{array}{r} 4)2087\text{far.} \\ 12)521 + 3\text{far.} \\ 20)43 + 5\text{d.} \\ \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \\ \text{£2} + 3\text{s.} \end{array}$$

Ans. £2 3s. 5d. 3far.

9. In 3189 farthings, how many pounds, etc.?

Ans. £3 6s. 5d. 1far.

10. Reduce 1785 pence to pounds, etc. *Ans.* £7 8s. 9d.

11. Change 6031 farthings to pounds, etc.

Ans. £6 5s. 7d. 3far.

12. In 2263 pence, how many pounds, etc.?

Ans. £9 8s. 7d.

13. Reduce 9771 farthings to pounds, etc.

Ans. £10 3s. 6d. 3far.

In the last six examples we have changed quantities from a *lower* denomination into a *higher*. This operation is called **REDUCTION ASCENDING**.

REDUCTION ASCENDING is the *reverse* of **REDUCTION DESCENDING**. Reduction Ascending is performed by *division*; Reduction Descending is performed by *multiplication*.

ART. 164. TROY WEIGHT.

TABLE.

24 grains (gr.) make 1 pennyweight, dwt.

20 pennyweights " 1 ounce, oz.

12 ounces " 1 pound, lb.

Troy weight is used in weighing *gold, silver, jewels, precious stones*, etc.

What part of a dwt. is one grain? What part of an oz. is one dwt? What part of a pound is one ounce?

Ex. 1. In 5lb. 4oz. 6dwt., how many grains?

Analysis. In 1lb. there are 12oz., and in 5lb. there are 5 times as many ounces = 60oz., and the 4oz. to be added make 64 ounces. Again, in 1 ounce there are 20dwt., and in 64oz. there are 64 times as many pennyweights = 1280dwt., and the 6dwt. to be added make 1286 pennyweights. Lastly, in 1dwt. there are 24 grains, and in 1286dwt. there are 1286 times as many grains = 30864 grains.

Ans. 30864 grains.

OPERATION.

lb.	oz.	dwt.
5	4	6
12		
64	ounces.	
20		
1286	pennyweights	
24		
5144		
2572		
30864	grains.	

2. In 4lb. 7oz. 3dwt. 9gr., how many grains?

Ans. 26481gr.

3. Reduce 7lb. 6oz. 8dwt. to pennyweights.

Ans. 1808dwt

4. Change 9lb. 6oz. 8dwt. to grains.

Ans. 54912gr.

5. Reduce 10oz. 7dwt. 9gr. to grains.

Ans. 4977gr

6. In 12lb. 6oz., how many pennyweights?

Ans. 3000dwt.

7. Change 9oz. 9dwt. 12gr. to grains.

Ans. 4548gr.

To which kind of Reduction do the above examples belong?

Why?

8. In 1866dwt. how many pounds, etc.?

Analysis. Since 20dwt. make 1oz., then in 1866dwt. there are $\frac{1}{20}$ as many ounces as pennyweights = 93 ounces, and 6dwt. over. Again, since 12oz. make one pound, then in 93oz. there are $\frac{1}{12}$ as many pounds as ounces = 7lb., and 9 ounces over. Hence, in 1866dwt there are 7lb. 9oz. 6dwt.

OPERATION.

20)1866dwt.
12)93 + 6dwt.
7lb. + 9oz.

Ans. 7lb. 9oz. 6dwt.

9. In 2107dwt., how many pounds, etc.?

Ans. 8lb. 9oz. 7dwt.

10. In 284oz., how many pounds, etc.?

Ans. 23lb. 8oz.

11. Reduce 3252 grains to ounces, etc.

Ans. 6oz. 15dwt. 12gr.

12. In 26030 grains, how many pounds, etc.?

Ans. 4lb. 6oz. 4dwt. 14gr.

13. Change 6292dwt. to pounds, etc.

Ans. 26lb. 2oz. 12dwt.

14. Reduce 62454 grains to pounds, etc.

Ans. 10lb. 10oz. 2dwt. 6gr.

To which kind of Reduction do the last 7 examples belong?

Why?

ART. 165. APOTHECARIES' WEIGHT.

TABLE.

20 grains (gr.)	make 1 scruple, $\mathfrak{D}$.
3 scruples	" 1 dram, $\mathfrak{z}$.
8 drams	" 1 ounce, $\mathfrak{z}$.
12 ounces	" 1 pound, lb

Apothecaries' Weight is used by druggists and physicians in *mixing* their drugs and medicines, but they buy and sell by Avoirdupois Weight.

What part of a scruple is one grain? What part of a dram is 1 scruple? What part of an ounce is a dram? What part of a pound is 1 ounce?

Ex. 1. In 4 lb 5 $\mathfrak{z}$ 6 $\mathfrak{z}$ 2 $\mathfrak{D}$, how many grains?

Analysis. In 1 pound there are 12 ounces, and in 4 pounds there are 4 times as many ounces = 48 ounces, and the 5 ounces to be added make 53 ounces. Again, in 1 ounce there are 8 drams, and in 53 ounces there are 53 times as many drams = 424 drams, and the 6 drams to be added make 430 drams. Again, in 1 dram there are 3 scruples, and in 430 drams there are 430 times as many scruples = 1290 scruples, and the 2 scruples to be added make 1292 scruples. Lastly, in 1 scruple there are 20 grains, and in 1292 scruples there are 1292 times as many grains = 25840 grains.

OPERATION.

$$\begin{array}{r}
 \text{lb } \mathfrak{z} \ \mathfrak{z} \ \mathfrak{D} \\
 4 \ 5 \ 6 \ 2 \\
 \hline
 12 \\
 53\mathfrak{z} \\
 \hline
 8 \\
 430\mathfrak{z} \\
 \hline
 3 \\
 1292\mathfrak{D} \\
 \hline
 20 \\
 \hline
 25840\text{gr.}
 \end{array}$$

Ans. 25840gr.

EXAMPLES.

- In 8 lb , how many grains? Ans. 46080gr.
- In 7 lb 5 $\mathfrak{z}$ 2 $\mathfrak{z}$, how many drams? Ans. 714 $\mathfrak{z}$.
- Reduce 10 lb 7 $\mathfrak{z}$ 3 $\mathfrak{z}$ 1 $\mathfrak{D}$ to scruples. Ans. 3058 $\mathfrak{D}$

5. Reduce 3lb 23 33 20 5gr. to grains. *Ans.* 18465gr.

6. Change 6lb 53 03 20 to grains. *Ans.* 37000gr.

To which kind of Reduction do the above examples belong?
Why?

7. In 24694gr. how many pounds, etc.?

Analysis. Since 20gr. make 1 scruple, then in 24694gr. there are $\frac{1}{20}$ as many scruples as grains = 1234 scruples, and 14gr. over. Again, since 3 scruples make 1 dram, then in 1234 scruples there are $\frac{1}{3}$ as many drams as scruples = 411 drams, and 1 scruple over. Again, since 8 drams make 1 ounce, then in 411 drams there are $\frac{1}{8}$ as many ounces as drams = 51 ounces, and 3 drams over. Lastly, since 12 ounces make 1 pound, then in 51 ounces there are $\frac{1}{12}$ as many pounds as ounces = 4lb, and 3 ounces over.

OPERATION.

$$\begin{array}{r} 20 \overline{)24694\text{gr.}} \\ 3 \overline{)1234} + 14\text{gr.} \\ 8 \overline{)411} + 10. \\ 12 \overline{)51} + 30. \\ \quad 4\text{lb} + 30. \end{array}$$

Ans. 4lb 33 33 10 14gr.

8. In 16753, how many pounds, etc.? *Ans.* 17lb 53 33.

9. In 27700, how many pounds, etc.? *Ans.* 9lb 73 33 10.

10. Reduce 31928gr. to pounds, etc.

Ans. 5lb 63 43 00 8gr.

11. Change 51440 to pounds, etc. *Ans.* 17lb 103 23 20.

12. In 26447gr. how many pounds, etc.

Ans. 4lb 73 03 20 7gr.

To which kind of Reduction do the last examples belong?
Why?

ART. 166. APOTHECARIES' FLUID MEASURE

TABLE.

60 minims (m)	make 1 fluidrachm, f3.
8 fluidrachms	" 1 fluidounce, f3.
16 fluidounces	" 1 pint, O.
8 pints	" 1 gallon, Cong.

This table is used by physicians and druggists in measuring liquids for medical prescriptions.

Minim is from the Latin *minus*, meaning *least*, because a minim is the *least* measure used by physicians; about two drops of water make a minim.

O, the symbol for *pint*, is the initial letter of the Latin word *octarius*, meaning *an eighth*, because a pint is $\frac{1}{8}$ of a gallon; and *cong.* is an abbreviation of *congius*, signifying a Roman measure, nearly equivalent to our gallon.

Ex. 1. Reduce 4cong. 5O 3f $\frac{3}{4}$ to minims.

Analysis. In 1 gallon there are 8 pints, and 4 gallons = 4 times as many = 32 pints; and 5 pints make 37 pints. Again, 1 pint = 16 fluidounces, and 37 pints = 37 times as many fluidounces = 592 fluidounces; and 3 fluidounces make 595 fluidounces. Again, 1 fluidounce = 8 fluidrachms, and 595 fluidounces = 595 times as many fluidrachms = 4760 fluidrachms. Lastly, 1 fluidrachm = 60 minims, and 4760 fluidrachms = 4760 times as many minims = 285600 minims.

OPERATION.

Cong. O. f $\frac{3}{4}$.

4 5 3

8

37O.

16

225

37

595f $\frac{3}{4}$.

8

4760f $\frac{3}{4}$.

60

285600 minims.

Ans. 285600m.

2. In 6cong. 4O 5 $\frac{3}{4}$, how many fluidrachms?

Ans. 6696f $\frac{3}{4}$.

3. How many fluidounces in 5cong. 6O 4 $\frac{3}{4}$? Ans. 740 $\frac{3}{4}$.

4. Reduce 3cong. 5O 2 $\frac{3}{4}$ 3 $\frac{3}{4}$ to minims. Ans. 223860m.

5. In 232260 minims, how many gals., etc.?

Ans. 3cong. 6O 3f $\frac{3}{4}$ 7f $\frac{3}{4}$.

6. In 5691 fluidrachms, how many gals. etc.?

Ans. 5cong. 4O 7f $\frac{3}{4}$ 3f $\frac{3}{4}$.

7. In 163020 minims, how many gals., etc.?

Ans. 2cong. 5O 3f $\frac{3}{4}$ 5f $\frac{3}{4}$.

ART. 167. AVOIRDUPOIS WEIGHT.

TABLE.

16 drams (dr.)	make 1 ounce,	oz.
16 ounces	“ 1 pound,	lb.
25 pounds	“ 1 quarter,	qr.
4 quarters	“ 1 hundredweight,	cwt.
20 hundredweight	“ 1 ton,	T.

This weight is used for weighing all coarse and heavy articles, such as iron, coal, hay and groceries.

Formerly, it was customary to reckon 28 pounds as a quarter, 112 pounds a hundredweight, and 2240 pounds a ton; but this custom is growing obsolete, as it should have done long ago.

What part of a ton is 1 hundredweight? What part of 1cwt. is 1qr.?

What part of 1qr. is 1lb.? What part of 1lb. is 1oz.?

What part of 1oz. is 1dr.? How many qr. in 2cwt.? In 3cwt.?

How many dr. in 2oz.? In 3oz.? How many cwt. in 12qr.? In 16qr.? In 20qr.?

Ex. 1. In 5cwt. 3qr. 4lb. 5oz., how many ounces?

Analysis. In 1cwt. there are 4qr., and in 5cwt. there are 5 times as many quarters = 20 quarters; and the 3 quarters to be added make 23 quarters. Again, in 1qr. there are 25lb., and in 23qr. there are 23 times as many pounds = 575 pounds; and the 4 pounds to be added make 579 pounds. Lastly, in 1 pound there are 16 ounces, and in 579 pounds there are 579 times as many ounces = 9264 ounces; and the 5 ounces to be added make 9269 ounces.

Ans. 9269 ounces.

OPERATION.

cwt.	qr.	lb.	oz.
5	3	4	5
	4		
	<u>23</u>		
	25		
	<u>119</u>		
	46		
	<u>579</u>		
	16		
	<u>3479</u>		
	579		
	<u>9269</u>		

9269 ounces.

EXAMPLES.

2. In 7cwt. 2qr. 10lb., how many pounds? *Ans.* 760lb
 3. In 8cwt. 0qr. 17lb., how many pounds? *Ans.* 817lb
 4. Reduce 3cwt. 3qr. 7lb. 8oz. to ounces. *Ans.* 6120oz
 5 In 3qr. 9lb. 6oz. 10dr., how many drams?

Ans. 21610dr

6. Reduce 4T. 7cwt. 2qr. 15lb. to pounds.

Ans. 8765 pounds

7. Reduce 3cwt. 2qr. 0lb. 5oz. to ounces. *Ans.* 5605oz.

To what kind of Reduction do the above examples belong?

8. In 8792 ounces, how many cwt., etc.?

Analysis. Since 16oz. make 1 pound,
 then in 8792oz. there are $\frac{1}{16}$ as many
 pounds as ounces = 549lb., and 8oz.
 over. Again, since 25lb. make 1qr.,
 then in 549lb. there are $\frac{1}{25}$ as many
 quarters as pounds = 21qr., and 24lb.
 over. Lastly, since 4qr. make 1cwt., then in 21qr. there are
 $\frac{1}{4}$ as many cwt. as there are quarters = 5cwt., and 1qr. over.
 Hence, in 8792 ounces there are 5cwt. 1qr. 24lb. 8oz.

OPERATION

16)8792oz.

25)549 + 8oz.

4)21 + 24lb.

5cwt. + 1qr.

Ans. 5cwt. 1qr. 24lb. 8oz.

9. In 24972 ounces, how many cwt., etc.?

Ans. 15cwt. 2qr. 10lb. 12oz.

10. In 8765lb., how many tons, etc.?

Ans. 4T. 7cwt. 2qr. 15lb.

11. Reduce 7698 ounces to cwt., etc.

Ans. 4cwt. 3qr. 6lb. 2oz.

12. Reduce 42597 drams to cwt., etc.

Ans. 1cwt. 2qr. 16lb. 6oz. 5dr.

13. Change 9208 pounds to tons, etc.

Ans. 4T. 12cwt. 0qr. 8lb.

14. Reduce 9367 ounces to cwt., etc.

Ans. 5cwt. 3qr. 10lb. 7oz.

To which kind of Reduction do the last seven examples belong?

ART. 168. LONG MEASURE.

TABLE.

12 inches (in.)	make	1 foot,	ft.
3 feet	"	1 yard,	yd.
5½ yards	"	1 rod,	r.
40 rods	"	1 furlong,	fur.
8 furlongs	"	1 mile,	mi.
3 miles	"	1 league,	L.

LONG MEASURE, frequently called LINEAR MEASURE, is used in measuring lengths, distances and heights.

A pendulum vibrating once a second, in the latitude of London, at the level of the sea, in a vacuum, with the thermometer at 60°, measures 39.139 inches. By subdividing this length, it becomes an easy problem to determine an inch.

Besides the denominations mentioned in the Table, other linear measures are sometimes met with, viz.:

The *line* = $\frac{1}{12}$ of an inch; the *hand* = 4 inches, used in measuring the height of horses; the *span* = 9 inches; the cubit = 18 inches; and the *fathom* = 6 feet, used in measuring depths at sea.

MENTAL QUESTIONS.

What part of 1ft. is lin.? What part of 1yd. is 1ft.? What part of 1fur. is 1r.? What part of 1mi. is 1fur.? What part of 1L. is 1mi.?

How many inches in 2 feet? In 3ft.? In 4ft.? In 5ft.?

How many feet in 2 yards? In 3yd.? In 5yd.? In 9yd.?

How many yards in 2 rods? In 4 rods? In 6 rods?

How many yards in 9 feet? In 12ft.? In 18ft.?

Ex. 1. In 5mi. 3fur. 8r., how many feet?

Analysis. In 1mi. there are 8fur., and in 5mi. there are 5 times as many furlongs = 40 furlongs, and the 3 furlongs to be added make 43 furlongs. Again, in 1fur. there are 40 rods, and in 43 furlongs there are 43 times as many rods = 1720 rods, and the 8 rods to be added make 1728 rods. Again, in 1 rod there are $5\frac{1}{2}$ yards, and in 1728 rods there are 1728 times as many yards = 9504 yards. Lastly, in 1 yard there are 3 feet, and in 9504 yards there are 9504 times as many feet = 28512 feet.

Ans. 28512ft.

OPERATION.

mi.	fur.	r.
5	3	8
8		
43	furlongs.	
40		
1728	rods.	
5 $\frac{1}{2}$		
8640		
864		
9504	yards.	
3		
28512	feet.	

EXAMPLES.

2. In 5yd. 2ft. 9in., how many inches? *Ans.* 213in.
3. In 8r. 2yd. 2ft. 5in., how many inches? *Ans.* 1685in.
4. Reduce 6r. 1yd. 2ft. to feet. *Ans.* 104ft.
5. How many feet in 6 miles? *Ans.* 31680ft.
6. How many yards in 8 miles? *Ans.* 14080yd.
7. Reduce 5fur. 6r. 2yd. 2ft. to feet. *Ans.* 3407ft.
8. Reduce 5mi. 4fur. 6r. 1yd. to feet. *Ans.* 29142ft.
9. In 1721 inches, how many rods, etc.?

Analysis. Since 12in. make 1 foot,

OPERATION.

then in 1721 inches there are $\frac{1}{12}$ as many feet as inches = 143ft. and 5in. over. Again, since 3 feet make 1yd., then in 143ft. there are $\frac{1}{3}$ as many yards as there are feet = 47yd. and 2ft. over. Lastly, since $5\frac{1}{2}$ yd. or $\frac{1}{2}$ yards make 1 rod, then in 47yd.

$$12)1721\text{in.}$$

$$3)143 + 5\text{in.}$$

$$5\frac{1}{2})47 + 2\text{ft.}$$

$$\frac{2}{2}$$

$$11)94$$

$$8\text{r.} + \frac{3}{2}\text{yd.} = 3\text{yd.}$$

there are as many rods as $\frac{1}{2}$ are contained times in 47. We change the 47 into halves, making 94 ; then $\frac{1}{2}$ are contained

m $\frac{3}{4}$, 8 times, making 8 rods; with $\frac{1}{2}$ of a yard = 3 yd. over. Hence, in 1721 inches there are 8r. 3yd. 2ft. 5in.

Ans. 8r. 3yd. 2ft. 5in.

10. In 215 inches, how many yards, etc?

Ans. 5yd. 2ft. 11in.

11. Reduce 1653in. to rods, etc. Ans. 8r. 1yd. 2ft. 9in.

12. Reduce 528 feet to rods. Ans. 32r.

13. Change 21120 feet to miles. Ans. 4mi.

14. Reduce 190080 inches to miles. Ans. 3mi.

ART. 169. CLOTH MEASURE.

TABLE.

2 $\frac{1}{4}$ inches (in.)	make 1 nail,	na.
4 nails	" 1 quarter,	qr.
4 quarters	" 1 yard,	yd.
3 quarters	" 1 Ell Flemish,	E. Fl.
5 quarters	" 1 Ell English,	E. E.
6 quarters	" 1 Ell French,	E. F.

CLOTH MEASURE was formerly used in measuring *cloth, silk, lace and ribbons*; *practically*, the Table has gone out of use, for merchants no longer measure muslins, calicoes, etc. in *inches* and *nails*, but in *halves, quarters, eighths* and *sixteenths* of a yard of LONG MEASURE.

ART. 170. SQUARE MEASURE.

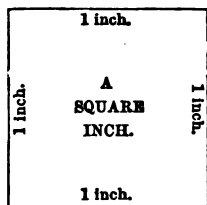
TABLE.

144 square inches (sq. in.)	make 1 square foot,	sq. ft.
9 " feet	" 1 " yard,	sq. yd.
30 $\frac{1}{4}$ " yards	" 1 " rod,	sq. rd.
40 " rods	" 1 " rood,	R.
4 roods	" 1 acre,	A.
640 acres	" 1 square mile,	sq. mi.

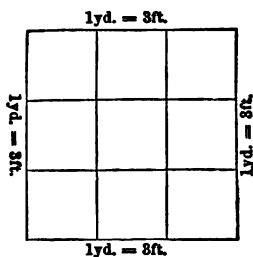
This measure is used in measuring the area or contents of such things as have *length* and *breadth*, without thickness, as flooring, plastering, etc. One of its most common uses is in measuring land, and hence it is frequently called **LAND MEASURE**.

A **SQUARE** is a figure that has four equal sides and four equal corners or right angles.

A **SQUARE** whose side is one inch is called a square inch; a square whose side is one foot is called a square foot; one whose side is one yard is called a square yard, etc.

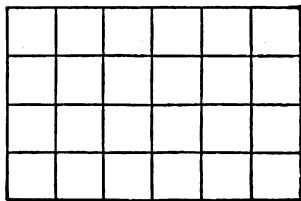


Our table says that 9 square feet make 1 square yard. This will be obvious by inspecting the figure on the right. Each of the sides is 1 yard, or 3 feet, in length, and it contains 9 small squares, each square having its side one foot. The area, or contents, therefore, $= 3 \times 3 = 9$ square feet.



In the same way, if we had a square, each of whose sides was 1 foot, or 12 inches, in length, we should find that it contained 144 small squares, each square having each of its sides 1 inch.

If we have a figure 6 inches *long* and 1 inch *wide*, it is obvious that the figure would contain 6 *square* inches; if it is 2 inches wide, instead of 1 inch, it would contain 12 square inches; if it is 3 inches wide, it would contain 18 square inches; if 4 inches wide, it will contain 24 square inches, and so on.



From these illustrations we learn that the area of a square, or of any other figure having its angles or corners like those of a square, is found by *multiplying the length by the breadth*.

Ex. 1. In 16 sq. rd. 8 sq. yd. 7 sq. ft. how many square feet?

Analysis. In 1 sq. rd. there are $30\frac{1}{4}$ sq. yards, and in 16 sq. rods there are 16 times as many sq. yards = 484 sq. yards; and the 8 sq. yards to be added make 492 sq. yards. Again, in 1 sq. yard there are 9 sq. feet, and in 492 sq. yards there are 492 times as many sq. feet = 4428 sq. feet; and the 7 sq. feet to be added make 4435 sq. feet.

OPERATION.

sq. rd.	sq. yd.	sq. ft.
16	8	7
$30\frac{1}{4}$		
484		
8		
492	sq. yd.	
9		
4435	sq. ft.	

Ans. 4435 sq. ft.

EXAMPLES.

2. In 8 sq. yd. 6 sq. ft. how many sq. inches?

Ans. 11232 sq. in.

3. In 20 sq. rd. 7 sq. yd. how many sq. feet?

Ans. 5508 sq. ft.

4. Reduce 18A. 3R. 10 sq. rd. to sq. rods.

Ans. 3010 sq. rd.

5. Reduce 9 acres to sq. yards.

Ans. 43560 sq. yd.

6. In 6 sq. miles how many sq. rods?

Ans. 614400. sq. rd.

7. Change 7 acres to sq. feet.

Ans. 304920 sq. ft.

8. Reduce 15 sq. yd. 7 sq. ft. to sq. inches.

Ans. 20448 sq. in.

9. In 24 sq. rd. 5 sq. yd. how many sq. feet?

Ans. 6579 sq. ft.

10. Reduce 15A. 2R. 8 sq. rd. to sq. rd.

Ans. 2488 sq. rd.

11. How many square inches in the floor of a room 18 feet long and 17 feet broad?

OPERATION

Analysis. If the room were 18 feet long and 1 foot broad, it would evidently contain 18 sq. feet; and if it were 17 feet broad, instead of 1 foot, it would contain 17 times as many sq. feet = 306 sq. feet; and since 1 sq. foot contains 144 sq. inches, then 306 sq. feet will contain 306 times as many sq. inches = 44064 sq. inches.

Ans. 44064 sq. in.

$$\begin{array}{r}
 18 \\
 17 \\
 \hline
 126 \\
 18 \\
 \hline
 306 \text{ sq. ft.} \\
 144 \\
 \hline
 1224 \\
 1224 \\
 \hline
 306 \\
 \hline
 44064 \text{ sq. in.}
 \end{array}$$

12. How many sq. in. in a table 10 feet long and 5 feet wide?

Ans. 7200 sq. in.

13. How many sq. feet in a carpet 8 yards long and 6 yards wide?

Ans. 432 sq. ft.

14. How many sq. inches in a marble slab 5ft. 8in. in length and 2ft. 10in. in breadth?

Ans. 2312 sq. in.

15. How many sq. rods in 6602 sq. ft.?

Analysis. Since 9 sq. ft. make 1 sq. yd., then in 6602 sq. ft. there are $\frac{1}{9}$ as many sq. yards as there are sq. feet = 733 sq. yards, and 5 sq. feet over. Again, since $30\frac{1}{4}$ sq. yards, or $12\frac{1}{4}$ sq. yards, make 1 sq. rod, then in 733 sq. yards there are as many square rods as $12\frac{1}{4}$ are contained times in 733. We change the 733 into *fourths*, making 2932 *fourths*. Then, $12\frac{1}{4}$ is contained in 2932 *fourths* 24 times, with a remainder of 28 *fourths* = 7 *whole ones*; therefore, in 733 sq. yards there are 24 sq. rods, and 7 sq. yards over. Hence, in 6602 sq. feet there are 24 sq. rd. 7 sq. yd. 5 sq. ft.

Ans. 24 sq. rd. 7 sq. yd. 5 sq. ft.

OPERATION.

$$\begin{array}{r}
 9 \overline{)6602} \text{ sq. ft.} \\
 30\frac{1}{4} \overline{)733} + 5 \text{ sq. ft.} \\
 \underline{4 \quad 4} \\
 121 \overline{)2932} (24 \text{ sq. rd.} \\
 \underline{242} \\
 512 \\
 \underline{484} \\
 28 \\
 \frac{28}{4} = 7 \text{ sq. yd}
 \end{array}$$

16. In 19328 sq. in., how many sq. yards, etc.?

Ans. 14 sq. yd. 8 sq. ft. 32 sq. in.

17. Reduce 1705 sq. rd. to acres, etc.

Ans. 10A. 2R. 25 sq. rd.

18. The carpet on a floor contains 54432 sq. inches; how many sq. yards in the carpet?

Ans. 42 sq. yd.

19. A tablecloth contains 18144 sq. inches; how many sq. yards in the cloth?

Ans. 14 sq. yd

ART. 171. CUBIC MEASURE.

TABLE.

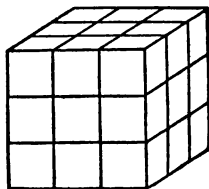
1728 cubic inches (cu. in.)	make 1 cubic foot,	cu. ft.
27 " feet	" 1 " yard,	cu. yd.
40 cu. feet of round, or	} " 1 ton or load,	T.
50 cu. feet of hewn timber		
128 cubic feet	" 1 cord of wood,	C.

CUBIC MEASURE, sometimes called SOLID MEASURE, is used in measuring things which have *length*, *breadth* and *thickness*, such as boxes, stone, wood and bales of goods.

ART. 172. A CUBE is a solid or body bounded by six equal squares, which are called its *faces*.

A CUBIC INCH is a cube one inch *long*, one inch *broad* and one inch *thick*. A CUBIC FOOT is a cube one foot *long*, one foot *broad* and one foot *thick*. A CUBIC YARD is a cube one yard *long*, one yard *wide* and one yard *thick*.

The figure on the right represents a cube, each side of which is one yard or 3 feet in length. Each one of its sides or faces contains $3 \times 3 = 9$ square feet; and if it were 1 foot *thick* it would contain $3 \times 3 \times 1 = 9$ cubic feet; but since it is 3 feet *thick*, it will contain



3 times as many cubic feet as when it is 1 foot thick $= 3 \times 3 \times 3 = 27$ cubic feet.

In the same way, if we had a cube each of whose edges was 1 foot or 12 inches, we should find its solid contents to be $12 \times 12 \times 12 = 1728$ cubic or solid inches. Hence,

To find the solid contents of a cubical body we multiply the *length, breadth and thickness together*.

ART. 173. A pile of wood 8 feet long, 4 feet wide and 4 feet high contains a cord of wood. A cord is thus seen to be 128 cubic feet.

Ex. 1. In 19 cu. yd. 4 cu. ft. 8 cu. in., how many cubic inches?

Analysis. In 1 cu. yd. there are 27 cu. ft., and in 19 cu. yards there are 19 times as many cu. feet $= 513$ cu. ft., and the 4 cu. ft. to be added make 517 cu. feet. Again, in 1 cubic foot there are 1728 cu. in., and in 517 cu. ft. there are 517 times as many cu. inches $= 893376$ cu. inches, and the 8 cu. in. to be added make 893384 cu. inches.

Ans. 893384 cu. in.

OPERATION.

cu. yd.	cu. ft.	cu. in.
19	4	8
27		
137		
38		
517	cu. ft.	
1728		
4144		
1034		
3619		
517		
893384	cu. in.	

EXAMPLES.

2. In 22 cu. ft. 78 cu. in., how many cu. inches?

Ans. 38094 cu. in.

3. In 18 cu. yd. 13 cu. ft., how many cu. inches?

Ans. 862272 cu. in.

4. In 12 tons of round timber, how many cu. in.?

Ans. 829440 cu. in.

5. How many cu. in. in 9 cords of wood?

Ans. 1990656 cu. in.

6. In 335469 cu. in., how many cu. yards, etc.?

Analysis. Since 1728 cu. in. make

1 cu. foot, then in 335469 cu. in. there are $\frac{1}{1728}$ as many cu. feet as there are inches = 194 cu. ft., and 237 cu. inches over. Again, since 27 cu. ft. make 1 cu. yd., then in 194 cu. ft. there are $\frac{1}{27}$ as many cu. yards as there are feet = 7 cu. yards, and 5 cu. ft. over. Hence, in 335469 cu. inches there are 7 cu. yd. 5 cu. ft. 237 cu. in.

Ans. 7 cu. yd. 5 cu. ft.

237 cu. in.

OPERATION.

$$\begin{array}{r}
 1728)335469(194 \text{ cu. ft.} \\
 \underline{1728} \\
 16266 \\
 \underline{15552} \\
 7149 \\
 \underline{6912} \\
 237 \text{ cu. in.} \\
 27)194(7 \text{ cu. yd.} \\
 \underline{189} \\
 5 \text{ cu. ft.}
 \end{array}$$

7. How many cu. feet, etc., in 45657 cu. in.?

Ans. 26 cu. ft. 729 cu. in.

8. In 1874 cu. ft. how many cu. yards, etc.?

Ans. 69 cu. yd. 11 cu. ft.

9. How many cords of wood in 4992 cu. feet?

Ans. 39C.

10. In a pile of wood 64 feet long, 12 feet broad and 8 feet high, how many cords?

Ans. 48C.

ART. 174. LIQUID MEASURE.

TABLE.

4 gills (gi.)	make 1 pint,	pt.
2 pints	" 1 quart,	qt.
4 quarts	" 1 gallon,	gal.
31½ gallons	" 1 barrel,	bbl.
2 barrels or 63 gallons	" 1 hogshead, hhd.	
2 hogsheads	" 1 pipe,	pi.
2 pipes	" 1 tun,	tun.

M

LIQUID MEASURE, more frequently called **WINE MEASURE**, is used in measuring nearly all liquids, as water, wine, oil, molasses, etc.

ART. 175. The standard wine gallon of the U. S. contains 231 cubic inches. The imperial gallon of Great Britain contains 277.274 cubic inches.

ART. 176. Besides the measures given in the Table, the *tierce* and *puncheon* are sometimes employed; the tierce contains 42 and the puncheon 84 gallons.

MENTAL QUESTIONS.

How many gills in 3pt.? In 5pt.? In 7pt.? In 10pt.?

How many pints in 3qt.? In 6qt.? In 8qt.? In 12qt.?

How many qt. in 3gal.? In 5gal.? In 7gal.? In 10gal.?

What part of 1pt. is 1gi.? What part of 1qt. is 1pt.?

What part of 1gal. is 1qt.? What part of 1hhd. is 1gal.?

Ex. 1. How many gills in 7gal. 3qt. 1pt. 2gi.?

Analysis. In 1gal. there are 4qt., and in 7gal. there are 7 times as many quarts = 28qt., and the 3qt. to be added make 31 quarts. Again, in 1qt. there are 2 pints, and in 31qt. there are 31 times as many pints = 62 pints, and the 1 pint to be added makes 63 pints. Lastly, in 1 pint there are 4 gills, and in 63 pints there are 63 times as many gills = 252 gills, and the 2 gills to be added make 254 gills.

OPERATION.

gal.	qt.	pt.	gi
7	3	1	2
4			
31	quarts.		
2			
63	pints.		
4			
254	gills.		

Ans. 254gi.

EXAMPLES.

2. In 9gal. 3qt. 1pt., how many gills?

Ans. 316gi.

3. Reduce 27gal. 3qt. 1pt. to pints.

Ans. 223pt.

4. Reduce 36gal. 2qt. 1pt. to gills.

Ans. 1172gi.

5. In 4 hogsheads, how many pints?

Ans. 2016pt.

6. Reduce 9hhd. 3gal. 0qt. 1pt. to pints. *Ans.* 4561pt.

7. In 43gal. 2qt. 1pt., how many gills? *Ans.* 1396gi.

8. In 767 gills, how many gallons, etc.?

Analysis. Since 4 gills make 1 pint, then *OPERATION.*

in 767 gills there are $\frac{1}{4}$ as many pints as gills = 191 pints, and 3 gills over. Again, since 2 pints make 1 quart, then in 191 pints there are $\frac{1}{2}$ as many quarts as pints = 95 quarts, and 1 pint over. Lastly, since 4 quarts make 1 gallon, then in 95 quarts there are $\frac{1}{4}$ as many gallons as quarts = 23gal., and 3qt. over.

Ans. 23gal. 3qt. 1pt. 3gi.

9. In 1151 gills, how many gallons, etc.?

Ans. 35gal. 3qt. 1pt. 3gi

10. In 349 pints, how many gallons, etc.?

Ans. 43gal. 2qt. 1pt.

11. Reduce 7639 pints to hogsheads, etc.

Ans. 15hhd. 9gal. 3qt. 1pt.

12. Reduce 18972 gills to hogsheads, etc.

Ans. 9hhd. 25gal. 3qt. 1pt.

ART. 177. BEER MEASURE.

TABLE.

2 pints (pt.)	make 1 quart,	qt.
4 quarts	“ 1 gallon,	gal.
36 gallons	“ 1 barrel,	bbl.
$1\frac{1}{2}$ barrels, or 54 gallons,	“ 1 hogshead,	hhd.

BEER MEASURE was formerly used in measuring ale, beer, malt liquors and milk. Its use has been almost entirely superseded by **LIQUID MEASURE**.

The *firkin* contains 9 gallons; the *kilderkin* contains 2 firkins.

ART. 178. DRY MEASURE.

TABLE.

2 pints (pt.)	make 1 quart, qt.
8 quarts	" 1 peck, pk.
4 pecks	" 1 bushel, bu.

DRY MEASURE is used for measuring all kinds of grain, seeds, salt, etc.

The standard bushel of the United States is the Winchester bushel of Great Britain, which is a cylindrical vessel $18\frac{1}{2}$ inches in diameter and 8 inches deep; it contains 2150.42 cubic inches.

Coal was formerly measured by the *chaldron*, which contains 36 bushels. it is now bought and sold by the *ton*.

MENTAL QUESTIONS.

How many pints in 3qt.? In 5qt.? In 7qt.? In 12qt.?
 How many quarts in 3pk.? In 5pk.? In 7pk.? In 10pk.?
 How many pecks in 3bu.? In 6bu.? In 9bu.? In 10bu.?
 How many bu. in 8pk.? In 12pk.? In 20pk.? In 24pk.?
 How many pk. in 16qt.? In 24qt.? In 32qt.? In 40qt.?
 What part of 1bu. is 1pk.? What part of 1pk. is 1qt.?

Ex. 1. In 23bu. 3pk. 7qt., how many pints?

Analysis. In 1 bushel there are 4 pecks, and in 23 bu. there are 23 times as many pecks = 92 pecks, and the 3 pecks to be added make 95 pecks. Again, in 1 peck there are 8 quarts, and in 95 pecks there are 95 times as many quarts = 760 quarts, and the 7 quarts make 767 quarts. Lastly, in 1 quart there are 2 pints, and in 767 quarts there are 767 times as many pints = 1534 pints.

OPERATION.

bu	pk	qt
23	3	7
	4	
95		pecks.
	8	
767		quarts.
	2	
1534		pints.

Ans. 1534pt.

EXAMPLES.

2. How many pints in 13bu. 3pk. 5qt. 1pt. ? *Ans.* 891pt.
 3. How many pints in 17 bushels? *Ans.* 1088pt.
 4. A boy sold 3 bushels of peanuts at 7 cents a pint; how much did he receive for them? *Ans.* \$13.41.
 5. A market-woman sold 3bu. 2pk. 5qt. of blackberries at 6 cents a quart; how much did she receive? *Ans.* \$7.02.
 6. In 959 pints how many bushels, etc. ?
Analysis. Since 2 pints make 1qt., then
 in 959 pints there are $\frac{1}{2}$ as many quarts as
 pints = 479 quarts, and 1pt. over. Again,
 since 8 quarts make 1 peck, then in 479
 quarts there are $\frac{1}{8}$ as many pecks as quarts
 = 59 pecks, and 7 quarts over. Lastly,
 since 4 pecks make 1 bushel, then in 59 pecks there are $\frac{1}{4}$ as
 many bushels as pecks = 14 bushels, and 3 pecks over.
Ans. 14bu. 3pk. 7qt. 1pt.
 7. In 895 pints how many bushels, etc. ?
Ans. 13bu. 3pk. 7qt. 1pt.
 8. Reduce 1483 pints to bushels, etc.
Ans. 23bu. 5qt. 1pt.
 9. Reduce 606 quarts to bushels, etc.
Ans. 18bu. 3pk. 6qt

ART. 179.

TIME.

TABLE.

60 seconds (sec.)	make 1 minute,	min.
60 minutes	" 1 hour,	hr.
24 hours	" 1 day,	da.
7 days	" 1 week,	wk.
365 days	" 1 common year,	yr.
366 days	" 1 leap year,	yr.
100 years	" 1 century,	C.

A **YEAR** is the length of time occupied by the earth in making one revolution round the sun.

A *calendar* month is an arbitrary period usually reckoned at 30 days.

The year is divided into 12 calendar months, the names of which are as follows: January, February, March, April, May, June, July, August, September, October, November, December.

The number of days belonging to each month may be easily remembered from the following lines:

Thirty days has September,
 April, June and November;
 All the rest have thirty-one,
 Except February alone,
 Which claims but twenty-eight, in fine,
 Till Leap Year gives it twenty-nine.

MENTAL QUESTIONS.

How many min. in 2 hours? In 3hr.? In 5hr.? In 6hr.?

How many hours in 2 days? In 3da.? In 4da.?

How many days in 2 weeks? In 3wk.? In 4wk.?

How many months in 2 years? In 3yr.? In 5yr.?

Ex. 1. How many minutes in 4wk. 5da.
 5hr.?

Analysis. In 1 week there are 7 days, and in 4 weeks there are 4 times as many days = 28 days, and 5 days make 33 days. Again, in 1 day there are 24 hours, and in 33 days there are 33 times as many hours = 792 hours, and 5 hours make 797 hours. Lastly, in 1 hour there are 60 minutes, and in 797 hours there are 797 times as many minutes = 47820 minutes.

Ans. 47820 minutes.

OPERATION.

wk.	da.	hr.
4	5	5
7		
33	days.	
24		
137		
66		
797	hours.	
60		
47820	minutes	

2. How many minutes in 3wk. 3da. 9hr. ? *Ans.* 35100min.

3. How many seconds in 3 days ? *Ans.* 259200sec.

4. How many minutes in 2 weeks ? *Ans.* 20160min.

5. How many hours in a leap year ? *Ans.* 8784hr.

6. How many minutes in the month of August ?

Ans. 44640min.

7. If a ship sail 9 miles an hour, how far could it sail at the same rate in 2wk. 4da. 7hr. ? *Ans.* 3951 miles.

8. The moon goes round the earth in $29\frac{1}{2}$ days; how many minutes does it occupy in one revolution ? *Ans.* 42480min.

9. In 47769 minutes, how many weeks, etc. ?

Analysis. Since 60min. make 1hr.,

OPERATION.

then in 47769min. there are $\frac{1}{60}$ as

60)47769min.

many hours as minutes = 796 hours,

24)796 + 9min.

and 9 minutes over. Again, in 1 day

7)33 + 4hr.

there are 24 hours, and in 796 hours

4wk. + 5da.

there are $\frac{1}{24}$ as many days as hours =

33 days, and 4 hours over. Lastly, since 7 days make 1 week,

then in 33 days there are $\frac{1}{7}$ as many weeks as days = 4 weeks,

and 5 days over.

Ans. 4wk. 5da. 4hr. 9min.

10. Reduce 35065 minutes to weeks, etc.

Ans. 3wk. 3da. 8hr. 25min.

11. In 1282 hours, how many weeks, etc. ?

Ans. 7wk. 4da. 10hr.

12. How many days, etc. in 479529 seconds ?

Ans. 5da. 13hr. 12min. 9sec

ART 180. CIRCULAR MEASURE.

TABLE.

60 seconds (")	make 1 minute,	'.
60 minutes	" 1 degree,	°.
30 degrees	" 1 sign,	§.
12 signs, or 360°,	" 1 circumference of a circle,	C.

CIRCULAR MEASURE is used in astronomy, geography and surveying, and in estimating the motions of the heavenly bodies.

ART. 181. The circumference of every circle, whether *great* or *small*, is supposed to be divided into 360 equal parts, called *degrees*. A *degree*, therefore, is not a quantity of invariable length, like a *foot* or a *yard*, but it is $\frac{1}{360}$ of the circumference of any circle. A degree will be larger or smaller according to the size of the circle to which it belongs.

Ex. 1. How many seconds in $9^{\circ} 27' 19''$?

OPERATION.

Analysis. In 1° there are 60', and in 9° there are 9 times as many minutes = 540', and 27' make 567'. Again, in 1' there are 60'', and in 567' there are 567 times as many seconds = 34020'', and 19'' make 34039''.

Ans. 34039''.

$$\begin{array}{r} 9^{\circ} \quad 27' \quad 19'' \\ 60 \\ \hline 567' \\ 60 \\ \hline 34020 \\ 19 \\ \hline 34039'' \end{array}$$

EXAMPLES.

2. How many seconds in $23^{\circ} 18' 25''$? *Ans.* 83905''.
3. How many minutes in a circumference. *Ans.* 21600'.
4. Reduce $28^{\circ} 17' 25''$ to seconds. *Ans.* 101845''.
5. How many seconds in 3 signs? *Ans.* 324000''.
6. In 47725'', how many degrees, etc.?

Analysis. Since 60 seconds make 1 minute, then in 47725'' there are $\frac{1}{60}$ as many minutes as seconds = 795', and 25'' over. Again, since 60 minutes make 1° , then in 795' there are $\frac{1}{60}$ as many degrees as minutes = 13° , and 15' over.

OPERATION.

$$\begin{array}{r} 60)47725'' \\ \hline 60)795' + 25'' \\ \hline 13^{\circ} + 15' \end{array}$$

- Ans.* $13^{\circ} 15' 25''$.
7. In 32887'', how many degrees, etc.?
8. How many degrees in 64800''?
9. Reduce 8565' to signs, etc.

Ans. $9^{\circ} 8' 7''$.

Ans. 18° .

Ans. 4S. $22^{\circ} 45'$.

ART. 182**PAPER MEASURE.****TABLE.**

24 sheets	make	1 quire.
20 quires	"	1 ream.
2 reams	"	1 bundle.
5 bundles	"	1 bale.

EXAMPLES.

1. How many sheets in a ream of paper? *Ans.* 480 sheets.
2. In a bale of paper, how many sheets?
Ans. 4800 sheets.
3. How many quires in 4 bales? *Ans.* 800 quires.
4. How many bales in 14400 sheets? *Ans.* 3 bales.
5. How many reams in 5760 sheets? *Ans.* 12 reams.
6. How many reams in 380 quires? *Ans.* 19 reams.

ART. 183.**COUNTING.****TABLE.**

12 single things	make	1 dozen.
12 dozen	"	1 gross.
12 gross	"	1 great gross.
20 units	"	1 score.

EXAMPLES.

1. How many steel pens in 9 gross? *Ans.* 1296 pens.
2. A merchant bought 13 great gross of needles; how many needles did he buy? *Ans.* 22464 needles.
3. The Bible says there were "six score thousand children" in a certain city; how many children were in the city?
Ans. —.
4. A merchant sold 4 great gross of buttons at the rate of 3 cents a dozen; how much did he get for them?
Ans. \$17 28.

5. A grocer sold 3744 bottles of London porter at the rate of \$3.25 per dozen bottles; how much did he receive for the porter? *Ans.* \$1014.

6. A farmer bought $14\frac{1}{2}$ score of sheep at \$1.45 apiece; how much did he pay for the sheep? *Ans.* \$413.25.

MISCELLANEOUS TABLE.

ART. 184. Many articles that were formerly sold by *measure* are now sold almost entirely by *weight*. The following Table embraces a number of the more important:

60 pounds of wheat			make 1 bushel.		
56	"	shelled corn	"	1	"
70	"	corn in the ear	"	1	"
48	"	corn-meal	"	1	"
60	"	peas	"	1	"
56	"	rye	"	1	"
32	"	oats	"	1	"
47	"	barley	"	1	"
60	"	Irish potatoes	"	1	"
55	"	sweet potatoes	"	1	"
60	"	white beans	"	1	"
45	"	castor beans	"	1	"
60	"	clover seed	"	1	"
46	"	timothy seed	"	1	"
56	"	flax seed	"	1	"
44	"	hemp seed	"	1	"
14	"	blue-grass seed	"	1	"
52	"	buckwheat	"	1	"
38	"	dried peaches	"	1	"
24	"	dried apples	"	1	"
57	"	onions	"	1	"
55	"	turnips	"	1	"
25	"	peanuts, or groundnuts,	"	1	"

32 pounds of	cotton seed	make 1 bushel.
20	“ bran	“ 1 “
80	“ stone coal	“ 1 “
80	“ unslacked lime	“ 1 “
8	“ plastering hair	“ 1 “
50	“ coarse salt	“ 1 “
55	“ fine salt	“ 1 “
196	“ flour	“ 1 barrel.
200	“ beef or pork	“ 1 “

REDUCTION.

PROMISCUOUS EXAMPLES.

1. Reduce £5 8s. 4d. 3far. to farthings. *Ans.* 5203far.
2. In 1942 far., how many pounds, etc. ?
Ans. £2 0s. 5d. 2far.
3. In 2584 pence, how many pounds, etc. ?
Ans. £10 15s. 4d.
4. Reduce £13 7s. 5d. to pence. *Ans.* 3209d.
5. In 6lb. 7oz. 8dwt., Troy, how many grains ?
Ans. 38112gr.
6. A goldsmith bought a lump of gold weighing 2lb. 5oz. 9dwt., and paid \$1.05 per pennyweight; what was the value of the gold ?
Ans. \$618.45
7. In 2054dwt., how many pounds, etc. ?
Ans. 8lb. 6oz. 14dwt.
8. Reduce 26597 grains, Troy, to pounds, etc.
Ans. 4lb. 7oz. 8dwt. 5gr.
9. A grocer bought 4T. 7cwt. 2qr. 9lb. of rice at 7 cents per pound; what was the cost of the rice ? *Ans.* \$613.13.
10. In 20672 ounces, avoirdupois, how many cwt., etc. ?
Ans. 12cwt. 3qr. 17lb.
11. A grocer sold 4 barrels of sugar, each barrel weighing

2cwt. 1qr. 12lb., at 13 cents per pound; how much did he receive for the sugar? *Ans.* \$123.24.

12. A druggist made 2lb 3 $\frac{3}{4}$ 2 $\frac{3}{4}$ 2 $\frac{3}{4}$ of morphine into pills, each pill containing *half* a grain; how many pills did he make? *Ans.* 26240 pills.

13. Reduce 3346 $\frac{3}{4}$ to pounds, etc. *Ans.* 11lb 7 $\frac{3}{4}$ 3 $\frac{3}{4}$ 1 $\frac{3}{4}$.

14. What would be the cost of putting up 24 miles of telegraph wire at 17 cents a foot? *Ans.* \$21542.40.

15. How many times would a carriage wheel 16 feet in circumference revolve in going 4mi. 3fur. 8r. 5yd. 1ft.? *Ans.* 1453 times.

16. Reduce 380160 inches to miles. *Ans.* 6mi.

17. In 4A. 3R. 24 sq. rd. how many square yards? *Ans.* 23716 sq. yd

18. A man bought 3A. 2R. 23 sq. rd. of meadow-land at 28 cents per square rod; what was the cost of the land? *Ans.* \$163.24

19. A tablecloth contains 10368 sq. inches; what would be the cost at \$2.15 per square yard? *Ans.* \$17.20.

20. At \$1.65 per square yard, how much would a carpet cost, sufficient to cover a room 30 feet long and 24 feet broad? *Ans.* \$132.

21. What would be the cost of a pile of wood 24 feet long, 12 feet broad and 8 feet high, at the rate of \$5.25 per cord? *Ans.* \$94.50.

22. In 459683 cu. in., how many cubic yards, etc.? *Ans.* 9 cu. yd. 23 cu. ft. 35 cu. in.

23. A grocer sold 3hhd. 12gal. 2qt. of Teneriffe wine at 21 cents a pint; what did he receive for the wine? *Ans.* \$338.52.

24. A vintner bought a hogshead of Madeira wine for \$156.25, and retailed it at 9 cents per gill; did he gain or lose on the sale, and how much? *Ans.* \$25.19 gain.

25. A merchant bought 5184 gills of claret wine at the rate of \$2.25 per gallon; what was the cost of the wine?

Ans. \$364.50.

26. At 13 cents a pint, what would be the cost of 3hhd. 18gal. 2qt. of sugar-house syrup?

Ans. \$215.80.

27. A boy bought 4 bushels of peanuts at \$3.15 per bushel, and afterward retailed them at 7 cents a pint; did he gain or lose, and how much?

Ans. \$5.32 gain.

28. At 6 cents a pint, what would be the cost of 13bu. 3pk. 3qt. of dried apples?

Ans. \$53.16.

29. In 679 pints, how many bushels, etc.?

Ans. 10bu. 2pk. 3qt. 1pt.

30. A boy sold 576 pints of blackberries at the rate of \$2.15 per bushel; how much did he get for the berries?

Ans. \$19.35.

31. A grocer bought 3bu. 2pk. of strawberries at the rate of \$2.12 a peck, and retailed them at 12 cents a pint; did he gain or lose, and how much?

Ans. \$2.80 loss.

32. How many seconds in the month of July?

Ans. 2678400sec.

33. How many minutes in a leap year? *Ans.* 527040min.

34. Reduce 2279365sec. to weeks, etc.

Ans. 3wk. 5da. 9hr. 9min. 25sec.

35. If a locomotive can run *half a mile* in a minute, how far could it run, at this rate, in 3da. 7hr. 25min.?

Ans. 2382½mi.

36. In 10° 15' 25'', how many seconds? *Ans.* 36925''.

37. In 26135'', how many degrees, etc.?

Ans. 7° 15' 35''.

38. At 33¼ cents a quire, what would be the cost of a bale of paper?

Ans. \$66.66¾.

39. How many reams of paper in 6240 sheets?

Ans. 13 reams.

40. A merchant bought 4 great gross of fish-hooks, and sold them at 7 cents a dozen; how much did he get for the hooks?

Ans. \$40.32.

41. A merchant bought a package of hair-pins containing 9 gross 7 dozen and four score; how many pins were in the package?

Ans. 1460 hair-pins.

COMPOUND ADDITION.

ART. 185. COMPOUND ADDITION is the process of uniting two or more COMPOUND quantities to find their SUM.

The principles which govern the addition of compound quantities are the same with those which regulate the addition of simple numbers, with the single exception, that in simple numbers we *invariably* carry 1 for every *ten*, but in *compound* quantities we carry *irregularly*—sometimes 1 for *less* than *ten*, and sometimes 1 for *more* than *ten*.

Ex. 1. What is the sum of 9lb. 6oz. 8dwt. 12gr., 6lb. 8oz. 7dwt. 16gr., 3lb. 7oz. 9dwt. 10gr., and 5lb. 9oz. 8dwt. 18gr.?

Analysis. We write the quantities, placing those of the same denomination in the same column. We begin to add on the right, and we find the sum of the grains to be 56. But 56 grains = 2 pennyweights, and 8 grains over. We write the 8 grains under the column of grains, and carry the 2dwt. to the next column.

OPERATION.

lb.	oz.	dwt.	gr.
9	6	8	12
6	8	7	16
3	7	9	10
5	9	8	18
25	7	14	8

We now add the numbers in the pennyweights' column, and including the 2dwt. carried, we find the sum to be 34dwt. = 1 ounce and 14dwt.; we write the 14dwt. under the column of pennyweights, and carry the 1 ounce to the next column. Adding the numbers in the column of ounces, and including the 1 ounce carried, we find the sum to be 31 ounces = 2

pounds and 7 ounces. We write the 7 ounces under the column of ounces, and carry the 2 pounds to the next column. Adding the numbers in the pounds' column, and carrying the 2 pounds, we find the sum to be 25 pounds, all of which we write under the pounds' column, as we cannot reduce it farther.

Ans. 25lb. 7oz. 14dwt. 8gr.

From the foregoing *Analysis* we derive the

RULE.

I. *Write the quantities, placing those of the same denomination in the same column.*

II. *Beginning at the right, add the numbers of the lowest denomination, and divide the sum by the number which is required of this denomination to make ONE of the next higher. Write the remainder under this denomination, and carry the quotient to the next column.*

III. *Add each column in the same way through all the denominations.*

EXAMPLES.

2. What is the sum of £5 6s. 8d. 3far., £7 5s. 9d. 2fa., £6 9s. 7d. 3far., and £4 8s. 6d. 2far.?

Ans. £23 10s. 8d. 2far.

3. Find the sum of 9s. 8d. 3far., 10s. 7d. 2far., 18s. 9d 3far., and 7s. 8d. 3far.

Ans. £2 6s. 10d. 3far.

4. Add together 8lb. 9oz. 8dwt. 12gr., 7lb. 8oz. 9dwt. 10gr., 6lb. 7oz. 13dwt. 18gr., and 5lb. 6oz. 10dwt. 15gr.

Ans. 28lb. 8oz. 2dwt. 7gr.

5. A silversmith has four silver goblets of the following weights: the first weighs 8oz. 9dwt. 15gr.; the second, 7oz. 12dwt. 13gr.; the third, 6oz. 16dwt. 12gr.; and the fourth, 9oz. 8dwt. 17gr.; what is the entire weight?

Ans. 2lb. 8oz. 7dwt. 9gr.

6. What is the sum of 9T. 7cwt. 3qr. 12lb. 18oz., 7T. 6cwt 3qr. 14lb. 10oz., and 8T. 15cwt. 2qr. 13lb. 12oz.?

Ans. 25T. 10cwt. 1qr. 16lb. 8oz.

7. Find the sum of 14cwt. 3qr. 15lb. 8oz. 12dr., 12cwt. 2qr. 13lb. 12oz. 14dr., and 18cwt. 3qr. 12lb. 9oz. 10dr.

Ans. 2T. 6cwt. 1qr. 16lb. 15oz. 4dr.

8. What is the sum of 7lb 8 $\frac{3}{4}$ 6 $\frac{3}{4}$ 2 $\frac{1}{2}$ 13gr., 8lb 5 $\frac{3}{4}$ 7 $\frac{3}{4}$ 2 $\frac{1}{2}$ 10gr., 12lb 6 $\frac{3}{4}$ 6 $\frac{3}{4}$ 1 $\frac{1}{2}$ 12gr., and 8lb 4 $\frac{3}{4}$ 7 $\frac{3}{4}$ 1 $\frac{1}{2}$ 13gr.?

Ans. 37lb 2 $\frac{3}{4}$ 4 $\frac{3}{4}$ 2 $\frac{1}{2}$ 8gr.

9. A grocer sold 3 casks of Malaga wine: the first contained 33gal. 3qt. 1pt. 2gi.; the second, 34gal. 2qt. 1pt. 3gi.; and the third, 37gal. 3qt. 1pt. 2gi. How many gallons in all?

Ans. 106gal. 2qt. 0pt. 3gi.

10. Add 8hhd. 21gal. 3qt. 1pt., 7hhd. 28gal. 2qt. 1pt., 9hhd. 26gal. 3qt., and 6hhd. 15gal. 1qt. 1pt.

Ans. 31hhd. 29gal. 2qt. 1pt.

11. A farmer sold four loads of corn: the first measured 36bu. 3pk. 7qt. 1pt.; the second, 34bu. 2pk. 6qt.; the third, 38bu. 2pk. 5qt.; and the fourth, 35bu. 2pk. 4qt. What was the whole number of bushels? *Ans.* 145bu. 3pk. 6qt. 1pt.

12. Find the sum of 7pk. 6qt. 1pt., 6pk. 5qt. 1pt., 5pk. 7qt., and 6pk. 3qt. 1pt. *Ans.* 6bu. 2pk. 6qt. 1pt.

13. Add 23da. 12hr. 15min. 12sec., 34da. 17hr. 23min. 35sec., and 27da. 22hr. 24min. 25sec.

Ans. 86da. 4hr. 3min. 12sec.

14. Find the sum of 7wk. 5da. 12hr. 20min. 15sec., 6wk. 4da 18hr. 25min. 36sec., 5wk. 6da. 23hr. 45min. 56sec., and 5wk. 7hr. 15min. 9sec.

Ans. 25wk. 3da. 13hr. 46min. 56sec.

15. What is the sum of 8mi. 5fur. 12r. 4yd. 2ft., 6mi. 7fur. 20r. 5yd. 1ft., and 10mi. 6fur. 25r. 3yd. 2ft.?

Ans. 26mi. 3fur. 19r. 2yd. 2ft.

16. A man has three rows of palings: the first measures

35yd. 2ft. 7in.; the second, 23yd. 1ft. 9in.; and the third, 17yd. 2ft. 10in. What is the entire length?

Ans. 77yd. 1ft. 2in.

17. A farmer has three fields: the first contains 89A. 3R. 27 sq. rd.; the second, 114A. 2R. 18 sq. rd.; and the third, 123A. 3R. 24 sq. rd. How many in the three fields?

Ans. 328A. 1R. 29 sq. rd.

18. Find the sum of 18 cu. yd. 23 cu. ft. 729 cu. in., 24 cu. yd. 25 cu. ft. 956 cu. in., and 27 cu. yd. 21 cu. ft. 823 cu. in.

Ans. 71 cu. yd. 16 cu. ft. 780 cu. in.

19. What is the sum of 9S. 15° 25' 35", 8S. 16° 18' 28", 15S. 34° 28' 30", and 8S. 7° 13' 15"?

Ans. 42S. 13° 25' 48"

COMPOUND SUBTRACTION.

ART. 186. Ex. 1. From £25 7s. 6d. 3far. take £22 4s. 9d. 2far.

Analysis. We write the less quantity under the greater, placing pounds under pounds, shillings under shillings, etc. We then begin at the right, and say: 2far. from 3far. leave 1far., which we write under the column of farthings. As we cannot take 9 pence from 6 pence, we borrow 1 shilling from the 7 shillings of the next column, and call the 1 shilling borrowed, 12 pence; adding the 12 pence borrowed to the 6 pence, we have 18 pence; then 9 pence from 18 pence leave 9 pence; which we write underneath the pence. As we borrowed 1 shilling from the 7 shillings, we, of course, left only 6 shillings remaining. Hence, 4 shillings from 6 shillings leave 2 shillings; which we place under the column of shillings; then, 22 pounds from 25 pounds leave 3 pounds.

OPERATION.

£	s.	d.	far.
25	7	6	3
22	4	9	2
3	2	9	1

Ans. £3 2s. 9d. 1far.

From the *Analysis* we derive the following

RULE.

I. Write the subtrahend under the minuend, so that the same denominations may stand in the same column.

II. Begin at the right hand, and subtract each denomination separately, and write the remainder under the number subtracted.

III. If any number in the subtrahend is greater than its corresponding number in the minuend, borrow a unit of the next higher denomination, and, reducing it to the next lower denomination, add it to the number in the minuend, and then subtract. Proceed in the same way with each denomination, remembering that the number from which a unit has been borrowed must be regarded ONE LESS than it stands.

PROOF. Add the remainder to the subtrahend, just as in simple Subtraction.

EXAMPLES.

2. From £65 4s. 8d. 3far. take £38 9s. 6d. 1far.

Ans. £26 15s. 2d. 2far.

3. Take £23 5s. 4d. 2far. from £36 3s. 8d. 3far.

Ans. £12 18s. 4d. 1far.

4. What is the difference between 32lb. 8oz. 3dwt. 10gr and 17lb. 5oz. 8dwt. 5gr. ?

Ans. 15lb. 2oz. 15dwt. 5gr.

5. From 48lb. 3oz. 9dwt. 12gr. take 29lb. 9oz. 6dwt. 5gr.

Ans. 18lb. 6oz. 3dwt. 7gr.

6. A druggist bought 19lb 5 $\frac{3}{4}$ 6 $\frac{3}{4}$ 2 $\frac{1}{2}$ 12gr. of Epsom salts, and after selling 12lb 9 $\frac{3}{4}$ 3 $\frac{3}{4}$ 1 $\frac{1}{2}$ 5gr., how much remained ?

Ans. 6lb 8 $\frac{3}{4}$ 3 $\frac{3}{4}$ 1 $\frac{1}{2}$ 7gr.

7. From 22lb 8 $\frac{3}{4}$ 4 $\frac{3}{4}$ 2 $\frac{1}{2}$ 8gr. take 12lb 5 $\frac{3}{4}$ 9 $\frac{3}{4}$ 1 $\frac{1}{2}$ 12gr.

Ans. 10lb 2 $\frac{3}{4}$ 3 $\frac{3}{4}$ 0 $\frac{1}{2}$ 16gr.

8. A merchant bought 23T. 6cwt. 3qr. 8lb. of coal; he sold 15T. 8cwt. 1qr. 17lb.; how much remained ?

Ans. 7T. 18cwt. 1qr. 16lb.

9. From 18cwt. 3qr. 10lb. 4oz. 8dr. take 12cwt. 2qr. 15lb 2oz. 10dr. *Ans.* 6cwt. 0qr. 20lb. 1oz. 14dr.

10. A man started on a journey of 75mi. 3fur. 9rd. 4yd., and after traveling 58mi. 6fur. 5rd. 2yd., how much farther had he to travel? *Ans.* 16mi. 5fur. 4rd. 2yd.

11. William had a fishing-line measuring 9yd. 1ft. 5in., and cut off 5yd. 2ft. 9in.; how much was left?

Ans. 3yd. 1ft. 8in.

12. A grocer had a cask of Teneriffe wine containing 35gal. 2qt. 1pt. 3gi., but by accident 7gal. 3qt. 1pt. 1gi. leaked out how much was left? *Ans.* 27gal. 3qt. 0pt. 2gi.

13. From 34hhd. 21gal. 3qt. 1pt. take 18hhd. 48gal. 1qt. 1pt. *Ans.* 15hhd. 36gal. 2qt.

14. A farmer raised 85bu. 2pk. 5qt. of white wheat, and 58bu. 3pk. 2qt. 1pt. of red wheat; how much more did he raise of white than of red? *Ans.* 26bu. 3pk. 2qt. 1pt.

15. A miller bought 256bu. 2pk. 3qt. of corn, and after selling 129bu. 3pk. 7qt. 1pt., how much remained?

Ans. 126bu. 2pk. 3qt. 1pt.

16. A gentleman owned 256A. 2R. 25 sq. rd. of land, and after giving 123A. 3R. 19 sq. rd. to his son, how much had he left? *Ans.* 132A. 3R. 6 sq. rd.

17. From 512A. 1R. 28 sq. rd. 25 sq. yd. subtract 284A 3R. 23 sq. rd. 18 sq. yd.

Ans. 227A. 2R. 5 sq. rd. 7 sq. yd.

18. From 36 cu. yd. 8 cu. ft. 456 cu. in. take 17 cu. yd 5 cu. ft. 936 cu. in. *Ans.* 19 cu. yd. 2 cu. ft. 1248 cu. in.

19. What is the difference between 92da. 5hr. 36min. 27sec and 45da. 12hr. 23min. 30sec.?

Ans. 46da. 17hr. 12min. 57sec.

20. Take 3wk. 6da. 10hr. 45min. 27sec. from 7wk. 4da 16hr. 25min. 48sec. *Ans.* 3wk. 5da. 5hr. 40min. 21sec.

21. A ship in latitude $48^{\circ} 23' 48''$ N. sails southward until

her latitude is $32^{\circ} 34' 26''$ N.; what difference of latitude has she made? *Ans.* $15^{\circ} 49' 22''$.

22. If one place is situated in longitude $65^{\circ} 15' 25''$ E., and another place is in longitude $42^{\circ} 25' 45''$ E., what is the difference of longitude of the two places? *Ans.* $22^{\circ} 49' 40''$.

23. A grocer bought 18bu. of potatoes, and after selling 7bu. 2pk. 5qt., how many had he left?

Analysis. Since there are no *pecks* and no *quarts* in the minuend, we must borrow 1 bushel from the 18 bushels, and call the 1bu. borrowed, 4 *pecks*; we now borrow 1 *peck* from the 4 *pecks*, and call the 1 *peck* borrowed, 8 *quarts*; the minuend will then stand thus: 17bu. 3pk. 8qt. Then, 5qt. from 8qt. leave 3qt.; 2pk from 3pk. leave 1pk.; and 7bu. from 17bu. leave 10bu.

OPERATION.

bu.	pk.	qt.
18	0	0
	7	2
		5
10	1	3

Ans. 10bu. 1pk. 3qt.

24. A merchant bought 45 tons of coal, from which he sold 28T. 12cwt. 2qr. 18lb.; how much was left?

Ans. 16T. 7cwt. 1qr. 7lb.

25. A lady bought a jar of honey which weighed 23lb., but the jar itself weighed 3lb. 10oz. 12dr.; what was the weight of the honey? *Ans.* 19lb. 5oz. 4dr.

26. A vintner bought a cask of wine which held exactly 36gal., but 4gal. 3qt. 1pt. 2gi. leaked out; how much remained? *Ans.* 31gal. 0qt. 0pt. 2gi.

ART. 187. *To find the difference of time between two given dates.*

In computing the interest on notes it is frequently necessary to find the interval of time between two given dates. In finding these intervals we represent each month by its number in the year; *i. e.*, January by 1, February by 2, March by 3, and so on. We also uniformly reckon 30 days to each month, and 12 months to the year.

27. What is the difference of time between April 27, 1869, and September 18, 1872?

Analysis. We write the earlier date under the later, placing the year first, the number of the month next, and the day of the month last. September being the ninth month, and April the fourth, we write 9 and 4 to represent those months. We begin at the right, as in other cases of Compound Subtraction. As we cannot take 27 days from 18 days, we borrow 1 month from the 9 months, calling the month borrowed, 30 days; adding the 30 days to the 18 days, we have 48 days; and by subtracting the 27 days from 48 days, the remainder is 21 days, which we write underneath. Now, as we borrowed 1 month from the 9 months, we, of course, left but 8 months; hence, taking 4 months from 8 months leaves 4 months; then 9 years from 12 years leave 3 years.

OPERATION.

yr.	mo.	da.
1872	9	18
1869	4	27
3	4	21

Ans. 3yr. 4mo. 21da.

28. What is the interval of time from August 25, 1868, to October 16, 1871?

Ans. 3yr. 1mo. 21da.

29. A man borrowed \$500, July 26, 1868, and returned it March 13, 1872; how long did he keep the money?

Ans. 3yr. 7mo. 17da.

30. What is the interval of time from November 18, 1867, to May 24, 1872?

Ans. 4yr. 6mo. 6da.

COMPOUND MULTIPLICATION.

ART. 188. Ex. 1. How much barley can be contained in 7 sacks, each sack holding 4bu. 2pk. 5qt. 1pt.?

Analysis. 7 sacks will hold 7 times as much as 1 sack; therefore we multiply by 7. We begin with the lowest denomination and say, 7 times 1 pint = 7 pints = 3qt., and 1 pint over. We write the 1 pint under

OPERATION.

bu.	pk.	qt.	pt
4	2	5	1
32	2	6	1

the pints' column, and carry the 3qt. to the next product. Then, 7 times 5qt. = 35qt., and the 3qt. we carry make 38qt. = 4pk., and 6qt. over. We place the 6qt. under the quarts' column, and carry the 4pk. to the next product. Then, 7 times 2pk. = 14pk., and the 4pk. we carry make 18pk. = 4bu., and 2pk. over. We write the 2pk. under the pecks' column, and carry the 4bu. to the next product. Lastly, 7 times 4bu. = 28bu., and the 4bu. we carry = 32 bushels, all of which we write in the bushels' column.

Ans. 32bu. 2pk. 6qt. 1pt.

From the *Analysis* we derive the following

RULE.

I. *Write the multiplier under the lowest denomination of the multiplicand, and multiply each denomination in succession.*

II. *Reduce each product, writing the remainder, and carrying the quotient exactly as in COMPOUND ADDITION.*

EXAMPLES.

2. What would be the cost of 6 pieces of cassimere, each piece costing £8 7s. 5d. 3far. ? *Ans.* £50 4s. 10d. 2far.

3. A silversmith made 7 pitchers, each weighing 2lb. 4oz. 12dwt. 8gr.; what was the weight of all the pitchers ?

Ans. 16lb. 8oz. 6dwt. 8gr.

4. How much wheat in 9 sacks, each sack holding 3bu. 3pk. 7qt 1pt. ?

Ans. 35bu. 3pk. 3qt. 1pt.

5. A grocer bought 13 bags of pepper, each bag containing 35lb. 7oz. 5dr.; what was the whole amount ?

Ans. 460lb. 15oz. 1dr.

6. A merchant sold 17 boxes of tobacco, each box weighing 3cwt. 1qr. 18lb.; how much was the entire weight ?

Ans. 2T. 18cwt. 1qr. 6lb.

7. A vintner bought 27 casks of Madeira wine, each cask

containing 35gal. 3qt. 1pt.; how many gallons, etc. did he buy? *Ans.* 968gal. 2qt. 1pt.

8. How much corn would grow on 43 acres of land, if each acre produced 27bu. 3pk. 5qt. 1pt.?

Ans. 1200bu. 2pk. 4qt. 1pt.

9. How far would a man travel in 15 hours, at the rate of 32mi. 5fur. 15r. per hour? *Ans.* 490mi. 0fur. 25r.

10. If a steamship sail 68L. 2mi. 3fur. 25r. in a day, how far could she sail in 13 days?

Ans. 894L. 1mi. 7fur. 5r.

11. How many acres in 12 lots of land, each lot containing 45A. 2R. 23 sq. rd.? *Ans.* 547A. 2R. 36 sq. rd.

12. If a man can travel 35mi. 4fur. 10r. 3yd. in a day, how far could he travel in 11 days at the same rate?

Ans. 390mi. 6fur. 36r.

13. The average daily motion of the moon is $13^{\circ} 10' 35''$; how far does it move in 10 days? *Ans.* $131^{\circ} 45' 50''$.

14. A druggist has 7 bottles of quinine, each bottle containing 2lb 3 $\frac{3}{4}$ 5 $\frac{3}{4}$ 2 $\frac{1}{2}$ 7gr.; how much quinine has he?

Ans. 16lb 2 $\frac{3}{4}$ 0 $\frac{3}{4}$ 1 $\frac{1}{2}$ 9gr.

15. How much kerosene oil in 9 barrels, each barrel containing 42gal. 3qt. 1pt. 3gi.? *Ans.* 386gal. 2qt. 1pt. 3gi.

16. If it requires 4da. 7hr. 15min. 30sec. for a man to travel a certain distance, how long a time would be required to go 9 times the distance? *Ans.* 38da. 17hr. 19min. 30sec.

17. A tropical year consists of 365da. 5hr. 48min. 48sec. how many days would there be in 12 tropical years?

Ans. 4382da. 21hr. 45min. 36sec.

18. What would be the weight of 9 silver tea-sets, each set weighing 4lb. 7oz. 9dwt. 7gr.? *Ans.* 41lb. 7oz. 3dwt. 15gr.

19. If a carload of guano weigh 6T. 7cwt. 3qr. 15lb., how much would 13 car-loads weigh at the same rate?

Ans. 83T. 2cwt. 2qr. 20lb.

COMPOUND DIVISION.

ART. 189. Ex. 1. If 5 pieces of cassimere cost £36 7s. 9d 3far., what would be the cost of 1 piece?

Analysis. It is plain that 1 piece will cost $\frac{1}{5}$ as much as 5 pieces. We must therefore take $\frac{1}{5}$ of £36 7s. 9d. 3far., which is the same as dividing by 5. We begin with the *highest* denomination, and say: $\frac{1}{5}$ of £36 is £7, with a remainder of £1; we write the £7 under the denomination divided, and reducing the remainder, £1, to shillings = 20s., and adding the 7s. of the dividend, we have 27s. Then, $\frac{1}{5}$ of 27s. is 5s., with a remainder of 2 shillings. We write the 5s. under the shillings' column, and reducing the remainder, 2s., to pence = 24d., and adding the 9d. of the dividend, we have 33d. Then, $\frac{1}{5}$ of 33 pence is 6 pence, with a remainder of 3 pence; we write the 6d. under the pence column, and reducing the remainder, 3d., to farthings = 12far., and adding the 3far. of the dividend, we have 15 farthings. Then, $\frac{1}{5}$ of 15far. is 3far., which we write in its proper place under farthings. Hence, we get £7 5s. 6d. 3far. as the cost of 1 piece.

OPERATION.

£	s.	d.	far.
5)36	7	9	3
£7	5s.	6d.	3far.

Ans. £7 5s. 6d. 3far

From the *Analysis* we derive the

RULE.

I. *Begin with the highest denomination, and divide as in simple numbers.*

II. *If a remainder occurs, reduce it to the next lower denomination, add to it the number of the same denomination in the dividend, if any, and divide as before.*

III. *Continue this operation until all the numbers have been divided.*

EXAMPLES.

2. A farmer sowed 7 acres of land in oats, which produced 167bu. 1pk. 6qt. 1pt.; what was the yield per acre?

Ans. 23bu. 3pk. 5qt. 1pt.

3. A silversmith made 6 tankards, weighing 15lb. 7oz. 12dwt. 6gr.; what was the weight of each tankard?

Ans. 2lb. 7oz. 5dwt. 9gr.

4. A grocer bought 9 casks of Teneriffe wine, holding 386gal. 2qt. 1pt. 3gi.; how much was in each cask?

Ans. 42gal. 3qt. 1pt. 3gi.

5. If 10 tierces of rice weigh 58cwt. 3qr. 10lb. 10oz., what is the weight of one cask? *Ans.* 5cwt. 3qr. 13lb. 9oz.

6. In 8 days the moon moves over an arc of $105^{\circ} 24' 40''$; how much is her average daily motion? *Ans.* $13^{\circ} 10' 35''$.

7. If 15 yards of English velvet cost £32 11s. 10d. 2far., what would be the cost per yard? *Ans.* £2 3s. 5d. 2far.

8. A planter sold 8 bales of cotton, the whole weighing 2T. 2cwt. 3qr. 21lb.; what was the average weight per bale?

Ans. 5cwt. 1qr. 12lb.

9. A tract of land, containing 294A. 3R. 24 sq. rd., was divided into 8 equal portions; how much was in each portion?

Ans. 36A. 3R. 18 sq. rd.

10. A ship sailed 2010mi. 5fur. 28r. in 9 days; what was the average distance per day? *Ans.* 223mi. 3fur. 12r.

11. If 12wk. 2da. 3hr. 7min. 30sec. are required for one man to do a piece of work, how long would 5 men require to do the same work? *Ans.* 2wk. 3da. 5hr. 25min. 30sec.

12. If 18 cars carry 168T. 2cwt. 3qr. 19lb. of coal, what is the average load on each car?

Analysis. When the divisor is more than 12, as in this example, it will be more convenient to employ the method of **LONG DIVISION**.

OPERATION.

	T.	cwt.	qr.	lb.	T.	cwt.	qr.	lb.	
18)	168	2	3		19	(9	6	3	8
	162								
		6	T. rem.						
		20							
		122	cwt. in 6T.	2cwt.					
		108							
			14	cwt. rem.					
			4						
			59	qr. in 15cwt.	3qr.				
			54						
				5	qr. rem.				
			25						
			144	lb. in 5qr.	19lb.				
			144						

Ans. 9T. 6cwt. 3qr. 8lb.

13. A wine-dealer bought 37 casks of Madeira wine, the whole containing 1579gal. 1qt. 1pt. 2gi.; what was the average quantity in each cask?

Ans. 42gal. 2qt. 1pt. 2gi.

14. A merchant bought 27 boxes of Havana sugar, the whole weighing 4T. 18cwt. 2qr. 5lb.; what was the average weight of each box?

Ans. 3cwt. 2qr. 15lb.

15. If 32 acres of land produce 698 bushels of corn, how much is the yield per acre?

Ans. 21bu. 3pk. 2qt.

ART. 190. In the foregoing examples the *divisor* has been a *simple* number; but cases often occur in practical business where it becomes necessary to divide one *compound* quantity by another.

16. A commission-merchant has 10T. 8cwt. 3qr. 5lb. of Peruvian guano, which he wishes to put into sacks, each to hold 2cwt. 1qr. 15lb.; how many sacks will it require?

Analysis. Here the divisor and the dividend are *compound* quantities of the same kind; and since each sack is to contain

2cwt. 1qr. 15lb., it is evident that as often as 2cwt. 1qr. 15lb. is contained in 10T. 8cwt. 3qr. 5lb., so many sacks will there be. We first reduce each compound quantity to the *same* denomination. Reducing 2cwt. 1qr. 15lb. to pounds, we have 240 pounds. Reducing 10T. 8cwt. 3qr. 5lb. to pounds, we find 20880 pounds. The process will be thus :

OPERATION.

$$\begin{array}{r}
 240)20880(87 \text{ sacks. } \textit{Ans.} \\
 \underline{1920} \\
 1680 \\
 \underline{1680}
 \end{array}$$

17. A silversmith has 12lb. 11oz. 17dwt. of silver, which he wishes to make into spoons, each dozen of spoons to weigh 1lb. 5oz. 6dwt. 8gr.; how many dozen can he make?

Ans. 9 dozen.

18 The average interval from one new moon to another is 29da 12hr. 43min. 12sec.; how many new moons, then, can occur in 177da. 4hr. 19min. 12sec.? *Ans.* 6 new moons.

19. A miller has 164bu. 1pk. 2qt. of meal, which he wishes to put into bags, each containing 3bu. 2pk. 7qt. 1pt.; how many bags will be required? *Ans.* 44 bags.

20 If a locomotive run 27mi. 6fur. 15r. in one hour, how long would it occupy in running 333mi. 4fur. 20r., at the same rate? *Ans.* 12hr.

PROMISCUOUS QUESTIONS IN COMPOUND QUANTITIES

Ex. 1. In 4hhd. 12gal. 3qt. 1pt., how many gills?

Ans. 8476gi.

2. In 7031 far., how many pounds, etc.?

Ans. £7 6s. 5d. 3far.

3. A grocer bought a cask of bacon weighing 15902 ounces; how many cwt., etc. were in it? *Ans.* 9cwt. 3qr. 18lb. 14oz.

4. A silver tea-set weighs 3lb. 9oz. 8dwt. 17gr.; how many grains does it contain? *Ans.* 21809gr.

5. Reduce 18mi. 3fur. 28r. to ft. *Ans.* 97482ft.

6. In 2273107sec., how many weeks, etc.? *Ans.* 3wk. 5da. 7hr. 25min. 7sec

7. Reduce 26A. 3R. 29 sq. rd. to sq. rd. *Ans.* 4309 sq. rd.

8. In 2427 pints, how many bushels, etc.? *Ans.* 37bu. 3pk. 5qt. 1pt

9. Reduce 5lb 8 $\frac{3}{4}$ 6 $\frac{3}{4}$ 2 $\frac{1}{2}$ to grains. *Ans.* 33040gr.

10. A silversmith has a tea-set weighing 2lb. 5oz. 9dwt 17gr.; a pitcher weighing 2lb. 7oz. 12dwt. 18gr.; and a goblet weighing 10oz. 15dwt. 19gr.; what is the entire weight? *Ans.* 5lb. 11oz. 18dwt. 6gr.

11. A merchant bought 75gal. 3qt. 1pt. 2gi. of Malaga wine, and after selling 48gal. 1qt. 1pt. 3gi., how much remained? *Ans.* 27gal. 1qt. 1pt. 3gi.

12. What is the sum of £8 7s. 5d. 3far., £12 5s. 9d. 2far., and £13 10s. 9d. 2far.? *Ans.* £34 4s. 0d. 3far.

13. From a car-load of coal weighing 9T. 12cwt. 2qr. 13lb. there were sold 5T. 16cwt. 1qr. 18lb.; how much was left? *Ans.* 3T. 16cwt. 0qr. 20lb.

14. What is the sum of 9mi. 3fur. 25r. 4yd. 2ft., 12mi. 7fur. 18r. 3yd. 1ft., and 15mi. 7fur. 24r. 5yd. 2ft.? *Ans.* 38mi. 2fur. 29r. 2yd. 2ft.

15. From 7wk. 5da. 12hr. 28min. 35sec., take 4wk. 6da. 9hr. 32min. 30sec. *Ans.* 2wk. 6da. 2hr. 56min. 5sec.

16. A farmer sold 9 sacks of barley, each sack containing 4bu. 3pk. 5qt. 1pt.; how much did he sell? *Ans.* 44bu. 1pk. 1qt. 1pt.

17. A merchant paid £45 15s. 8d. 1far. for 7 pieces of velvet; what was the price per piece? *Ans.* £6 10s. 9d. 3far

18. A druggist has 6 jars of rhubarb, each containing 2lb 9 $\frac{3}{4}$ 7 $\frac{5}{8}$ 2 $\frac{1}{2}$ 8gr.; what is the entire amount?

Ans. 16lb 11 $\frac{3}{4}$ 6 $\frac{3}{8}$ 2 $\frac{1}{2}$ 8gr.

19. A wine merchant bought 9 casks of Sherry wine, the whole quantity being 323gal. 2qt. 1pt. 3gi.; what was the average quantity in each cask? *Ans.* 35gal. 3qt. 1pt. 3gi.

20. If one car can convey 9T. 12cwt. 3qr. 14lb. of iron, how much could be carried on 15 cars?

Ans. 144T. 13cwt. 1qr. 10lb.

21. A tract of land containing 1790A. 3R. 16 sq. rd. was equally divided among 8 persons; what was the share of each?

Ans. 223A. 3R. 17 sq. rd.

22. What would it cost to carpet a room 36 feet long and 24 feet broad, at \$1.35 per square yard? *Ans.* \$129.60.

23. A market woman sold 5bu. 3pk. 7qt. of tomatoes at 6 cents a pint; how much did she receive? *Ans.* \$22.92.

24. A wine merchant sold 42gal. 3qt. 1pt. of Teneriffe wine at 8 cents a gill; how much did he get for the wine?

Ans. \$109.76.

25. If a ship sail 54L. 2mi. 5fur. 24r. in a day, how many days would she require to sail 658L. 2mi. 3fur. 8r.?

Ans. 12da.

DENOMINATE FRACTIONS.

ART. 191. A DENOMINATE FRACTION is either a common fraction, or a decimal whose *unit* belongs to some denomination of a compound quantity. Thus, $\frac{3}{4}$ of a bushel is a denominate fraction, the bushel being the *unit*; $\frac{1}{2}$ of a shilling is also a denominate fraction, the shilling being the *unit*. In the same way, .25 of a gallon and .375 of an hour are denominate fractions.

ART. 192. DENOMINATE FRACTIONS admit of various operations, such as Reduction, Addition, Subtraction, etc.

CASE I.

ART. 193. *To reduce a compound quantity to a common fraction of a higher denomination.*

Ex. 1. Reduce 7gal. 2qt. 1pt. to the fraction of a hogshead.

Analysis. This question is just the same as if we had asked, "What part of a hogshead is 7gal. 2qt. 1pt.?" Reducing 7gal. 2qt. 1pt. to pints, we find 61 pints; and reducing 1 hogshead to pints, we have 504 pints. Our question *now* might read thus: "What part of 504 pints is 61 pints?" Since 504 pints make 1 hogshead, then 1 pint is $\frac{1}{504}$ of a hogshead, and 61 pints are 61 times as much = $\frac{61}{504}$.

Ans. $\frac{61}{504}$ hhd.

It will be observed that in this example we first reduced the given compound quantity to the lowest denomination mentioned, and this became the numerator of the fraction. We also reduced a unit of the higher denomination to the same denomination with the numerator, and this became the denominator of the fraction. Hence, we have the

RULE.

Reduce the given compound quantity to its lowest denomination for the numerator, and a unit of the required denomination for the denominator, of the required fraction.

EXAMPLES.

2. Reduce 2ft. 9in. to the fraction of a yard.

Ans. $\frac{3}{4}$ = $\frac{3}{4}$ yd

3 Reduce 3qt. 1pt. 3gi. to the fraction of a gallon.

Ans. $\frac{13}{16}$ gal.

4 Reduce 12s. 9d. 3far. to the fraction of a pound sterling

Ans. $\frac{15}{16}$ = $\frac{15}{16}$ £

5. What part of a cwt. is 3qr. 5lb. 8oz.? Ans. $\frac{1}{16}$ cwt.

6. What part of an acre is 3R. 24 sq. rd.? Ans. $\frac{1}{16}$ A.

7. What fraction of a bushel is 3pk. 5qt. 1pt.?

Ans. $\frac{59}{16}$ bu.

8. Reduce 9d. 3far. to the fraction of a shilling.

Ans. $\frac{1}{8}$ s.

9. Express 5da. 12hr. 36min. as the fraction of a week.

Ans. $\frac{221}{840}$ wk.

10. What fraction of a pound, Troy, is 8oz. 12dwt. 12gr.?

Ans. $\frac{3}{16}$ lb.

11. What part of a bushel is 3pk. 6 $\frac{3}{4}$ qt.?

Analysis. Reducing 3pk. 6 $\frac{3}{4}$ qt. to quarts, we obtain 30 $\frac{3}{4}$ quarts, and reducing a bushel to quarts, we have 32 quarts. The question now is, "What part of 32 quarts is 30 $\frac{3}{4}$ quarts?" With the numbers in their present shape we can express the answer only as a *complex* fraction. To avoid this, we reduce 30 $\frac{3}{4}$ quarts to *thirds* of a quart, and we have $\frac{2}{3}$ quarts. We also reduce the 32 quarts to *thirds*, and we have $\frac{96}{3}$ quarts. Our question now reads thus: "What part of $\frac{96}{3}$ quarts is $\frac{2}{3}$ quarts?" We say, $\frac{1}{3}$ of a quart is $\frac{1}{96}$ part of $\frac{96}{3}$ quarts, and $\frac{2}{3}$ quarts will be 92 times as much = $\frac{92}{3}$ or $\frac{31}{1}$.

Ans. $\frac{31}{1}$ bu.

12. Reduce 2ft. 7 $\frac{1}{2}$ in. to the fraction of a yard.

Ans. $\frac{7}{12}$ yd

13. What part of a pound is 10d. 2 $\frac{1}{2}$ far.? Ans. $\frac{57}{1120}$

14. What fraction of a gallon is 3qt. 1 $\frac{1}{2}$ pt.?

Ans. $\frac{3}{8}$ gal.

CASE II.

ART. 194. To reduce a compound quantity to a DECIMAL of a higher denomination.

Ex. 1. Reduce 3qt. 1pt. 3gi. to the decimal of a gallon.

Analysis. Reducing 3qt. 1pt. 3gi. to gills, we have 31 gills; and reducing 1 gallon to gills, we have 32 gills. Since 32 gills = 1 gallon, 1 gill is $\frac{1}{32}$ of a gallon, and 31 gills will be

$\frac{3}{4}$ of a gallon; and changing $\frac{3}{4}$ to a decimal (Art. 147), we have .96875 of a gallon. *Ans.* .96875gal.

From the foregoing *Analysis* we derive the

RULE.

Reduce the compound quantity to a common fraction of the required denomination; then reduce this fraction to a DECIMAL.

EXAMPLES.

2. Reduce 3pk. 4qt. to the decimal of a bushel.

Ans. .875bu.

3. Reduce 3qr. 10lb. 12oz. to the decimal of a cwt.

Ans. .8575cwt.

4. Reduce 7s. 6d. 3far. to the decimal of a pound.

Ans. £.378125.

5. Reduce 18hr. 36min. to the decimal of a day.

Ans. .775da.

6. Reduce 9oz. 15dwt. 18gr. to the decimal of a pound.

Ans. .815625lb.

7. Reduce 4yd. 2ft. 6in. to the decimal of a rod.

Ans. .878787 + r.

8. Reduce 3R. 25 sq. rd. to the decimal of an acre.

Ans. .90625A.

9. Reduce 5qt. 1pt. to the decimal of a peck.

Ans. .6875pk.

10. Express 7mi. 4fur. 20r. 2yd. decimally, making the mile the unit.

Ans. 7.5636363 + mi.

11. Express 35gal. 2qt. 1pt. decimally, making the gallon the unit.

Ans. 35.625gal.

12. Express 29cwt. 2qr. 6lb. 4oz. decimally, making the cwt. the unit.

Ans. 29.5625cwt.

CASE III.

ART. 195. *To reduce a fraction of a HIGHER denomination to its value in whole numbers of LOWER denominations.*

Ex. 1. Reduce $\frac{2}{3}$ of a pound, Troy, to its value in ounces, etc.

Analysis. Since 1 pound is equal to 12 ounces, $\frac{2}{3}$ of a pound is $\frac{2}{3}$ of 12 ounces = 8 ounces = 6 ounces, and $\frac{2}{3}$ of an ounce over. Again, since 1 ounce is equal to 20dwt., $\frac{2}{3}$ of an ounce is $\frac{2}{3}$ of 20dwt. = $13\frac{1}{3}$ dwt., or 13dwt. and $\frac{1}{3}$ of a dwt. over. Lastly, since 1dwt. is equal to 24 grains, $\frac{1}{3}$ of a dwt. is $\frac{1}{3}$ of 24 grains = 8 grains, or 8 grains. Hence, the value of $\frac{2}{3}$ of a pound, Troy, is 6oz. 13dwt. 8gr. *Ans.* 6oz. 13dwt. 8gr.

NOTE.—The teacher will find it a very profitable exercise to make his pupils write each step of the Analysis in a series of Equations, thus:

$$\begin{aligned}
 1\text{lb.} &= 12 \text{ ounces.} \\
 \frac{2}{3}\text{lb.} &= 8 \text{ ounces.} \\
 \frac{2}{3}\text{lb.} &= 6 \text{ ounces and } \frac{2}{3} \text{ of an ounce.} \\
 \text{Again, } 1\text{oz.} &= 20\text{dwt.} \\
 \frac{2}{3}\text{oz.} &= 13\frac{1}{3}\text{dwt.} \\
 \frac{2}{3}\text{oz.} &= 13\text{dwt. and } \frac{1}{3} \text{ of a dwt.} \\
 \text{Lastly, } 1\text{dwt.} &= 24\text{gr.} \\
 \frac{1}{3}\text{dwt.} &= 8\text{gr.} \\
 \frac{1}{3}\text{dwt.} &= 8\text{gr.} = 8\text{gr.}
 \end{aligned}$$

Ans. 6oz. 13dwt. 8gr.

EXAMPLES.

2. Reduce $\frac{3}{4}$ of a pound sterling to its value in shillings, etc. *Ans.* 15s. 6d. $2\frac{1}{2}$ far
3. What is the value of $\frac{1}{8}$ of a cwt.? *Ans.* 3qr. 18lb. 12oz.
4. What is the value of $\frac{1}{7}$ of a week? *Ans.* 5da. 10hr. 40min.
5. What is the value of $\frac{2}{10}$ of a hogshead? *Ans.* 56gal. 2qt. $1\frac{1}{2}$ pt.
6. What is the value of $\frac{1}{4}$ of a pound, Troy? *Ans.* 8oz. 11dwt. $10\frac{1}{2}$ gr.

7. What is the value of $\frac{1}{2}$ of a day? *Ans.* 21hr. 20min.
 8. Reduce $\frac{1}{11}$ of a furlong to its value in rods, etc.
Ans. 21r. 4yd. 1ft. 6in.
 9. What is the value of $\frac{1}{4}$ of a bushel?
Ans. 3pk. 3qt. $1\frac{7}{8}$ pt.
 10. What is the value of $\frac{1}{2}$ of a pound, Troy?
Ans. 9oz. 12dwt
 11. What is the value of $\frac{1}{2}$ of a degree? *Ans.* 33' 20''

CASE IV.

ART. 196. *To reduce a DECIMAL of a HIGHER denomination to whole numbers of a LOWER denomination.*

Ex. 1. Reduce .724 of a pound, Troy, to its value in ounces, etc.

Analysis. In .724 of a pound there are 12 times as many *thousandths* of an ounce, which gives 8 whole ounces, and 688 *thousandths* of an ounce. Then in 688 *thousandths* of an ounce there are 20 times as many *thousandths* of a pennyweight, which gives 13 whole pennyweights, and 760 *thousandths* of a pennyweight. Then in 760 *thousandths* of a pennyweight there are 24 times as many *thousandths* of a grain, which gives 18 whole grains, and 240 *thousandths* of a grain over. We have, therefore, as the result,

OPERATION.

.724lb.
12
8.688oz.
20
13.760dwt.
24
3040
1520
18.240gr.

Ans. 8oz. 13dwt. 18.240gr.

EXAMPLES.

2. Reduce .785da. to its value in hours, etc.
Ans. 18hr. 50min. 24sec.
 3. Change .965gal. to quarts, etc.
Ans. 3qt. 1pt. 2.88gi.

4. Reduce .742cwt. to quarters, etc.

Ans. 2qr. 24lb. 3oz. 3.2dr.

5. Reduce .9625bu. to pecks, etc. *Ans.* 3pk. 6qt. 1.6pt

6. Reduce .6875wk. to days, etc.

Ans. 4da. 19hr. 30min.

7. Change £.89 to shillings, etc. *Ans.* 17s 9d. 2.4far.

8. Change .495T. to cwt., qr., etc. *Ans.* 9cwt. 3qr 15lb.

9. What is the value of £8.785?

Ans. £8 15s. 8d. 1.6far.

10. What is the value of 15.9375cwt.?

Ans. 15cwt. 3qr. 18lb. 12oz.

CASE V.

ART. 197. *To reduce a fraction of a LOWER Denomination to another of a HIGHER denomination.*

Ex. 1. Reduce $\frac{5}{8}$ of a gallon to the fraction of a hogshead.

Analysis. This question is just the same as if we had asked, "What part of a hogshead is $\frac{5}{8}$ of a gallon?" Since 63 gallons = 1hhd., then 1 gallon is $\frac{1}{63}$ of 1 hogshead; and $\frac{1}{8}$ of a gallon is $\frac{1}{8}$ as much = $\frac{1}{504}$ of a hogshead; and $\frac{5}{8}$ of a gallon will be 5 times as much = $\frac{5}{504}$ of a hogshead. *Ans.* $\frac{5}{504}$ hhd.

2. What part of a mile is $\frac{7}{8}$ of a yard?

Analysis. Reducing 1 mile to yards, we have 1760 yards. Then 1 yard is $\frac{1}{1760}$ of a mile; and $\frac{1}{8}$ of a yard is $\frac{1}{8}$ as much = $\frac{1}{15840}$ of a mile; and $\frac{7}{8}$ of a mile will be 7 times as much = $\frac{7}{15840}$ of a mile. *Ans.* $\frac{7}{15840}$ mi.

The above example might have been exhibited under the form of a series of Equations, thus:

OPERATION.

$$1760\text{yd.} = 1\text{mi.}$$

$$1\text{yd.} = \frac{1}{1760}\text{mi.}$$

$$\frac{1}{8}\text{yd.} = \frac{1}{15840}\text{mi.}$$

$$\frac{7}{8}\text{yd.} = \frac{7}{15840}\text{mi.}$$

EXAMPLES.

- 3 Reduce $\frac{3}{4}$ of a pint to the fraction of a bushel.
Ans. $\frac{3}{448}$ bu.
4. What part of a shilling is $\frac{3}{4}$ of a farthing?
Ans. $\frac{6}{112}$ s.
5. What part of a pound, Troy, is $\frac{1}{16}$ of a dwt.?
Ans. $\frac{1}{288}$ lb.
6. What part of a day is $\frac{5}{6}$ of a minute?
Ans. $\frac{1}{1728}$ da.
7. What fraction of a ton is $\frac{3}{11}$ of a pound?
Ans. $\frac{1}{3750}$ T.
8. What fraction of a pound sterling is $\frac{1}{2}$ of a penny?
Ans. $\frac{1}{240}$ £.
9. What part of an acre is $\frac{1}{2}$ of a square yard?
Ans. $\frac{1}{6000}$ A.

CASE VI.

ART. 198. To reduce a fraction of a HIGHER denomination to another of a LOWER denomination.

Ex. 1. Reduce $\frac{1}{1920}$ of a mile to the fraction of a yard.

Analysis. This question is the same as to ask, "What part of a yard is $\frac{1}{1920}$ of a mile?" Reducing 1 mile to yards, we have 1760 yards. Then $\frac{1}{1920}$ of a mile is the same as $\frac{1}{1920}$ of 1760 yards = $\frac{1760}{1920}$ yd. = $\frac{11}{120}$ yd. *Ans.* $\frac{11}{120}$ yd.

2. Reduce $\frac{7}{1728}$ of a pound sterling to the fraction of a penny.

Analysis. Reducing 1 pound to pence we have 240 pence. Then $\frac{7}{1728}$ of a pound is the same as $\frac{7}{1728}$ of 240 pence; but $\frac{7}{1728}$ of 240 pence is $\frac{240}{1728}$ of a penny, and $\frac{7}{1728}$ is 7 times as much = $\frac{1680}{1728}$ of a penny = $\frac{35}{36}$ of a penny. *Ans.* $\frac{35}{36}$ d.

OPERATION.

$$£1 = 240d.$$

$$£\frac{7}{1728} = \frac{240}{1728}d.$$

$$£\frac{7}{1728} = \frac{1680}{1728}d. = \frac{35}{36}d.$$

EXAMPLES.

3. Reduce $\frac{3}{512}$ of a bushel to the fraction of a pint.
Ans. $\frac{3}{8}$ pt.
4. Reduce $\frac{5}{1612}$ of a hogshead to the fraction of a quart.
Ans. $\frac{5}{8}$ qt.
5. Reduce $\frac{11}{2880}$ of a pound, Troy, to the fraction of a dwt.
Ans. $\frac{1}{2}$ dwt.
6. Reduce $\frac{15}{1296}$ of a yard to the fraction of an inch.
Ans. $\frac{5}{12}$ in.
7. Reduce $\frac{10}{1048}$ of an acre to the fraction of a sq. rd.
Ans. $\frac{5}{262}$ sq. rd.
8. Reduce $\frac{5}{7776}$ of a day to the fraction of a minute.
Ans. $\frac{5}{1296}$ min.
9. Reduce $\frac{5}{864}$ of a shilling to the fraction of a farthing.
Ans. $\frac{5}{1728}$ far.

ADDITION OF DENOMINATE FRACTIONS.

ART. 199. Ex. 1. What is the sum of $\frac{7}{8}$ of a pound, Troy, $\frac{1}{2}$ of an ounce and $\frac{7}{8}$ of a pennyweight?

Analysis. In their present shape we cannot add them. We may, however, reduce them to integral quantities of lower denominations, and then add them. By CASE III., $\frac{7}{8}$ of a pound is equal to 9oz. 6dwt. 16gr., $\frac{1}{2}$ of an ounce is 6dwt. 16gr., and $\frac{7}{8}$ of a pennyweight is 21 grains. Hence, the question is reduced simply to finding the sum of 9oz. 6dwt. 16gr., 6dwt. 16gr., and 21 grains. The process may be exhibited thus:

OPERATION.			
	oz.	dwt.	gr.
$\frac{7}{8}$ lb. =	9	6	16
$\frac{1}{2}$ oz. =		6	16
$\frac{7}{8}$ dwt. =			21
	9oz.	14dwt.	5gr.

Ans 9oz. 14dwt. 5gr

2. What is the sum of .3675 of a day, .875 of an hour and .45 of a minute?

Analysis. By CASE IV. (Art. 196), .3675da. may be reduced to 8hr. 49min. 12sec., .875hr. may be changed to 52min. 30sec., and .45min. to 27sec., after which the several quantities may be added according to the Rule for Compound Addition. The whole process may be thus shown :

OPERATION.

	hr.	min.	sec.
.3675da. =	8	49	12
.875hr. =		52	30
.45min. =			27
	9hr.	42min.	9sec.

Ans. 9hr. 42min. 9sec.

EXAMPLES.

3. Add $1\frac{3}{8}$ cwt. $7\frac{1}{10}$ qr. and $\frac{3}{4}$ lb. *Ans.* 3qr. 24lb. 8oz.

4. Add $\frac{5}{8}$ hhd. $1\frac{1}{8}$ gal. and $\frac{7}{8}$ qt. *Ans.* 53gal. 2qt. 0pt. 1gi.

5. Add £ $\frac{5}{8}$ $1\frac{3}{8}$ s. and $\frac{3}{4}$ d. *Ans.* 13s. 4d. 2far.

6. Add $\frac{5}{8}$ wk. $1\frac{3}{8}$ da. and $\frac{7}{8}$ hr. *Ans.* 6da. 16hr. 16min. 40sec.

7. Add £.3125 .5625s. and .75d. *Ans.* 6s. 10d. 2far.

3. Add .375hhd. .4375gal. and .875qt. *Ans.* 24gal. 1qt. 0pt. 1gi.

SUBTRACTION OF DENOMINATE FRACTIONS.

ART. 200. The preliminary operations required in the *Subtraction* of Denominate Fractions are precisely the same with those required in the *Addition* of those fractions. We first reduce the fractions to whole numbers of lower denominations, and then subtract as in Compound Subtraction.

Ex. 1. From $\mathcal{L}1\frac{1}{2}$ take $\frac{1}{2}$ s.

OPERATION		
	s.	d. far.
$\mathcal{L}1\frac{1}{2} =$	9	4 2
$\frac{1}{2}$ s. =	9	3
	8s.	6d. 3far.

Ans. 8s. 6d. 3far.

2. From $\frac{3}{4}$ lb., Troy, take $\frac{1}{4}$ oz. Ans. 9oz. 15dwt. 20gr.

3. From $\frac{1}{2}$ cwt. take $\frac{1}{4}$ qr. Ans. 1qr. 15lb. 10oz.

4. From $\frac{1}{2}$ da. take $\frac{1}{4}$ hr. Ans. 12hr. 27min. 30sec.

5. From $\frac{1}{2}$ mi. take 3fur. 25r. Ans. 3fur. 35r.

6. From .9375bu. take 2pk. 3qt. 1pt.
Ans. 1pk. 2qt. 1pt.

LONGITUDE AND TIME.

ARTICLE 201. The sun in his apparent daily course from east to west passes over 360° of longitude in a day, or 24 hours; then, in 1 hour, he passes over $\frac{1}{24}$ of 360° , $= \frac{1}{24} \times 360^\circ = 15^\circ$; in 1 minute he passes over $\frac{1}{60}$ of 15° , $= \frac{1}{60} \times 15^\circ = \frac{1}{4}^\circ = 15'$; and in 1 second he passes over $\frac{1}{60}$ of $15'$, $= \frac{1}{60} \times 15' = \frac{1}{4}' = 15''$. Hence, we have the following

TABLE OF LONGITUDE AND TIME.

15° of longitude $=$ 1 hour of time.

$15'$ of longitude $=$ 1 minute of time.

$15''$ of longitude $=$ 1 second of time.

When the sun is directly over the meridian of any given place, it is noon at that place, and to all places *east* of this the time is past noon, and to those *west* of it it is before noon.

Hence, if we know the time at any place, to find the time at any other locality we *add* the difference of time if the place is *east*, or *subtract* when it is *west*.

CASE I.

ART. 202. *To find the difference of longitude between two places when their difference of time is given.*

Ex. 1. The difference of time between two places is 5hr 3min. 2sec.; what is their difference of longitude?

Analysis. By the table we see that 1 second of time = $15''$ of longitude, and therefore 2sec. of time = $2 \times 15'' = 30''$ of longitude. Also, from the table, 1min. of time = $15'$ of longitude; hence, 3min. of time = $3 \times 15' = 45'$ of longitude; and as 1 hour = 15° of longitude, then 5 hours = $5 \times 15^\circ = 75^\circ$ of longitude. Hence, our result is $75^\circ 45' 30''$.

OPERATION.		
5hr. 3min. 2sec.		
		15
		75° 45' 30''

Therefore, *to change time into longitude we simply multiply by 15 as in other compound quantities.*

EXAMPLES.

2. The difference of time between two places is 2hr. 10min. 22sec.; what is their difference of longitude?

Ans. $32^\circ 35' 30''$.

3. The difference of time between Springfield, Mass., and Indianapolis is 58 minutes; what is their difference of longitude?

Ans. $14^\circ 30'$.

4. The difference of time between Boston and Savannah is 40m. 16sec.; what is their difference of longitude?

Ans. $10^\circ 4'$.

5. If Boston is in longitude $71^\circ 4' W.$, what is the longitude of Savannah?

Ans. $81^\circ 8' W.$

6. The longitude of Philadelphia is $75^\circ 10' W.$, and the dif-

ference of time between it and Montgomery is 44m. 52sec.; what is the longitude of Montgomery? *Ans.* $86^{\circ} 23' W.$

7. A ship sailed from Norfolk, longitude $76^{\circ} 19' W.$, toward Liverpool. Out on the ocean the captain took an observation, and found the local time to be 20 minutes past 5 P. M., while his chronometer, which showed Norfolk time, indicated 15min. after 3 P. M.; what was the longitude of the ship?

Ans. $45^{\circ} 4' W.$

8. Vicksburg is in longitude $90^{\circ} 54' W.$, and the difference of time between it and Albany, N. Y., is 1hr. 8m. 36sec.; what is the longitude of Albany? *Ans.* $73^{\circ} 45' W.$

9. The difference of time between Greenwich and Portland, Me., is 4hr. 41min. 4sec.; what is the longitude of Portland?

Ans. $70^{\circ} 16' W.$

10. The difference of time between Philadelphia and Jacksonville, Fla., is 20min. 2sec.; what is their difference of longitude? *Ans.* $5^{\circ} 0' 30''.$

11. Two persons observed an eclipse of the moon. One noticed its commencement, at 38min. past 8 P. M., and the other saw it 52min. past 9 P. M.; what was the difference of longitude of the two observers? *Ans.* $18^{\circ} 30'.$

CASE II.

ART. 203. *To find the difference of time between two places when their difference of longitude is given.*

Ex. 1. The difference of longitude between two places is $60^{\circ} 30' 45''$; what is their difference of time?

Analysis. By the table, 15° of longitude = 1 hour, hence in 60° of longitude there will be $\frac{1}{15}$ as many hours = 4 hours. Again, as $15'$ of longitude = 1 min. of time, then $30'$ of longitude = $\frac{1}{15}$ as many min. of time = 2 min. of time; and

OPERATION.

$$\begin{array}{r} 15 \overline{) 60^{\circ} 30' 45''} \\ \underline{4 \text{ hr. } 2 \text{ m. } 3 \text{ sec.}} \end{array}$$

as $15''$ of longitude = 1sec. of time, then $45'' = \frac{1}{18}$ as many sec. of time, = 3 seconds. Hence, we obtain 4hr. 2min. 3sec. as our result. Hence, *to change longitude into time we simply divide by 15.*

EXAMPLES.

2. The difference of longitude of two places is $48^{\circ} 20'$; what is their difference of time? *Ans.* 3hr. 13m. 20sec.

3. The longitude of Burlington, Vt., is $73^{\circ} 15' W.$, and Denver is $105^{\circ} 4' W.$; what is the difference in time of the two places? *Ans.* 2hr. 7min. 16sec.

NOTE.—First find the difference of longitude, and then change this into time.

4. When it is 10 o'clock A. M. at Burlington, what is the time at Denver? *Ans.* 52min. 44sec. past 7 A. M.

5. When it is half-past 11 A. M. at Denver, what is the time at Burlington? *Ans.* 37min. 16sec. after 1 P. M.

6. The longitude of Boston is $71^{\circ} 4' W.$, and that of Savannah is $81^{\circ} 8' W.$; when it is noon at Savannah, what is the time at Boston? *Ans.* 40min. 16sec. after noon.

7. New Haven is in longitude $72^{\circ} 57' W.$, and San Francisco is $122^{\circ} 26' 15'' W.$; what is the time at San Francisco when it is 40 minutes after 9 A. M. at New Haven?

Ans. 22min. 3sec. after 6 A. M.

8. Louisville, Ky., is in longitude $85^{\circ} 52' W.$, and Portland, Me., is $70^{\circ} 16' W.$ If a man should set his watch to Portland time, and travel to Louisville, would his watch be too fast or too slow for the Louisville time, and how much?

Ans. Too fast by 1hr. 2m. 24sec.

9. The longitude of St. Petersburg is $30^{\circ} 45' E.$, and Galveston is $94^{\circ} 50' W.$; how much earlier will the sun reach the meridian of St. Petersburg than that of Galveston?

Ans. 8hr. 22min. 20sec.

NOTE.—When one place is in longitude E., and another in longitude W., their *difference* of longitude will be found by *adding* the two longitudes.

10. The Cape of Good Hope is in longitude $18^{\circ} 29'$ E., and Washington is $77^{\circ} 1'$ W.; what is the difference of time between the two places? *Ans.* 6hr. 22min.

11. Pike's Peak is in longitude $104^{\circ} 59'$ W., and Augusta, Ga., is in $81^{\circ} 54'$ W. If a man should set his watch to the time of the Peak, and travel to Augusta, would his watch, as compared with the Augusta time, be too fast or too slow, and how much? *Ans.* Too slow by 1hr. 32min. 20sec.

PERCENTAGE.

ARTICLE 204. PER CENT. is from the Latin phrase *per centum*, and means *by the hundred*; thus, 7 *per cent.* signifies 7 of every hundred, and may mean 7 *dollars* of every hundred *dollars*, or 7 *bushels* of every hundred *bushels*, or 7 *horses* of every hundred *horses*, etc.

ART. 205. The symbol, %, is very generally used by merchants and bankers to represent the words *per cent.*; thus, 6% signifies 6 *per cent.*; 9% is read 9 *per cent.*

ART. 206. PER CENT. may be expressed either as a common fraction or as a decimal. Thus, 8 *per cent.* may be written either $\frac{8}{100}$ or .08; 5 *per cent.* may be expressed as $\frac{5}{100}$ or .05. The usual custom is to express it decimally.

1	<i>per cent.</i>	is expressed thus,	.01.
2	"	"	" .02.
3	"	"	" .03.
5	"	"	" .05.
10	"	"	" .10.

14	<i>per cent.</i>	is expressed thus,	.14.
28	"	"	.28.
45	"	"	.45.
75	"	"	.75.
100	"	"	1.00.
150	"	"	1.50.
225	"	"	2.25.
$\frac{1}{2}$	"	"	.005.
$\frac{1}{4}$	"	"	.0025.
$3\frac{1}{2}$	"	"	.035.

Sometimes the parts of 1 *per cent.* are written as common fractions joined to the decimals; thus,

$5\frac{1}{2}$	<i>per cent.</i>	may be written	.05 $\frac{1}{2}$.
$8\frac{3}{4}$	"	"	.08 $\frac{3}{4}$.
$12\frac{1}{2}$	"	"	.12 $\frac{1}{2}$.
$16\frac{2}{3}$	"	"	.16 $\frac{2}{3}$.
$24\frac{1}{2}$	"	"	.24 $\frac{1}{2}$.

ART. 207. RATE PER CENT. is the number which expresses how many *hundredths* of a quantity are to be taken.

ART. 208. PERCENTAGE is the amount obtained by taking a certain *per cent.* of any given quantity.

ART. 209. Under Percentage are included Commission, Brokerage, Insurance, Interest, Banking, Discount, etc.

Ex. 1. A gentleman had \$975, and spent 9% of it for board; how much did he spend?

Analysis. Since 9% is the same as $\frac{9}{100}$, or .09, it is plain that the gentleman must have spent $\frac{9}{100}$ of \$975; but $\frac{1}{100}$ of \$975 is \$9.75, and $\frac{9}{100}$ is 9 times as much = \$87.75.

OPERATION.

\$975
.09
\$87.75

Ans. \$87.75.

NOTE.—In this example 9 is the RATE PER CENT., and \$87.75 is the PERCENTAGE.

2. A farmer shipped 475 bushels of wheat to market, but on the way 7% of it was wasted; how many bushels were wasted?

OPERATION.
475bu.
.07

33.25bu.

Analysis. 1% of 475bu. is 4.75bu., and 7% is 7 times as many bushels = 33.25 bushels.

Ans. 33.25bu.

NOTE.—Here 7 is the RATE PER CENT., and 33.25bu. is the PERCENTAGE.

Hence, to find the PERCENTAGE on any quantity we have the

RULE.

Multiply the quantity by the rate per cent., expressed decimally, and point off the product as in Decimals.

EXAMPLES.

3. A cotton-buyer invested \$3875 in cotton, and sold it at a profit of 7%; what was his entire profit? *Ans.* \$271.25.

4. A gentleman paid 9% for the rent of a house valued at \$5675; what was the rent? *Ans.* \$510.75.

5. A dealer in coal bought 9325 bushels, but in getting it to market 5% was wasted; how much was wasted?

Ans. 466.25bu.

6. A ship loaded with 2275 boxes of goods was overtaken by a storm, when the sailors threw overboard 16% of the cargo; how many boxes were saved? *Ans.* 1911 boxes.

7. A drover at Cincinnati bought 725 hogs, but on the way to market lost 4% of them; how many were left?

Ans. 696 hogs.

8. A gentleman borrowed \$1825, and paid 7% for the use of it; how much did he pay? *Ans.* \$127.75.

9. A grocer bought 4275 bananas, but 12% of them spoiled; how many were left? *Ans.* 3762 bananas.

10. In a barn 6296 bushels of wheat were stored. The

barn having taken fire, $12\frac{1}{2}\%$ of the wheat was lost; how much was saved? *Ans.* 5509bu.

11. A shepherd having 445 sheep lost 20% of them by disease during the year; how many had he remaining?

Ans. 356 sheep.

12. A merchant with 5625 dollars went to Philadelphia, where he invested 17% in groceries, 43% in dry goods, 12% in clothing and the remainder in shoes; how much did he invest in each article?

Ans. \$956.25 groceries; \$2418.75 dry goods; \$675 clothing; \$1575 shoes.

13. A locomotive drawing 324 passengers had a collision, by which $16\frac{3}{4}\%$ of the passengers were injured; how many were injured?

Ans. 54 passengers.

14. A planter has 480 acres in cultivation— $37\frac{1}{2}\%$ is planted in corn, $43\frac{3}{4}\%$ in cotton and the remainder in wheat; how many acres of each article?

Ans. 180 corn; 210 cotton; 90 wheat.

15. A merchant bought 5 casks of Madeira wine, each cask containing 45gal., at \$2.75 per gallon, and sold it at a loss of 8% ; what was the entire loss?

Ans. \$49.50.

16. What must be a clerk's salary that 12% of it may be \$144?

Analysis. If 12% of his salary is \$144, then 1% of it is $\frac{1}{12}$ of \$144 = $\$12$; and if 1% is \$12, then 100% , or the whole salary, is 100 times as much = \$1200.

Ans. \$1200.

17. If 8% of a number is 128, what is the number?

Ans. 1600.

18. If 15% of a merchant's stock in trade is \$975, how much is his entire stock?

Ans. \$6500.

19. If 7% of an army is 2961 men, what is the entire strength of the army?

Ans. 42300 men.

20. A planter sold 52 bales of cotton, which was 13% of his entire crop; what was his whole crop? *Ans.* 400 bales.

21. Of the trees in an orchard, 56 are peach trees, and these are 8% of the whole; how many trees in the orchard?

Ans. 700 trees.

22. What is that number of which 168 is 14%?

Ans. 1200.

23. A grocer bought 840 pineapples. During the first week he sold $16\frac{2}{3}\%$ of his stock, and in the second he sold 25% of the remainder; how many had he left?

Analysis. $16\frac{2}{3}\%$ of 840 pineapples is 140 pineapples. He therefore sold 140 pineapples the first week. Taking 140 from 840, we see that he had 700 pineapples left. During the second week, he sold 25% of what remained; 25% of 700 pineapples is 175 pineapples. Taking 175 pineapples from 700 pineapples, we see he had 525 pineapples left.

Ans. 525 pineapples.

24. A gentleman deposited \$750 in bank; at one time he drew out 18%, and afterward drew out 20% of the remainder; how much was still left in the bank?

Ans. \$492.

25. In an orchard containing 540 fruit trees, 10% died the first year, and $16\frac{2}{3}\%$ of the remainder died during the second year; how many trees still remained?

Ans. 405 trees.

26. A farmer had 720 bushels of wheat. At one time he sold 10% of his stock; at another time he sold $16\frac{2}{3}\%$ of the remainder; and finally, 20% of what still remained; how many bushels had he left?

Ans. 432bu.

27. Of the trees in an orchard, $22\frac{1}{2}\%$ bear cherries, $12\frac{1}{2}\%$ bear figs, 50% bear peaches, and the remaining 45 bear plums; how many trees in the orchard?

Analysis. As 100% represents the *whole* of a thing, and as the cherries, figs and peaches make up 85%, hence the plums must be 15%. Then, if 15% of the orchard is 45 trees, 1%

of the orchard is $\frac{1}{8}$ of 45 trees = $\frac{45}{8}$ trees, or 3 trees; and 100%, or the entire orchard = 100 times as many trees = 300 trees.

Ans. 300 trees.

28. A gentleman being asked what was the value of his property, said that he had 30% of it in bank, $33\frac{1}{3}\%$ in a factory, $16\frac{2}{3}\%$ in real estate, and the remainder, worth \$6950, was invested in land; what was the value of his property?

Ans. \$34750.

29. A farmer has 35% of his land planted in corn, $12\frac{1}{2}\%$ in wheat, 13% in oats, $27\frac{1}{2}\%$ in cotton, and the remainder, containing 54 acres, is devoted to pasturage; how many acres in the farm?

Ans. 450 acres.

30. Five gentlemen jointly built a school-house. The first contributed 17% of the money; the second, 28%; the third, 32%; the fourth, 15%; and the fifth paid \$144; what was the cost of the house?

Ans. \$1800.

31. 12% of 450 is 15% of what number?

Analysis. 12% of 450 is 54. Then, since 15% of a certain number is 54, 1% of the number is $\frac{1}{15}$ of 54 = $\frac{54}{15}$, and 100%, or the whole number, is 100 times as much = $\frac{54 \times 100}{15}$, or 360.

Ans. 360.

32. 12% of 525 is 21% of what number? *Ans.* 300.

33. $16\frac{2}{3}\%$ of 432 is 18% of what number? *Ans.* 400.

34. $33\frac{1}{3}\%$ of 168 is 16% of what number? *Ans.* 350.

35. 15% of how many pounds will produce 372 pounds?

Ans. 2480 pounds.

36. 8% of how many dollars will produce \$78.72?

Ans. \$984.

37. 7% of how many pounds will make 47.25 pounds?

Ans. 675 pounds.

38. A gentleman owes \$425, and has only \$255 with which to pay it; what per cent. of the debt can he pay?

Analysis. If he pays \$1, he pays $\frac{1}{425}$ of the debt, and if

he pays \$255, he pays 255 times as much $= \frac{255}{\frac{5}{8}} = \frac{2}{3}$, and reducing $\frac{2}{3}$ to a decimal, we have $.60 = 60$ per cent.

Ans. 60%.

39. What % of an acre is 48r.?

Ans. 30%.

40. What % of a bushel is 18qt.?

Ans. $56\frac{1}{4}\%$.

41. What % of a yd. is 2ft. 3in.?

Ans. 75%.

42. What % of a gal. is 3qt. 1 pt.?

Ans. $87\frac{1}{2}\%$.

43. What % of a day is 8hr. 40min.?

Ans. $36\frac{1}{3}\%$.

44. What % of £1 is 8s. 6d.?

Ans. $42\frac{1}{2}\%$.

45. What % of a mile is 88yd.?

Ans. 5%.

46. What % of \$5 is 40 cents?

Ans. 8%.

47. What % of $\frac{5}{8}$ is $\frac{1}{4}$?

Analysis. $\frac{5}{8} = \frac{1}{2}\frac{5}{4}$; $\frac{1}{4} = \frac{1}{2}\frac{1}{4}$. Then $\frac{1}{4}$ is $\frac{1}{15}$ of $\frac{1}{2}\frac{5}{4}$, and $\frac{1}{2}\frac{5}{4}$ is $\frac{5}{15}$, which, reduced to *hundredths*, gives $.53\frac{1}{3} = 53\frac{1}{3}\%$.

Ans. $53\frac{1}{3}\%$.

48. What % of $\frac{4}{5}$ is $\frac{1}{4}$?

Ans. $56\frac{1}{4}\%$.

49. What % of $\frac{4}{5}$ is $\frac{2}{3}$?

Ans. $83\frac{1}{3}\%$.

50. What % of $\frac{7}{8}$ is $\frac{7}{10}$?

Ans. 80%.

51. What % of $\frac{3}{5}$ is $\frac{2}{3}$?

Ans. $71\frac{2}{3}\%$.

52. What % of $2\frac{2}{3}$ is $2\frac{1}{4}$?

Ans. $84\frac{2}{3}\%$.

53. What % of $\frac{4\frac{2}{3}}{5\frac{2}{3}}$ is $\frac{3\frac{1}{2}}{5\frac{1}{4}}$?

Ans. 80%.

54. A ship laden with 1060 barrels of flour was overtaken by a storm, and 371 barrels were thrown overboard; what per cent. of the flour was saved?

Ans. 65%.

55. A farmer had 450 sheep, and after a certain time they had increased to 531; what was the rate % of increase?

Ans. 18%.

56. A man had \$876 deposited in bank, and drew out a certain % of his money, when only \$730 remained; what % did he draw out?

Ans. $16\frac{2}{3}\%$.

57. At what rate % of itself must 924 be decreased to become 785.4?

Ans. 15%.

COMMISSION AND BROKERAGE.

ART. 210. AN AGENT, FACTOR OR COMMISSION MERCHANT is a person who buys or sells goods or who transacts business for others.

ART. 211. COMMISSION is the fee or percentage allowed to an agent, factor or commission merchant.

ART. 212. A BROKER is a person who deals in money, stocks, bills of exchange, etc. The commission paid to a broker is called BROKERAGE.

ART. 213. COMMISSION and BROKERAGE are usually reckoned at a certain per cent. on the amount of money employed in any particular transaction.

Ex. 1. An agent in Boston buys \$4276 worth of silks for a merchant in Atlanta; what is the agent's commission, at $2\frac{1}{4}\%$?

Analysis. If on \$1 he receives $2\frac{1}{4}\%$, or $.02\frac{1}{4}$, then on \$4276 he receives 4276 times as much $= \$4276 \times .02\frac{1}{4} = \$95.71$.

2. Find the commission on \$4096 at $2\frac{3}{4}\%$.

Ans. \$112.64.

3. A broker sells a bill of exchange worth \$6448; what is his brokerage at $1\frac{3}{8}\%$?

Ans. \$88.66.

4. A lawyer collects \$3575.40 for a client; what is his commission at 5% ?

Ans. \$178.77.

5. A broker sells a bill of exchange worth £5150; estimating the pound sterling at \$4.86, and allowing the broker $\frac{3}{4}\%$, what is the amount of his brokerage?

Ans. \$187.71 $\frac{1}{4}$.

6. A commission merchant sells 152 boxes of tobacco, at \$37 a box; how much is his commission at $3\frac{1}{4}\%$?

Ans. \$182.78.

7. What is the brokerage on \$9352 at $\frac{3}{8}\%$? *Ans.* \$35.07.

8. A factor sells 345 bales of cotton, averaging 512lb., at $9\frac{3}{8}$ cents a pound; what is his commission at $1\frac{3}{4}\%$?

Ans. \$289.80.

9. An agent receives \$35.04 commission for buying shoes at an allowance of 4% on the purchase; what amount of shoes does he purchase?

Analysis. Let $100\% =$ Amount of purchase.

$$4\% = \$35.04 = \text{Agent's commission.}$$

$$1\% = \$8.76.$$

$$100\% = \$8.76 \times 100 = \$876.$$

Proof. $\$876 \times .04 = \$35.04.$

10. An agent receives \$18.75 for selling fruit at 3% commission; what is the value of the fruit? *Ans.* \$625.

11. If at 5% the administrator of an estate receives \$473.25, how much is the estate? *Ans.* \$9465.

12. A broker received \$216 for buying stocks at $\frac{3}{4}\%$ brokerage; how much stock did he buy? *Ans.* \$57600.

13. A manufacturer in Lowell sends \$988 to his agent in Savannah to buy cotton and pay his commission on the purchase; what is the agent's commission at 4%, and how much does he expend in cotton?

Analysis. Let $100\% =$ Amt. expended for cotton.

$$4\% = \text{Agent's commission,}$$

$$100\% + 4\% = 104\% = \$988 = \text{Entire amt. sent.}$$

$$1\% = \$\frac{988}{104} = \$9.50.$$

$$100\% = \$9.50 \times 100 = \$950.$$

$$4\% = \$9.50 \times 4 = \$38 = \text{Agent's com.}$$

14. A college in Georgia sends to its agent in New York \$750 with which to purchase apparatus, including, also, the agent's commission at 5%; how much does the agent expend for apparatus? *Ans.* \$714.28 $\frac{1}{4}$.

15. A factor receives \$1611.95 with which to purchase sugar. After deducting his commission of 3%, how much does he lay out in sugar? *Ans.* \$1565.

16. What rate % does a commission merchant charge who receives \$30.24 for selling \$756 worth of fruit?

Analysis. If on \$756 the commission is \$30.24, then on \$1 it is $\frac{1}{756}$ of \$30.24 = $\frac{30.24}{756} = \$0.04 = 4\%$. *Ans.* 4%.

17. If an agent receives \$33.74 for selling \$964 worth of flour, what is his rate %? *Ans.* $3\frac{1}{2}\%$.

18. If the brokerage for selling 64 shares of railroad stock at \$97.50 per share is \$85.80, what is the rate %? *Ans.* $1\frac{3}{8}\%$.

19. A factor receives \$50.40 for selling 336 barrels of flour at \$6 $\frac{3}{4}$ a barrel; what is the rate %? *Ans.* $2\frac{1}{4}\%$.

20. A commission merchant sells 185 barrels of apples at \$3.20 per barrel, and, reserving his commission, remits \$572.76; what is the rate of his commission? *Ans.* $3\frac{1}{4}\%$.

21. An agent receives \$126 for selling 96 bales of cotton at \$52.50 a bale; what is the rate %? *Ans.* $2\frac{1}{2}\%$.

INSURANCE.

ART. 214. **INSURANCE** is security against loss or damage, guaranteed by one party to another, in consideration of a stipulated price paid by the latter to the former.

ART. 215. The written contract between the parties is called a **POLICY**.

ART. 216. The price paid for insurance is called the **PREMIUM**.

The **PREMIUM** paid for insurance is usually reckoned at a certain per cent. on the property insured.

Ex. 1. What would be the annual premium for insuring a house valued at \$9756, at 2 per cent.?

<i>Analysis.</i> If on \$1 there is paid 2 per cent.,	OPERATION.
or .02, then on \$9756 there must be paid 9756	\$9756
times as much = $\$9756 \times .02 = \$195.12$.	<u>.02</u>
<i>Ans.</i> \$195.12.	\$195.12

2. What annual premium must be paid for insuring a house worth \$7850, at $2\frac{1}{4}$ per cent. ? *Ans.* \$176.62 $\frac{1}{2}$.

3. What annual premium at $1\frac{1}{2}$ per cent. must be paid for insuring a house valued at \$4275, and furniture worth \$975 ?

Ans. \$78.75.

4. A ship from New York, valued at \$52650, was insured for a whaling-cruise at $8\frac{1}{4}$ per cent. ; what was the premium ?

Ans. \$4343.62 $\frac{1}{2}$.

5. A ship worth \$42865, with her cargo of molasses valued at \$23263, was insured for a voyage from St. Thomas to New York at $1\frac{1}{2}$ per cent. ; how much was the premium ?

Ans. \$743.94.

6. A cargo of tobacco, consisting of 462 hogsheads, worth \$256.50 per hogshead, was insured for a voyage from New York to Liverpool at $2\frac{3}{4}$ per cent. ; what was the premium ?

Ans. \$3258.83 $\frac{1}{2}$.

7. A gentleman has a house worth \$16500, and furniture valued at \$3675, both of which are insured for $\frac{2}{3}$ of their value at $1\frac{1}{2}$ per cent. ; what annual premium does he pay ?

Ans. \$201.75.

8. A gentleman pays an annual premium of \$98.25 for insuring his barn, at 3 per cent. ; what is the amount insured ?

Analysis. Since 3 cents will insure \$1 for a year, 1 cent will insure $\frac{1}{3}$ as much = $\frac{1}{3}$, and 9825 cents will insure 9825 times as much = \$3275, or \$3275.

Ans. \$3275.

9. When the rate of insurance is 4% annually, what amount can be insured for \$94.75 ?

Ans. \$2368.75.

10. A gentleman pays an annual premium of \$85.75 for insuring his mill, at 2% ; how much does he insure ?

Ans. \$4287.50.

11. At an annual rate of 5 per cent., what amount can be insured for \$212.25 ?

Ans. \$4245.

12. A commission-merchant ships a cargo of cotton from

Mobile to Liverpool, paying a premium of \$1303.92 at 4% for the voyage; what is the amount insured? *Ans.* \$32598.

13. The annual premium for insuring a factory at $2\frac{1}{4}\%$ is \$1137.15; what is the amount insured? *Ans.* \$45486.

ART. 217. Sometimes it is desirable to insure for a sum sufficient to cover the value of the property and the premium paid, in the event of a loss.

14. A gentleman owns a paper-mill worth \$6545, which he wishes to insure for a sum sufficient to cover the value of the mill and the premium of insurance, the rate being 3%; for what sum must he insure it?

Analysis. The *whole* amount to be insured may be represented by $\frac{100}{100}$, or 100 per cent., and since the premium is 3 per cent., the value of the mill would be represented by 100 per cent., *diminished* by 3 per cent. = 97 per cent. Then, as 97 per cent. = \$6545, 1 per cent. is $\frac{1}{97}$ of \$6545 = $\$6747.42$, and 100 per cent. is 100 times as much = $\$674742$ = \$6747.42 +.

Ans. \$6747.42 +.

15. For what sum must a gentleman insure his house and furniture, worth \$2450, in order to cover the value and the premium, the rate of insurance being 2%? *Ans.* \$2500.

16. For what sum must a stock of groceries, worth \$9720, be insured at 4% premium, so as to include the value and the premium? *Ans.* \$10125.

17. A college insures its library, worth \$7425, for a sum sufficient to cover its value and the premium; allowing 1% as the rate of insurance, for what sum must it be insured?

Ans. \$7500.

STOCKS.

ART. 218. STOCK is a general name for money or capital invested in manufactures, railroads, telegraphs, banking, etc., etc.

STOCK is usually divided into *shares*, and the persons who own these shares are called STOCKHOLDERS.

ART. 219. Shares are usually reckoned at \$100 each, but sometimes they are *more* and sometimes *less*.

ART. 220. The original valuation of a share is called its **NOMINAL** or **PAR VALUE**; when it sells for more than the nominal value it is said to be **ABOVE PAR**, or **AT A PREMIUM**; when it sells for less than the nominal value it is **BELOW PAR**, or **AT A DISCOUNT**.

Thus, if \$100 worth of stock sells for \$100, it is at par; if it sells for \$107, it is above par, or at a premium of 7 per cent.; if it sells for \$97, it is below par, or at a discount of 3 per cent.

ART. 221. Stocks are bought and sold just as cotton or sugar, and the price varies according to the demand.

NOTE.—In the succeeding examples \$100 will be regarded as the nominal value of a share, unless otherwise specified.

EXAMPLES.

Ex. 1. A man buys 15 shares of Georgia railroad stock at a premium of 5 per cent.; what does the whole cost?

Analysis. As the stock is 5 per cent. above par, each share is worth \$105, and 15 shares will be worth 15 times as much
 $= \$105 \times 15 = \$1575.$

Ans. \$1575.

2. A gentleman buys 23 shares of stock in a paper-mill at a discount of 6 per cent.; what does he pay for the stock?

Analysis. As the stock is 6 per cent. below par, each share is worth only \$94; then 23 shares will be worth 23 times as much
 $= \$94 \times 23 = \$2162.$

Ans. \$2162.

3. If I buy 36 shares of bank stock at an advance of 8 per cent., what does the stock cost me?

Ans. \$3888.

4. If I purchase 45 shares of stock in an iron-foundry at a discount of 9 per cent., what do I pay for the stock?

Ans. \$4095.

5. A man bought 39 shares of stock in a telegraph company

at 3 per cent. discount, the par value being \$50 per share; what was the cost? *Ans.* \$1891.50.

6. A gentleman buys 35 shares of bank stock at a discount of 2%, and afterward sells it at an advance of 3%; how much does he make by the operation? *Ans.* \$175.

7. If I purchase 52 shares of stock at a premium of 4%, and afterward sell it at a discount of 3%, how much do I lose on the transaction? *Ans.* \$364.

8. A man buys 34 shares of stock in a telegraph company at a discount of 3%, the par value being \$50 per share, and afterward sells it at a premium of $3\frac{1}{2}\%$; what does he gain? *Ans.* \$110.50.

9. A dividend of 9 per cent. is declared on the stock of the C. R. R. Company; what sum should a stockholder receive who owns 47 shares?

Analysis. The par value of each share being \$100, there will be a dividend of \$9 on one share, and on 47 shares there will be 47 times as many dollars = $\$9 \times 47 = \$423$.

Ans. \$423.

10. If a bank declares a dividend of $8\frac{1}{2}\%$ per cent., what sum would a stockholder receive who owns 57 shares?

Ans. \$484.50.

11. A paper-mill declares a dividend of $4\frac{1}{2}\%$ per cent.; how much would a man receive who owns 75 shares, the par value of a share being \$50? *Ans.* \$168.75.

12. If the S. W. Railroad pays $3\frac{1}{2}\%$ semi-annually, how much would a stockholder receive during the year who owns 65 shares? *Ans.* \$455.

13. How much would a stockholder receive who owns 25 shares in a manufacturing company, the par value of a share being \$500, and a dividend of 9% being declared?

Ans. \$1125.

14. If a stockholder in a railroad receives \$378 as his share

of the dividends at 7%, how many shares does he own, the par value of a share being \$100?

Analysis. The stockholder receives \$7 on every \$100; therefore to receive \$1 requires *one seventh* of \$100 = ~~\$100~~, and to receive \$378 will require 378 times as much stock = ~~\$37800~~, or \$5400, which will be 54 shares. *Ans.* 54 shares.

15. If the profits of a telegraph company for one year are 7 per cent., how many shares does a stockholder own who receives \$252 as his part of the dividends, the par value of a share being \$50? *Ans.* 72 shares.

16. If a dividend of 9% is declared on the stock of a bank, how many shares are owned by a stockholder who receives \$567, the par value being \$100 per share? *Ans.* 63 shares.

17. If the profits of a railroad are $3\frac{1}{2}\%$ semi-annually, how many shares does a stockholder own who receives \$336, the par value of a share being \$100? *Ans.* 48 shares.

18. A mining-company, with a capital of \$75456, makes a clear profit of \$5281.92 in a year; what rate of dividend may it declare?

Analysis. Since a capital of \$75456 clears \$5281.92, then \$1 clears $\frac{1}{75456}$ of \$5281.92 = $\frac{5281.92}{75456} = \$0.07$, or 7 per cent. *Ans.* 7%.

19. An iron-foundry, employing a capital of \$45844, clears \$3667.52 in a year; what rate of dividend may be declared? *Ans.* 8%.

20. A glass-factory, with a capital of \$25375, makes a clear profit of \$2283.75; what per cent. may it divide among the stockholders? *Ans.* 9%.

21. The gross earnings of a flouring-mill, costing \$32575, are \$6136.75, and the expenses are \$3856.50; what rate per cent. of dividend may be declared? *Ans.* 7%.

22. How much bank stock, at 3 per cent. discount, may be bought for \$3395, no charge being made for brokerage?

Analysis. The par value of a share being \$100, and the discount 3 per cent., it is obvious that a share will cost \$97; then, if \$97 will buy 1 share, \$1 will buy $\frac{1}{97}$ of a share, and \$3395 will buy 3395 times as many shares = $\frac{3395}{97}$ shares, or 35 shares.

Ans. 35 shares.

23. How many shares of factory stock could be bought for \$5985 when the stock is at a discount of 5 per cent.?

Ans. 63 shares.

24. When railroad stock is at a discount of 2 per cent., how many shares could be bought for \$5292? *Ans.* 54 shares.

25. How much railroad stock, at a premium of 9 per cent., could be bought for \$3815, brokerage not included?

Analysis. The par value being \$100 per share, it is plain that with a premium of 9% a share will cost \$109; then, if \$109 will buy 1 share, \$1 will buy $\frac{1}{109}$ of a share, and \$3815 will buy 3815 times as many shares = $\frac{3815}{109}$ shares, or 35 shares.

Ans. 35 shares.

26. When bank stock is at a premium of 5%, how many shares could be bought for \$3885? *Ans.* 37 shares.

27. In February, 1869, Central Railroad stock was 30% above par; how many shares could have been bought for \$11050? *Ans.* 85 shares.

28. In 1869, when gold was at a premium of 32%, how many dollars in gold could have been purchased with \$1155 in greenbacks? *Ans.* \$875.

29. When silver is at a premium of 12%, how many dollars in silver would \$846.72 in current funds purchase? *Ans.* \$756.

30. When gold commands a premium of 13%, how many dollars in greenbacks would buy \$956 in gold? *Ans.* \$1080.28.

31. When silver is at a premium of 9%, how much in greenbacks would \$845 in silver be worth? *Ans.* \$921.05

For more full information on Stocks consult "Sanford's Higher Analytical Arithmetic."

BANKRUPTCY.

ART. 222. When a person fails in business and is unable to pay his debts, he is called a *bankrupt*. If he owns any property whatever, it is distributed among his creditors in proportion to the amount he owes to each.

The rule which apportions the sum due to each creditor is called BANKRUPTCY.

Ex. 1. A bankrupt is indebted as follows, viz.: to A, \$600; to B, \$448; and to C, \$960. He owns property amounting to \$753; how much can he pay to each creditor?

Analysis. By the conditions of the question a debt of \$2008 is to be paid with \$753; then, \$1 of debt is worth only $\frac{1}{2008}$ of \$753 = $\frac{\$753}{2008}$, or $\frac{3}{8}$ of a dollar, and \$600 (A's debt) will be worth 600 times as much = $\$1800$, or \$225. Again, since \$1 of debt is worth only $\frac{3}{8}$, then \$448 (B's debt) will be worth 448 times as much = $\$1344$, or \$168; and since \$1 of debt is worth only $\frac{3}{8}$, then \$960 (C's debt) will be 960 times as much = $\$2880$, or \$360.

Ans. A, \$225; B, \$168; C, \$360.

EXAMPLES.

2. A farmer, failing in business, owes the following sums, viz.: to P, \$720, to R, \$1125, and to S, \$2475. His only property is a lot of land worth \$2400; how much can he pay to each creditor? Ans. P, \$400; R, \$625; S, \$1375.

3. A merchant, meeting with losses, owes the following sums, viz.: to E, \$2476, to H, \$3684, and to K, \$4856; his only property is a stock of groceries valued at \$2754; how much can he pay to each creditor?

Ans. E, \$619; H, \$921; K, \$1214

4. A bank failed, being indebted to the amount of \$325485. On winding up its affairs it was found to possess a banking-house valued at \$65750, office furniture worth \$13875, notes of other banks to the amount of \$30560, and \$3734.75 in silver; how much can it pay on the dollar? *Ans.* 35 cents.

PROFIT AND LOSS.

ART. 223. In carrying on business, men sometimes make a *profit*, at other times they *lose*, in trading. The Rule of "Profit and Loss" teaches them how to ascertain their gain or loss in buying or selling. It also teaches how to adjust the prices of goods so as to *gain* or to *lose* a certain per cent.

CASE I.

ART. 224. *To find the per cent. gained or lost when we know the cost of an article and the selling price.*

Ex. 1. A grocer buys a cask of syrup for \$65, and sells it for \$78; what is his gain per cent.?

Analysis. As he buys the syrup for \$65 and sells it for \$78, he *gains* \$13 on the whole. Then, as he gains \$13 on \$65, on \$1 he gains $\frac{1}{5}$ of \$1 = $\frac{1}{5}$ of a dollar, or 20 cents on each dollar, which is 20 per cent. *Ans.* 20%.

2. A man bought a buggy for \$225, and afterward sold it for \$191.25; what per cent. did he lose?

Analysis. As he bought it for \$225 and sold it for \$191.25, he evidently lost \$33.75 on the sale. If on \$225 he lost \$33.75, then on \$1 he lost $\frac{1}{15}$ of \$33.75 = $\frac{1}{15}$ of a dollar = 15 cents on the dollar, or 15 per cent. *Ans.* 15%.

3. If I buy muslin at 60 cents, and sell it for 75 cents per yard, what per cent. do I gain? *Ans.* 25%.

4. A merchant buys oil at 75 cents per gallon, and sells it at 60 cents; what per cent. does he lose? *Ans.* 20%.

5. A grocer buys a box of lemons for \$9.25, and sells them for \$10.73; what per cent. does he make? *Ans.* 16%.

6. A merchant buys cassimere at \$2.40 per yard, and sells it at \$2.10; what per cent. does he lose? *Ans.* $12\frac{1}{2}\%$.

7. A grocer buys cheese at 16 cents, which he sells at 19 cents per pound; what per cent. does he realize?

Ans. $18\frac{1}{4}\%$.

8. A gentleman bought a rockaway for \$325, and afterward sold it for \$260; what per cent. did he lose? *Ans.* 20% .

9. If I buy Hyson tea at 60 cents, and sell it at 80 cents per pound, what per cent. do I gain? *Ans.* $33\frac{1}{3}\%$.

10. If I buy tea at 80 cents, and sell it at 60 cents per pound, what per cent. do I lose? *Ans.* 25% .

11. A wine merchant bought a cask of Teneriffe wine containing 48 gallons, for \$120, and afterward retailed it at 85 cents per quart; did he gain or lose, and what per cent.?

Ans. Gained 36% .

CASE II.

ART. 225. *To find the whole amount of profit or loss when we know the cost of an article and the per cent. gained or lost.*

Ex. 1. A miller bought a load of wheat for \$65, and sold it at a profit of 15 per cent.; what was his whole profit?

Analysis. If on \$1 he gains 15 cents, or \$0.15, then on \$65 he gains 65 times as much
 $= \$65 \times .15 = \$9.75$.

OPERATION.

\$65
.15
325
65
\$9.75

Ans. \$9.75.

2. A grocer bought a lot of tobacco for \$225, and sold it for 13 per cent. less than the cost; what was his entire loss?

Ans. \$29.25.

3. A merchant bought a hogshead of sugar for \$145, and sold it at a profit of 17 per cent.; what was his whole profit?

Ans. \$24.65.

4. A miller bought 348 bushels of wheat at \$1.25 per

bushel, and sold it at $12\frac{1}{2}$ per cent. profit; how much was his whole profit?

Ans. \$54.37 $\frac{1}{2}$.

5. A merchant bought 5 casks of Malaga wine, each cask containing 43 gallons, at \$1.25 per gallon, and sold it so as to lose 14 per cent.; what was the entire loss?

Ans. \$37.62 $\frac{1}{2}$.

CASE III.

ART. 226. *Knowing the cost of an article, to find at what price to sell it so as to gain or lose a certain per cent.*

Ex. 1. A vintner bought a cask of Madeira wine for \$145, at what price must he sell it so as to gain 22 per cent.?

Analysis. The selling price must evidently be equal to the cost, added to the profit made on the sale; to find the profit, we proceed as in CASE II., and say, If on \$1 he gains 22 per cent., or .22 of a dollar, then on \$145 he will gain 145 times as much = \$31.90 profit, which, added to the cost, will give \$176.90 as the selling price.

Ans. \$176.90.

2. A merchant bought a crate of crockery for \$75, but in consequence of breakage lost 18% on the sale of it; for how much did he sell it?

Analysis. The selling price in this case is equal to the cost, diminished by the loss in selling. To find the loss, we say: If on \$1 he loses 18 per cent., or .18 of a dollar, then on \$75 he loses 75 times as much = \$13.50; and this, subtracted from the cost, gives \$61.50 as the selling price.

Ans. \$61.50.

3. A man bought a buggy for \$225, and sold it so as to gain 15%; what was the selling price?

Ans. \$258.75.

4. A merchant bought a box of boots for \$154, and lost $12\frac{1}{2}$ % on the sale; what was the selling price?

Ans. \$134.75.

5. If alpaca is bought at 75 cents a yard, what must be the selling price so as to gain 20%?

Ans. 90 cents.

6. If I buy cheese at 16 cents per pound, at what price must I sell it so as to lose 25%? *Ans.* 12 cents.

7. If wine is bought at \$2.25 per gallon, at what price must it be sold to gain 33 $\frac{1}{3}$ %? *Ans.* \$3.

8. If eggs are bought at 36 cents per dozen, what must be the selling price to realize a profit of 16 $\frac{2}{3}$ %? *Ans.* 42 cents.

9. A miller bought wheat at \$1.70 per bushel, and sold it so as to lose 12 $\frac{1}{2}$ %; what was the selling price? *Ans.* \$1.48 $\frac{1}{2}$.

CASE IV.

ART. 227. *Given the selling price and the per cent. gained or lost, to find the original cost.*

Ex. 1. A merchant sold a piece of silk for \$147.50, and thereby made a profit of 18 per cent.; what was the cost of the silk?

Analysis. As he gains 18 cents on every 100 cents, it is plain that what he sold for 118 cents must have cost 100 cents, and hence what he sold for 1 cent must have cost $\frac{1}{118}$ of 100 cents = $\frac{100}{118}$ of a cent, and what he sold for \$147.50, or 14750 cents, must have cost 14750 times as much = $\frac{14750 \times 100}{118}$ cents, or \$125. *Ans.* \$125.

2. A grocer sold a barrel of kerosene oil for \$44.45, which was 25% more than the cost; what was the cost? *Ans.* \$35.56.

3. A provision dealer sold a cask of bacon for \$158.94, thereby realizing a profit of 20%; what was the cost? *Ans.* \$132.45

4. Sold salt at \$2.52 per sack, making a profit of 12 per cent.; what did it cost? *Ans.* \$2.25.

5. A merchant sold calico at 18 cents per yard, and made 20% profit; what was the cost of the calico? *Ans.* 15 cents.

6. A grocer sold a cask of wine for \$63.75, and thereby lost 15 per cent.; what was the cost of the wine?

Analysis. As the grocer *loses* 15 cents on every 100 cents, it is plain that what he sold for 85 cents must have cost 100 cents, and what he sold for 1 cent must have cost $\frac{1}{85}$ of 100 cents = $\frac{100}{85}$ of a cent; and hence, what he sold for \$63.75, or 6375 cents, must have cost 6375 times as much = $\frac{6375 \times 100}{85}$ cents, or \$75. *Ans.* \$75.

7. A merchant sold a lot of tobacco for \$292.50, which was 10 per cent. less than the cost; what was the cost?

Ans. \$325.

8. By selling muslin at 72 cents a yard, a merchant loses 20%; what is the cost of the muslin? *Ans.* 90 cents.

9. A man sold a rockaway for \$280.56, and thereby lost 16 per cent.; what did the rockaway cost? *Ans.* \$334.

10. By selling butter at 18 cents a pound, a grocer loses 25%; what was the original cost of the butter?

Ans. 24 cents.

11. A commission-merchant sold a bale of cotton for \$104.76, which was 3 per cent. less than the cost; what was the cost?

Ans. \$108.

CASE V.

ART. 228. *To find the cost when we know the whole amount of profit or loss, and also the rate per cent. of profit or loss.*

Ex. 1. A merchant sells a lot of laces at a profit of 12 per cent., thereby realizing an entire profit of \$38.88; what was the original cost of the laces?

Analysis. As the merchant gains 12 cents on every 100 cents invested, it is plain that to gain 1 cent he must invest $\frac{1}{12}$ of 100 cents = $\frac{100}{12}$ of a cent, and to gain \$38.88, or 3888 cents, he must invest 3888 times as much = $\frac{3888 \times 100}{12}$ cents, or \$324. *Ans.* \$324.

2. A commission-merchant gains \$87.75 by selling a lot of tobacco at a profit of 9% ; what was the cost of the tobacco?

Ans. \$975.

3. A broker sold railroad stock at an advance of 7%, thereby realizing a profit of \$106.75; what was the cost of the stock?

Ans. \$1525.

4. A grocer makes a profit of \$319.28 by selling a lot of flour at a profit of 13% ; what did he give for the flour?

Ans. \$2456.

5. A vintner lost \$31.36 by selling a hogshead of wine for 14% less than cost; what did he give for the wine?

Analysis. As he loses 14 cents on every 100 cents invested, it is plain that to lose 1 cent he must invest $\frac{1}{14}$ of 100 cents = $\frac{100}{14}$ of a cent, and to lose 3136 cents he must invest 3136 times as many cents = $\frac{313600}{14}$ cents, or \$224.00.

Ans. \$224.

6. A miller lost \$35.49 by selling a car-load of corn for 15% less than cost; what was the cost?

Ans. \$236.60.

7. A gentleman lost \$744.75 by selling a house at 9% less than cost; what was the cost?

Ans. \$8275.

8. A cotton-broker lost \$690.55 by selling a lot of cotton at 7 per cent. less than the cost; how much did he give for the cotton?

Ans. \$9865.

9. A commission-merchant lost \$163.75 by selling a cargo of coal at 5 per cent. less than the cost; what was the cost?

Ans. \$3275

INTEREST.

ART. 229. Persons who carry on business frequently borrow money. When a person borrows money from another for a specified time, at the end of that period he must not only pay back the sum of money borrowed, but he must also pay an additional sum for the use of the money borrowed.

This additional sum is called **INTEREST**. Hence,

ART. 230. *INTEREST is money paid for the use of money.*

INTEREST is estimated at a certain **RATE PER CENT.** *per annum* ;* that is, at a certain number of dollars for the *use* of 100 *dollars*, or a certain number of cents for the *use* of 100 *cents*, or a certain number of *shillings* for the *use* of 100 *shillings*, for one year, etc.

ART. 231. The **PRINCIPAL** is the sum of money borrowed.

ART. 232. The **RATE PER CENT.** is the price paid for the use of \$100, or £100, etc. for *one* year.

ART. 233. The **AMOUNT** is the **PRINCIPAL** and **INTEREST** added together. Thus, if I borrow \$100 for 1 year, and promise to pay 8 per cent. for the use of the money, then at the end of the year I must pay \$108. In this case \$100 is the **PRINCIPAL**, \$8 is the **INTEREST**, \$108 is the **AMOUNT**, and 8 is the **RATE PER CENT.**

ART. 234. The **RATE** of interest in most countries is regulated by law, and this is called the *legal rate*.

ART. 235. **SIMPLE INTEREST** is interest upon the **PRINCIPAL** only.

TO COMPUTE INTEREST.

CASE I.

ART. 236. *To find the interest on any given principal, at any rate per cent., for any number of years.*

Ex. 1. What is the interest of \$425 for 5 years, at 7 per cent. ?

Analysis. The interest of \$1 at 7 per cent. for *one* year is 7 cents, or .07 of a dollar; then the interest of \$425 must be 425 times as much = 2975 cents, or \$29.75; and if the interest for *one* year is \$29.75, then for 5 years it must be *five* times as much = \$148.75. *Ans.* \$148.75.

* *Per annum* is a Latin phrase, and means *by the year*.

OPERATION.

$$\begin{array}{r}
 \$425 \quad \text{Principal.} \\
 .07 \quad \text{Rate.} \\
 \hline
 \$29.75 = \text{Interest for 1 year.} \\
 5 \\
 \hline
 \$148.75 = \text{Interest for 5 years.}
 \end{array}$$

2. What is the interest of \$365.56 for 3 years, at 8 per cent.?

Analysis. As the interest of \$1 for one year is 8 cents, or .08 of a dollar, then the interest of \$365.56 must be 365.56 times as much = \$29.2448; and if for *one* year the interest is \$29.2448, then for 3 years it must be 3 times as much = \$87.7344.

Ans. \$87.7344.

OPERATION.

$$\begin{array}{r}
 \$365.56 \quad \text{Principal.} \\
 8 \\
 \hline
 \$29.2448 = \text{Interest for 1 year.} \\
 3 \\
 \hline
 \$87.7344 = \text{Interest for 3 years.}
 \end{array}$$

From the foregoing *Analysis* we derive the following

RULE.

Multiply the principal by the rate, and the product will be the interest for one year; then multiply this product by the number of years.

To find the AMOUNT, add the PRINCIPAL to the INTEREST.

EXAMPLES.

3. What is the interest of \$575 for 5 years, at 7 per cent.?

Ans. \$201.25.

4. What is the interest of \$986 for 4 years, at 6 per cent.?

Ans. \$236.64.

5. What is the amount of \$545 for 3 years, at 5 per cent. ?
Ans. \$626.75.
6. What is the interest of \$324.56 for 4 years, at 6 per cent. ?
Ans. \$77.8944.
7. What is the interest of \$675.48 for 3 years, at 7 per cent. ?
Ans. \$141.8508.
8. What is the amount of \$1573 for 4 years, at 8% ?
Ans. \$2076.36.
9. At 7%, what is the interest of \$2245.85 for 5 years ?
Ans. \$786.0475.
10. To what sum will \$956.24 amount in 2 years at 9% ?
Ans. \$1128.3632.
11. If a man borrows \$586.65 for 3 years at 7%, what amount should he return at the end of that period ?
Ans. \$709.8465.
12. What is the interest of \$842.75 for 4 years, at 6 per cent. ?
Ans. \$202.26.

CASE II.

ART. 237. *To find the interest on any given principal, at any rate per cent., for any number of YEARS and MONTHS.*

Ex. 1. What is the interest of \$636 for 3yr. 10mo., at 6% ?

Analysis. Since the interest of \$1 for a year is .06 of a dollar, then on \$636 it is 636 times as much = \$38.16; and for 3 years the interest is 3 times as much = \$114.48. Again, since the interest of \$636 for 1 year is \$38.16, then for 6 months it is *one half* as much as for 1 year = \$19.08; and for 3 months it is *one half* as much as for 6 months = \$9.54; and for 1 month it is *one third* as much as for 3 months = \$3.18. Then adding the interest for the 3 years and 6 months and 3 months and 1 month, we obtain the interest for the *whole* time of 3yr. 10mo. = \$146.28.

Ans. \$146.28.

OPERATION.

\$636 Principal.

.06 Rate.

\$38.16 = Interest for 1 year.

3

\$114.48 = Interest for 3yr.

19.08 = " " 6mo.

9.54 = " " 3mo.

3.18 = " " 1mo.

\$146.28 = " " 3yr. 10mo.

6mo. = $\frac{1}{2}$ yr.

3mo. = $\frac{1}{4}$ of 6mo.

1mo. = $\frac{1}{3}$ of 3mo.

2. What is the interest of \$672 for 4yr. 5mo., at 7%?

OPERATION.

\$672 Principal.

.07 Rate.

\$47.04 = Interest for 1yr.

4

\$188.16 = Interest for 4yr.

15.68 = " " 4mo.

3.92 = " " 1mo.

\$207.76 = " " 4yr. 5mo.

Ans. \$207.76

From the foregoing explanations we deduce the

RULE.

I. *Multiply the principal by the rate per cent., and multiply this product by the number of years.*

II. *For MONTHS take such fractional parts of the interest for a year as are expressed by the number of months.*

EXAMPLES.

3. What is the interest of \$924 for 3yr. 7mo., at 6%?

Ans. \$198.66.

4. What is the interest of \$954 for 4yr. 8mo., at 7%?

Ans. \$311.64.

5. What is the interest of \$504.72 for 3yr. 10mo., at 8% ?
Ans. \$154.7808.
6. To what sum will \$786.72 amount in 5yr. 5mo., at 5% ?
Ans. \$999.79.
7. What is the interest of \$1836.36 for 4yr. 8mo., at 6% ?
Ans. \$514.1808.
8. What is the amount of \$504.84 for 3yr. 2mo., at 7% ?
Ans. \$616.7462.
9. What is the interest of \$640.75 for 3yr. 4mo., at 9% ?
Ans. \$192.225.
- 10 To what sum will \$784 amount in 2yr. 9mo., at 7% ?
Ans. \$934.92.

CASE III.

ART. 238. *To find the interest on any given principal, at any rate per cent., for any number of YEARS, MONTHS and DAYS.*

Ex. 1. What is the interest of \$756 for 2yr. 5mo. 20da., at 6 per cent. ?

Analysis. Since the interest of \$1 for a year is .06 of a dollar, then the interest of \$756 is 756 times as much = \$45.36, and for 2 years it is *twice* as much = \$90.72. Again, since the interest for 1 year is \$45.36, for 4 *months* it will be *one third* as much as for a year = \$15.12, and for 1 *month* it will be *one fourth* as much as for 4 *months* = \$3.78. Lastly, since the interest for 1 *month* is \$3.78, then for 15 *days* it will be *one half* as much as for 1 *month* = \$1.89, and for 5 *days* it will be *one third* as much as for 15 *days* = \$0.63. Adding these separate items of interest for 2yr., and for 4mo., and for 1mo., and for 15 days and 5 days, we have the whole interest for the required time of 2yr 5mo. 20da. = \$112.14.

Ans. \$112.14.

OPERATION.

	\$756	Principal.
	.06	Rate.
	\$45.36	= Interest for 1yr.
	2	
	\$90.72	= Interest for 2yr.
4mo. = $\frac{1}{3}$ yr.	15.12	= " " 4mo.
1mo = $\frac{1}{4}$ of 4mo.	3.78	= " " 1mo.
15da. = $\frac{1}{2}$ of 1mo.	1.89	= " " 15da.
5da. = $\frac{1}{3}$ of 15da.	.63	= " " 5da.
	\$112.14	= " " 2yr. 5mo. 20da.

From the preceding *Analysis* we derive the

RULE.

- I. Find the interest for YEARS and MONTHS, as in CASE II.
 II. For DAYS take such fractional parts of the interest for a month as are expressed by the number of days.

EXAMPLES.

2. What is the interest of \$630 for 3yr. 10mo. 10da., at 7 per cent. ?
Ans. \$170.275.
3. What is the interest of \$378 for 4yr. 5mo. 20da., at 6 per cent. ?
Ans. \$101.43.
4. What is the interest of \$576 for 5yr. 7mo. 25da., at 8% ?
Ans. \$260.48.
5. What is the amount of \$1020.96 for 3yr. 7mo. 18da., at 5% ?
Ans. \$1206.4344.
6. What is the amount of \$656.84 for 4yr. 10mo. 15da., at 6% ?
Ans. \$848.9657.
7. What is the interest of \$840.60 for 3yr. 5mo. 6da., at 7% ?
Ans. \$202.0242.
8. What is the amount of \$1248.72 for 4yr. 11mo. 15da. at 8% ?
Ans. \$1744.0456.
9. What is the interest of \$1296.36 for 5yr. 7mo. 20da., at 9% ?
Ans. \$657.9027

10. To what sum will \$369.60 amount in 6yr. 10mo. 18da., at 7%? *Ans.* \$547.6856.

11. What is the interest of \$618.75 for 5yr. 5mo. 23da., at 8%? *Ans.* \$271.2875.

12. To what sum will \$672.24 amount in 2yr. 2mo. 25da., at 6%? *Ans.* \$762.4322.

13. What is the interest of \$583.20 for 6yr. 11mo. 13da., at $4\frac{1}{2}$ per cent.? *Ans.* \$182.4687.

14. A man bought a lot of land for \$336.42 on a credit of 2yr. 8mo. 20da., with interest at 7 per cent.; what sum of money was required to pay the debt? *Ans.* \$400.5267.

ART. 239. It is frequently necessary to find the interest on a sum of money for a period of time between two given dates—i. e., from a certain day of a month in one year to a particular day of a month in the same year or in another year. In such cases we first find the interval of time by subtracting the earlier date from the later (Art. 154); we then proceed to compute the interest according to the methods already given.

15. What is the interest of \$504 from February 25, 1870, to August 5, 1872, at 7 per cent.?

OPERATION.

	yr.	mo.	da.
	1872	8	5
	1870	2	25
Time	2	5	10
	\$504	Principal.	
	.07	Rate.	
	\$35.28	= Interest for 1yr.	
	2		
	\$70.56	= Interest for 2yr.	
4mo. = $\frac{1}{3}$ yr.	11.76	=	" " 4mo.
1mo. = $\frac{1}{4}$ of 4mo.	2.94	=	" " 1mo.
10da. = $\frac{1}{4}$ of 1mo.	.98	=	" " 10da.
	\$86.24	= Interest for the given time	

16. What is the interest of \$540.36 from April 25, 1869, to December 15, 1872, at 6 per cent. ? *Ans.* \$117.9786.

17. What is the amount of \$201.60 from March 19, 1868, to June 9, 1871, at 8 per cent. ? *Ans.* \$253.568.

18. What is the amount of \$360.48 from August 12, 1869, to March 27, 1872, at 8% ? *Ans.* \$436.1808.

ART. 240. In the preceding examples we have reckoned 12 months as a year and 30 days as a month, thereby making the year to consist of 360 instead of 365 days. This method, although not *perfectly accurate*, is sanctioned by custom. If, however, it is desired to obtain the interest for *days* with *absolute exactness*, we have only to multiply the interest for a year by as many 365ths as there are days in the given time; for, since 1 day is $\frac{1}{365}$ of a year, 18 days will be $\frac{18}{365}$ of a year, and so on.

Ex. 1. What is the interest of \$912.50 for 21 days, at 7 per cent. ?

OPERATION.

$$\begin{array}{r}
 \$912.50 = \text{Principal.} \\
 .07 = \text{Rate.} \\
 \hline
 63.8750 = \text{Interest for 1 yr.} \\
 .21 \\
 \hline
 638750 \\
 1277500 \\
 365 \overline{)1341.3750} (3.6750. \quad \text{Ans} \\
 \underline{1095} \\
 2463 \\
 \underline{2190} \\
 2737 \\
 \underline{2555} \\
 1825 \\
 \underline{1825} \\
 00
 \end{array}$$

It is evident that the interest for *one day* is $\frac{1}{365}$ of the interest for *one year*, and for 21 days it must be 21 times as much $= \frac{21}{365}$ of the interest for a year.

2. What is the interest of \$642.72 for 25 days, at 6 per cent. ? Ans. \$2.6413 +.

3 What is the interest of \$756.84 for 23 days, at 7 per cent. ? Ans. \$3.3383 +.

4. What is the amount of \$1113.25 for 73 days, at 7 per cent. ? Ans. \$1128.8355.

CALCULATION OF NOTES.

ART. 241. When a person borrows money from another, the *borrower* gives to the *lender* a promise in writing to pay to the *latter*, or to his *order*, or to the *bearer*, a certain sum of money *at or before* a specified time.

This written promise is called a **PROMISSORY NOTE**, or sometimes simply a **NOTE**.

ART. 242. The person who signs a note is called the **MAKER**; the sum *promised* to be paid is called the **PRINCIPAL**, or the **FACE** of the note, and should always be expressed in *words*.

ART. 243. In calculating the interest on notes, the day on which the note is dated should not be reckoned as a part of the time.

ART. 244. In calculating the amount of a note, we first find the length of time it runs on interest, and then the calculations may be made by the methods given in the preceding pages.

NOTE.—In the following examples 7 per cent. is understood to be the rate of interest, unless otherwise specified.

(1.)

BOSTON, MASS., May 13, 1871.

\$936. On or before the 25th of December next I promise

to pay Messrs. Steinway & Sons, or order, the sum of nine hundred and thirty-six dollars, for value received, with interest from date.

R. W. F.

What was the amount of this note December 25, 1871?

Ans. \$976.404.

(2.)

NASHVILLE, TENN., April 5, 1871.

\$336 $\frac{24}{100}$. For value received, I promise to pay James D. Gray, or order, three hundred and thirty-six dollars on demand, with interest.

T. J. B.

What was the amount of this note July 15, 1872?

Ans. \$366.9684 +.

(3.)

PHILADELPHIA, PA., August 10, 1870.

\$672. One day after date, I promise to pay E. H. Butler & Co., or order, six hundred and seventy-two dollars, for value received, with interest at 6 per cent.

R. W. S.

What was the amount of this note December 25, 1871?

Ans. \$727.44.

(4.)

NEW YORK, May 20, 1870.

\$432 $\frac{72}{100}$. For value received, I promise to pay Sidney Root, or bearer, four hundred and thirty-two dollars and seventy-two cents, on demand, with interest.

F. H. T.

What was the amount of this note January 5, 1872?

Ans. \$481.9419.

(5.)

RUTLAND, VT., August 10, 1871.

\$576. Three months after date, I promise to pay Joel M. Haven, or bearer, five hundred and seventy-six dollars, with interest at 6 per cent., for value received.

B. R. W.

How much was due October 25, 1872? *Ans* \$617.76.

PARTIAL PAYMENTS.

ART. 245. When notes are running on interest, payments are frequently made on them, which gradually reduce them until they are finally paid off. These payments are called **PARTIAL PAYMENTS**, and as they are usually written on the *back* of the note, they are sometimes called *endorsements*.*

For computing the balance due on a note on which partial payments have been made, a majority of the States of the Union have adopted a method founded on a decision of the Supreme Court of the United States.

UNITED STATES RULE.

I. *"The rule for casting interest when partial payments have been made is to apply the payment, in the first place, to the discharge of the interest then due.*

II. *"If the payment exceeds the interest, the surplus goes toward discharging the principal, and the subsequent interest is to be computed on the balance of the principal remaining due.*

III. *"If the payment be less than the interest, the surplus of interest must not be taken to augment the principal, but the interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied toward discharging the principal, and the interest is to be computed on the balance as aforesaid."*

(6.)

COLUMBUS, GA., June 10, 1869.

\$504. For value received, I promise to pay James B. Estes, or bearer, five hundred and four dollars, on demand, with interest.

S. W. H.

* *Endorsement*, from the Latin words *in* and *dorsum*—meaning, literally, upon the back.

On this note were the following endorsements:

January 25, 1870, received eighty-four dollars.

May 15, 1870, received one hundred dollars.

February 20, 1871, received two hundred dollars.

What was the balance due on this note at 6 per cent., July 5, 1871?

Ans. \$166.50 +.

OPERATION.

Principal, bearing interest from June 10, 1869..	\$504.00
Interest to first payment (7mo. 15da.).....	18.90
Amount to time of first payment.....	<u>\$522.90</u>
Deduct first payment.....	84.00
Balance due after first payment.....	<u>\$438.90</u>
Interest on balance to second payment (3mo. 20da.).....	8.04
Amount to time of second payment.....	<u>\$446.94</u>
Deduct second payment.....	100.00
Balance due after second payment.....	<u>\$346.94</u>
Interest on balance to third payment (9mo. 5da.).....	15.90
Amount to time of third payment.....	<u>\$362.84</u>
Deduct third payment.....	200.00
Balance due after third payment.....	<u>\$162.84</u>
Interest on balance to July 5, 1871 (4mo. 15da.).....	3.66
Balance due July 5, 1871.....	<u>\$166.50 +</u>

Observation.—By following the successive steps in the above operation, we see that the whole process consists in finding the *amount* of the principal to the time of the *first* payment—then deducting this payment; then calling the remainder a new principal, we find its amount to the time of the *second* payment—then deduct this second payment; and continue this process to the time of final settlement. This rule will *always* hold good whenever each one of the payments *equals* or *exceeds* the interest. But instances often occur in which some

of the payments are *less* than the interest. In such cases the foregoing method will not *literally* apply. We will give an example to illustrate the rule applicable in such cases.

(7.)

NEW HAVEN, CONN., March 15, 1868.

\$1200. Four months after date, I promise to pay Benjamin Silliman, or order, twelve hundred dollars, with interest, at 6 per cent., value received. T. H. S.

The following credits were endorsed on this note :

November 15, 1868, received two hundred and forty dollars.

June 15, 1869, received twenty-four dollars.

April 30, 1870, received three hundred and sixty dollars.

January 30, 1871, received thirty dollars and forty-five cents.

November 10, 1871, received four hundred and fifty dollars.

What was the balance due on taking up this note, March 25, 1872?

Ans. \$303.718.

OPERATION.

Principal, bearing interest from March 15, 1868.	\$1200.00
Interest to first payment (8mo.).....	48.00
Amount to time of first payment.....	\$1248.00
Deduct first payment.....	240.00
Balance due after first payment.....	\$1008.00
Interest from November 15, 1868, to second payment (7mo.).....	\$35.28
Second payment (less than interest)....	24.00
Leaving interest unpaid.....	\$11.28
Interest on same principal from June 15, 1869, to April 30, 1870 (10mo. 15da.).....	52.92
Aggregate of interest unpaid to third payment.	64.20
Amount due on note to time of third payment.	\$1072.20
Deduct third payment.....	360.00
Balance due after third payment.....	\$712.20

Brought forward.....	\$712.20
Interest from third payment to fourth payment (9mo.).....	\$32.05
Fourth payment (less than interest)...	30.45
Leaving interest unpaid.....	\$1.60
Interest from fourth payment to fifth payment (9mo. 10da.).....	33.235
Aggregate of interest unpaid to November 10, 1871.....	34.835
Amount due on note to time of fifth payment..	\$747.035
Deduct fifth payment.....	450.000
Balance due after fifth payment.....	\$297.035
Interest from fifth payment to final settlement (4mo. 15da.).....	6.683
Balance due March 25, 1872.....	\$303.718 +

Observation.—In the above example we observe that two of the payments were *less* than the interest due at the time, and in each instance we subtracted the *payment* from the *interest*, and we set aside, *temporarily*, the interest which was still unpaid, and proceeded to calculate the interest on the *same* principal to the time of the next payment. The student may perhaps inquire: “Why did you not add the interest to the principal, and then deduct the payment, as you did before?” The answer is: That the principal would have been increased by the difference between the payment and the interest, and in computing interest on this increased principal we should have violated a rule sanctioned by the decisions of courts—viz., that “*Interest must not be calculated upon interest.*”

NOTE.—The teacher would find the time well spent to make the pupil examine each step in the working of the last example until he is perfectly master of the principles involved. A half hour thus occupied will save much labor to both teacher and pupil, in the future.

(8.)

NEW ORLEANS, October 12, 1869.

\$875. For value received, we jointly and severally promise to pay B. M. Palmer, or order, eight hundred and seventy-five dollars, on demand, with interest at 8 per cent.

B. W. P.

R. S. M.

On this note were the following endorsements:

July 12, 1870, received four hundred and one dollars.

March 27, 1871, received one hundred and seventy-nine dollars and seventy-five cents.

September 12, 1871, received sixty-five dollars.

March 12, 1872, received one hundred and thirty dollars

What was the balance due July 27, 1872?

Ans. \$214.6609 +.

Observation.—This note, being signed by two parties, is called a JOINT NOTE.

(9.)

CINCINNATI, April 5, 1869.

\$1560. On or before the 25th day of December next, I promise to pay Messrs. Putnam, Hooker & Co., or bearer, one thousand five hundred and sixty dollars, for value received, with interest at 6 per cent.

L. S. T.

On this note were the following credits:

November 15, 1869, received four hundred and seventeen dollars and twenty cents.

April 21, 1870, received twenty-five dollars.

January 9, 1871, received three hundred and fifty-seven dollars and eighty cents.

October 24, 1871, received thirty dollars.

April 4, 1872, received six hundred dollars.

What was the balance due August 4, 1872?

Ans. \$343.485.

(10.)

RICHMOND, VA., February 8, 1868.

\$720. For value received, I promise to pay James Thomas, or order, seven hundred and twenty dollars, in three months with interest afterward at 6 per cent. B. R. W

The following credits were endorsed :

December 23, 1868, received two hundred and seven dollars.

October 3, 1869, received twenty dollars.

August 13, 1870, received eighteen dollars.

December 28, 1870, received three hundred dollars.

What was the balance due April 28, 1871 ?

Ans. \$272.595.

(11.)

MONTGOMERY, ALA., January 6, 1869.

\$725. Due, from date hereof, to Joseph Hodgson, or order, seven hundred and twenty-five dollars, for value received, with interest at 8 per cent. R. P. S.

Observation.—The above is called a DUE-BILL.

The following credits were endorsed :

October 6, 1869, received ninety-six dollars and fifty cents.

March 21, 1870, received three hundred and thirty-six dollars and sixty-four cents.

November 6, 1870, received ninety dollars.

April 16, 1871, received two hundred dollars.

What was the balance due July 16, 1871 ?

Ans. \$100.2048.

ART. 246. When a note is paid up within a year, or any time less than a year, the following method of calculating the balance due, and which is known as the VERMONT RULE, is very frequently employed :

VERMONT RULE.

I. *Find the amount of the principal from the time when interest commenced to the time of settlement.*

II. Find the amount of each payment from the time it was made to the time of settlement.

III. Subtract the sum of the amounts of the payments from the amount of the principal.

(12.)

BALTIMORE, February 18, 1871.

\$768. For value received, I promise to pay Edward A Richardson, or bearer, seven hundred and sixty-eight dollars, with interest from date at 6 per cent. S. A. R.

The following payments were endorsed :

May 18, 1871, received one hundred and twenty dollars.

July 3, 1871, received three hundred dollars.

November 18, 1871, received two hundred dollars.

What was the balance due February 18, 1872, when the note was paid? Ans. \$——.

OPERATION.

Face of the note.....	\$768.00
Interest for 1 year.....	46.08
Amount of note for 1 year.....	<u>\$814.08</u>
First payment.....	\$120.00
Interest on first payment for 9mo.....	5.40
Second payment.....	300.00
Interest on second payment for 7mo. 15da.	11.25
Third payment.....	200.00
Interest on third payment for 3mo.....	<u>3.00</u>
Sum of the amounts of all the payments.....	639.65
Balance due February 18, 1872.....	<u>\$174.43</u>

The pupil will readily comprehend the *reason* of the above operation by a little inspection. We computed the interest on \$768 for a year, but in doing so we calculated it on \$120 *too much* for 9 months; but we corrected this error at the close of the operation by subtracting the \$120, together with its interest for 9 months. We also computed the interest on a principal *too large* by \$300 (the second payment) for 7

months and 15 days. This error we corrected at the close by subtracting the \$300, with its interest for 8mo. 15da. Lastly, we made a third error by calculating the interest on a principal *too much* by \$200 for 3 months. This error we also corrected at the termination by subtracting the \$300, with its interest for 3 months.

Compute the following by the Vermont Rule :

AUGUSTA, GA., May 5, 1871.

\$1008. For value received, I promise to pay Jos. B. Cumming, or bearer, at the Georgia Railroad Bank, one thousand and eight dollars, with interest at 8 per cent. H. B. O

The following payments were endorsed :

July 5, 1871, received three hundred and sixty dollars.

September 20, 1871, received four hundred and fifty dollars.

December 15, 1871, received one hundred and eighty dollars.

The balance was paid May 5, 1872; what was the balance then due?

Ans. \$46.54.

(13.)

RALEIGH, N. C., April 10, 1871.

\$432⁷²/₁₀₀. On demand, for value received, I promise to pay Charles Manley, or order, four hundred and thirty-two dollars and seventy-two cents, with interest at 10 per cent.

R. J. S.

The following payments were endorsed :

July 13, 1871, received sixty dollars.

September 10, 1871, received one hundred and fifty dollars

January 20, 1872, received one hundred dollars.

The balance was paid May 25, 1872; what was the balance then due?

Ans. \$152.104.

CASE IV.

ART. 247. *To compute interest when the principal is in pounds, shillings, etc.*

Ex. 1. What is the interest of £84 8s. 6d. for 5 years, at 7 per cent.?

OPERATION.

£84 8s. 6d. is the same as £84.425

$$\begin{array}{r}
 .07 \\
 \hline
 £5.90975 \text{ Interest for 1 year.} \\
 5 \\
 \hline
 £29.54875 \text{ Interest for 5 years.} \\
 20 \\
 \hline
 s.10.97500 \\
 12 \\
 \hline
 d.11.70000 \\
 4 \\
 \hline
 far.2.80000
 \end{array}$$

Ans. £29 10s. 11d. 2.8far.

We first reduce the 8s. 6d. to the decimal of a pound (Art. 194), then multiply by the rate per cent., just as in Federal Money; and, lastly, multiply this result by the number of years, taking aliquot parts for months and days.

2. What is the interest of £375 14s. 6d. for 4 years, at 6 per cent.?

Ans. £90 3s. 5d. 3.04far.

3. To what sum would £225 15s. 9d. amount in 3yr. 9mo. 18da., at 8 per cent.?

Ans. £294 8s. 6d. 1.824far.

PROBLEMS IN INTEREST.

ART. 248. In all computations respecting interest, *four* terms enter into the calculations; these are, the *principal*, the *rate per cent.*, the *time* and the *interest*. These terms are so related that any *three* of them being known, the remaining *one* may be found

PROBLEM I.

ART. 249. *Given the principal, the interest and the time, to find the RATE PER CENT.*

Ex. 1. A man borrowed \$375 for 3 years, and paid \$78.75 interest; what was the rate per cent.?

Analysis. If in 3 years he paid \$78.75 interest, then in 1 year he paid *one third* of \$78.75 = \$26.25. Then, if \$375 in 1 year gain \$26.25, \$1 will gain $\frac{1}{375}$ of \$26.25 = $\frac{26.25}{375}$ = \$0.07, or 7 cents. But the interest of \$1 for 1 year is the rate per cent.; therefore the rate per cent. is 7. *Ans.* 7%.

EXAMPLES.

2. If \$103.68 interest is received on a principal of \$432 for 4 years, what is the rate per cent. ? *Ans.* 6%.

3. If I lend \$564 for 3 years, and receive \$118.44 interest on the same, what is the rate per cent. ? *Ans.* 7%.

4. At what rate per cent. must \$756 be put at interest for 4 years to yield \$241.92 ? *Ans.* 8%.

5. A man lent \$484 for 5 years, and received \$181.50 for the interest; what was the rate per cent. ? *Ans.* $7\frac{1}{2}\%$.

6. At what rate per cent. must \$672 be put at interest for 2 years to yield an interest of \$89.60 ? *Ans.* $6\frac{2}{3}\%$.

7. A gentleman borrowed \$864 for 3yr. 6mo., and paid \$181.44 interest; what was the rate per cent. ? *Ans.* 6%.

PROBLEM II.

ART. 250. *Given the principal, the interest and the rate per cent., to find the TIME.*

Ex. 1. In what time will \$504 yield \$68.04 interest, at 6% ?

Analysis. The interest of \$504 for 1 year, at 6%, is \$30.24; hence, if to gain 3024 cents requires 1 year, to gain 1 cent requires $\frac{1}{3024}$ part of a year, and to gain 6804 cents will require 6804 times as long = $\frac{6804}{3024}$ years = 2.25 years, or 2yr. 3mo. *Ans.* 2yr. 3mo.

EXAMPLES.

2. How long a time would be required for \$525 to gain \$110.25, at 7 per cent. ? *Ans.* 3yr.

3. If the interest of \$576 for a certain time, at 6%, is \$120.96, what is the length of time ? *Ans.* 3yr. 6mo.

4. In what time would \$624 yield \$148.20 interest, at 5 per cent. ? *Ans.* 4yr. 9mo.

5. If the interest of \$972 for a certain time, at 6%, is \$211.41, what is the length of time ? *Ans.* 3yr. 7mo. 15da.

6. How long a time would it require for \$625 to amount to \$756.25, at 7 per cent. ? *Ans.* 3yr.

Observation.—In this example the *interest* is not given, but can be found by subtracting the principal from the amount.

PROBLEM III.

ART. 251. *Given the time, the rate per cent. and the interest, to find the PRINCIPAL.*

Ex. 1. The interest on a certain principal for 4 years, at 6%, is \$92.16; what is the principal?

Analysis. Since the interest for 4 years is \$92.16, then for 1 year the interest will be *one fourth* of \$92.16 = \$23.04. Now, since an interest of 6 cents for 1 year requires \$1 principal, then 1 cent interest requires *one sixth* as much principal = $\frac{1}{6}$ principal, and 2304 cents' interest will require 2304 times as much principal = $\frac{2304}{6}$, or \$384 principal.

Ans. \$384.

EXAMPLES.

2. What principal, at 7 per cent., will yield \$134 32 in 3 years ? *Ans.* \$642.

3. What sum of money must be put at interest for 5 years, at 6 per cent., to yield \$237.90 ? *Ans.* \$793.

4. What principal, at 8 per cent., will yield \$227.22 in 3 years? *Ans.* \$946.75.

5. The interest on a certain principal for 2yr. 3mo., at 6%, is \$98.28; what is that principal? *Ans.* \$728.

6. The interest on a note for 3yr. 6mo., at 7%, was \$314.58; what was the face of the note? *Ans.* \$1284

PROBLEM IV.

ART. 252. *Given time, the rate per cent. and the amount, to find the PRINCIPAL.*

Ex. 1. What principal in 3yr. 4mo., at 6%, will amount to \$547.20?

Analysis. The amount of \$1 in 3yr. 4mo., at 6%, is \$1.20. Now, since an amount of \$1.20, or 120 cents, requires a principal of \$1, an amount of 1 cent requires a principal $\frac{1}{120}$ as large = $\frac{1}{120}$, and an amount of \$547.20, or 54720 cents, will require a principal 54720 times as large = $\frac{54720}{120}$, or \$456. *Ans.* \$456.

2. What principal in 2yr. 6mo. will amount to \$901.60, at 6 per cent.? *Ans.* \$784.

3. What principal, at 5%, will amount to \$972.44 in 3 years? *Ans.* \$845.60.

4. What principal, at 8 per cent., will amount to \$1269.84 in 3yr. 9mo.? *Ans.* \$976.80.

5. What principal will amount to \$629.50 in 4yr. 2mo., at 6 per cent.? *Ans.* \$503.60

COMPOUND INTEREST.

ART. 253. If I borrow \$300 for a year at 6 per cent., at the end of the year I must pay not only the \$300 borrowed, but also \$18 interest, making the whole amount \$318. If, however, I prefer to keep the money still longer, then for the *second* year I must pay interest on \$318, which would be

\$19.08; and the amount due at the end of the second year would be \$337.08. If I still prefer to keep the money a *third* year, I must pay interest upon \$337.08, and so on. This is called *compounding* interest. Hence,

ART. 254. COMPOUND INTEREST is interest on both principal and interest.

SIMPLE INTEREST is interest on the principal only.

Ex. 1. What is the compound interest of \$400 for 3 years, at 6 per cent.?

OPERATION.

\$400	Principal for first year.
.06	
\$24.00	Interest.
400.00	
\$424.00	Amount for first year.
.06	
25.4400	Interest for second year.
424.00	
\$449.44	Amount for second year.
.06	
26.9664	Interest for third year.
449.44	
\$476.4064	Amount for third year.
400.00	Deduct first principal.
\$76.4064	Compound interest for three years.
	Ans. \$76.4064

From the preceding operation we deduce a

RULE.

I. Find the amount of the given principal, at the given rate, for one year, and make it the principal for the SECOND year.

II. Find the amount of this new principal for the second year, and make it the principal for the third year; and so proceed through the whole number of years.

III. *Subtract the given principal from the last amount, and the remainder will be the compound interest.*

NOTE.—If the interest is payable *semi-annually* or *quarterly*, find the amount for *half* a year or a *quarter* of a year, and thus proceed through the whole time. It is not necessary to carry the result beyond *four* places of decimals.

EXAMPLES.

2. What is the compound interest of \$750 for 4 years, at 8 per cent. ?
Ans. \$270.366 +.

3. What is the compound interest of \$800 for 5 years, at 5 per cent. ?
Ans. \$221.025.

4. Find the amount of \$600 for 2yr. 6mo., at 6 per cent., compound interest.
Ans. \$694.3848.

5. Find the amount of \$500 for 3yr. 3mo., at 6 per cent., compound interest.
Ans. \$604.44 +.

6. What is the amount of \$450 in 2 years, at 8%, compound interest, payable *semi-annually* ?
Ans. \$526.4363 +.

7. What is the difference between the *simple* and the *compound* interest of \$1250 for 4 years, at 6 per cent. ?
Ans. \$28.0962.

DISCOUNT.

ART. 255. It often happens that a man gives his note payable at some future period, without interest until that period. If, however, the *holder* of the note prefers to receive the money *immediately*, instead of waiting for the note to become due, he should make some *deduction* from the *face* of the note as a compensation for the ready payment of the money. This *deduction* is called DISCOUNT. Hence,

ART. 256. DISCOUNT is a *deduction* made for the payment of money before it becomes due.

ART. 257. The PRESENT WORTH of a debt payable at some future time without interest is such a sum as, being put at

interest, would amount to the debt at the time when the debt becomes due. Hence,

The DISCOUNT is the *difference* between the *debt* and the *present worth*.

ART. 258. In all computations of discount it is necessary first to find the *present worth*.

Ex. 1. What is the discount on a debt of \$457.92, payable in 9 months, when money is worth 8 per cent.?

Analysis. The amount of \$1 in 9 months at 8% is \$1.06; therefore \$1.06 due in 9 months is worth \$1 *now*. Hence, if the sum of 106 cents due 9 months *hence* is worth \$1 *now*, then 1 cent due 9 months *hence* is worth $\frac{1}{106}$ of \$1 *now* = $\frac{1}{106}$; and \$457.92, or 45792 cents, will be worth 45792 times as much *now* = $\frac{45792}{106} = \$432$, which is the *present worth*. Subtracting \$432, the *present worth*, from the *debt*, we have \$25.92 as the *discount*. Ans. \$25.92.

The foregoing *Analysis* illustrates the reason for the following

RULE.

I. Divide the given debt by the amount of one dollar for the given time, at the given rate; the quotient will be the PRESENT WORTH.

II. Subtract the PRESENT WORTH from the DEBT, and the remainder will be the DISCOUNT.

EXAMPLES.

2. What is the discount of \$897.82, payable in 3 years, when money is worth 7 per cent. ? Ans. \$155.82.

3. What is the present worth of \$787.75, due in 2yr. 6mo., when money is worth 6% ? Ans. \$685.

4. What is the discount on a note for \$1174.32, due in 3yr. 3mo., money being worth 8% ? Ans. \$242.32.

5. A note for \$1380.06 becomes due in 15 months; what

deduction should be made for the immediate payment of the money, supposing money to be worth 8%? *Ans.* \$125.46.

6. A merchant of Macon buys in Boston a bill of dry goods amounting to \$2513.79 on a credit of 4 months; a bill of crockery for \$469.68, payable in 6 months; and a bill of groceries for \$1691.75, due in 2 months; how much ready cash will pay all the bills, discounting at 6 per cent.?

Ans. \$4595.50.

7. Which is worth most—\$750 cash down, \$765 payable in 3 months, or \$780 payable in 6 months, money being worth 8%?

Ans. —.

BANK DISCOUNT.

ART. 259. A bank is an institution one of whose chief objects is to lend money. When money is borrowed from a bank, the interest is always paid *in advance*, or at the time of receiving the money. Thus, if a man gives his note for \$500, payable in 3 months, at 7 per cent., he will receive from the bank only \$491.25; as \$8.75 would be retained to pay the interest on the \$500 for the 3 months. This \$8.75 is called **BANK DISCOUNT**. Hence,

ART. 260. **BANK DISCOUNT** is really simple interest paid in advance.

ART. 261. A note is said to be *discounted* at a bank when the note is received by the bank, and the holder of the note receives the money for it after the bank discount is deducted. That which is left after deducting the discount from the *face* of the note is called the **NET PROCEEDS**.

ART. 262. It is the custom with banks to allow *three* days more time in which to pay a note than is specified on the face of the note. These three days are called "**DAYS OF GRACE**." In calculating the discount the three "days of grace" are always taken into the account. As bank discount is nothing

more than simple interest, we have for its computation the following

RULE.

I. Calculate the interest on the face of the note for the specified time, together with the three "days of grace;" this will give the BANK DISCOUNT.

II. Subtract the bank discount from the face of the note; the remainder will be the NET PROCEEDS.

EXAMPLES.

1. What is the bank discount on a note for \$1250 payable in 3mo., with interest at 6%? *Ans.* \$19.375.

2. A note for \$2016, payable in 4 months, was discounted at a bank at 8%; what was the bank discount?

Ans. \$55.104.

3. What are the proceeds of a note for \$1650, due in 2 months, discounted at a bank at 6%? *Ans.* \$1632.675.

4. A merchant in New York sells a grocer of Atlanta a bill of sugar amounting to \$2260, for which he receives a note payable in 3mo., with interest at 6%. If the merchant discounts his note at the "First National Bank of New York," what sum will he receive from the bank? *Ans.* \$2224.97.

ART. 263. In the practical operations of business it is often necessary to know for what particular amount to draw a bankable note, payable at some definite period, so that when discounted at a bank it may yield a specified sum. The principles of Bank Discount enable us to solve this problem.

Ex. 1. For what sum must I draw a note, payable in 3mo. 15da., so that, when discounted at a bank at 6%, the proceeds may be \$725?

Analysis. According to the preceding article, the bank discount on \$1, at 6%, for 3mo. 15da., together with the three "days of grace," is \$0 018. Subtracting this bank discount

from \$1, we obtain \$0.982 as the *proceeds* of a note for \$1. We see, therefore, that for each \$0.982 of *proceeds* the note must contain \$1; hence, the note must contain as many dollars as \$0.982 is contained times in the *proceeds*. Therefore, dividing \$725 by \$0.982, we have \$738.289 as the *face* of the note.

Ans. \$738.289.

From the foregoing explanation we deduce a

RULE.

Divide the given sum by the PROCEEDS of \$1 for the given rate and time; the quotient will be the FACE of the note

EXAMPLES.

2. For what sum must a note be drawn so that, discounted at a bank for 3mo. 15da., at 8%, the *proceeds* may be \$864?

Ans. \$885.245 +.

3. A grocer in Eufaula, Ala., owes a merchant in Augusta, Ga., \$1275; how large must the grocer draw his note, payable in 8mo. 9da., that, discounted at the Georgia Railroad Bank at 9%, the money received will just pay the Augusta merchant?

Ans. \$1360.725 +.

4. What must be the face of a note, payable in 5mo. 3da., so that, discounted at a bank at 6%, the maker may receive \$1240?

Ans. \$1273.10 +.

ART. 264. A very simple and convenient method of finding the *face* of a bankable note has been devised by J. H. Sawyer, Esq., Assistant Cashier of Citizens' Savings Bank, Columbia, S. C., which is as follows:

If we divide 360 (bankers' year) by 1 per cent., or .01, we get the quotient 36000; dividing by 2%, we have 18000; by 3%, 12000; 4%, 9000; and so on. Calling these several quotients REPRESENTATIVES of their corresponding per cents., we have the following

TABLE.

RATE PER CENT.	REPRESENTATIVE.	RATE PER CENT.	REPRESENTATIVE.
1.	36000.	6.	6000.
2.	18000.	7.	5143.
3.	12000.	8.	4500.
4.	9000.	9.	4000.
5.	7200.	10.	3600.

To show the application, let us take an example :

Ex. 1. What must be the face of a note which, discounted at 8% for 3 months, will give \$675 as the net proceeds?

Ans. \$689.244.

Reference to the above Table shows 4500 to be the *representative* of 8 per cent. From 4500 we subtract 93 (the number of days the note is to run, and three "days of grace" added); the remainder, 4407, we take for a *divisor*. We now multiply \$675 (the net proceeds) by 4500; the product is 3037500, which we call a *dividend*. Dividing 3037500 by 4407, we have as the quotient \$689.244; and this is the *face* of the note.

The process employed may be summed up in the following

RULE.

I. Subtract the number of days the note has to run, including the three days of grace, from the REPRESENTATIVE of the rate of interest for a DIVISOR.

II. Multiply the given sum by the representative of the rate of interest for a DIVIDEND. Divide the dividend by the divisor, and the quotient will be the FACE of the note.

EXAMPLES.

2. Required the face of a note payable in 5mo. 15da., so that, discounted at 6%, the holder may receive \$450.

Ans. \$462.962.

By the Table, 6000 is the *representative* of 6%. From 6000 we subtract 168 (the number of days the note is to run, with three days of grace), leaving 5832 as our *divisor*. Then we multiply \$450 by 6000 = \$2700000 = the *dividend*. Then $\$2700000 \div 5832 = \$462.962$.

3. What sum payable in 6mo. 15da., if discounted at a bank at 9%, will give \$824 as the net proceeds?

Ans. \$866.912 +.

4. What must be the face of a note that, when discounted at 6%, for 3mo., the holder may receive \$600?

Ans. \$609.446 +.

5. A merchant of Nashville owes a Savannah house \$462.70. How large a note, payable in 6mo. 15da., when discounted at the Bank of Savannah, at 7%, will just cancel the debt?

Ans. \$481.22 +.

6. What must be the face of a note that, when discounted at 10%, for 4mo. 15da., the holder may receive \$925?

Ans. \$961.871 +.

Observation.—The foregoing method is here inserted, by permission of Mr. Sawyer, for its *practical* utility. Calculations can be made by it easily and rapidly. The Rule is susceptible of a rigorous mathematical demonstration, but to understand it requires some knowledge of Algebra.

TAXES.

ART. 265. A TAX is a specified sum of money which each citizen is required to pay for the support of the government.

ART. 266. Taxes are of three kinds: 1st. *Capitation tax*; 2d. *Property tax*; 3d. *Income tax*.

CAPITATION tax (from *caput*, meaning head) is so much per head on all male citizens 21 years of age. It is also called POLL tax, from a Dutch word *poll*, meaning *head*. Each person liable to a capitation tax is called a POLL. PROPERTY tax

is a certain percentage on all one's property; and INCOME tax is a certain percentage on all his income.

ART. 267. There is always a government officer whose duty it is to assess the taxes. He first ascertains how much money will be required to carry on the affairs of the State for a year. Knowing the number of citizens in the State, he learns what the whole capitation tax will amount to, at so much per head. He then deducts this from the *whole* amount of tax to be raised; the remainder will be the sum to be raised by *property* tax. He then learns how much taxable property there is in the State. By dividing the whole *property* tax by the whole amount of taxable property, he finds how much tax is to be paid on \$1 of property; and therefore, to find each man's tax, he multiplies the value of his taxable property by the per cent. or tax on \$1, and to this adds his poll tax.

Ex. 1. A certain town wishes to build an academy by levying a tax of \$1445.51. The taxable property of the town is valued at \$140845; and there are 425 polls, each to pay \$0.75. What will be A's tax, whose property is valued at \$5825, and who pays for 2 polls?

Analysis. 425 polls at \$0.75 = \$318.75, the amount of the poll-taxes; and subtracting this from \$1445.51, the remainder, \$1126.76, will be the sum to be assessed on property. Now, since \$140845 of property is to raise a tax of \$1126.76, then \$1 of property must raise $\frac{1}{140845}$ of \$1126.76 = $\frac{1126.76}{140845}$, or \$0.008—i. e., 8 mills on \$1, or $\frac{8}{1000}$ of 1 per cent.; and since \$1 of property pays a tax of \$0.008, then \$5825 (A's property) will pay 5825 times as much = $5825 \times \$0.008 = \$46.60$. Adding to this 2 polls = \$1.50, we have \$48.10, A's entire tax.

Ans. \$48.10.

ART. 268. Having determined the tax to be paid on \$1, ASSESSORS of taxes usually make out a table containing the tax on \$1, \$2, \$3, etc., and then employ this table in calcu-

lating the tax of each person. We present a form of this table below, using the per cent. derived from the Example above:

TABLE.

\$1 pays	\$0.008	\$30 will pay	\$0.24	\$500 will pay	\$4.00
2 will pay	\$0.016	40 "	0.32	600 "	4 80
3 "	0.024	50 "	0.40	700 "	5 60
4 "	0.032	60 "	0.48	800 "	6 40
5 "	0.04	70 "	0.56	900 "	7 20
6 "	0.048	80 "	0.64	1000 "	8 00
7 "	0.056	90 "	0.72	2000 "	16 00
8 "	0.064	100 "	0.80	3000 "	24 00
9 "	0.072	200 "	1.60	4000 "	32 00
10 "	0.08	300 "	2.40	5000 "	40 00
20 "	0.16	400 "	3.20	6000 "	48 00

2. According to the foregoing table, what would be B's tax, whose property is estimated at \$3675, and who pays for 3 polls?

OPERATION.

B's tax on	\$3000	=	\$24.00
" "	600	=	4.80
" "	70	=	.56
" "	5	=	.04
" "	<u>\$3675</u>	=	<u>\$29.40</u>
" 3 polls		=	2.25
" entire tax		=	<u>\$31.65</u>

Ans. \$31.65

3 By the table, what would be E's tax, whose property is valued at \$2385, and who pays for 2 polls? *Ans.* \$20.58.

4. A township owning property valued at \$420960 wishes to raise \$5726.52 by taxation. There are 540 polls, each assessed at \$1.25. What is the tax on \$1, and how much must D pay, whose property is assessed at \$4265, and who pays for 1 poll? *Ans.* Tax on \$1 is \$0.012; D pays \$52.43.

CUSTOM-HOUSE BUSINESS.

ART. 269. DUTIES, or CUSTOMS, are taxes levied by governments upon foreign goods imported into the country.

ART. 270. Duties are either *specific* or *ad valorem*.*

SPECIFIC DUTY is a fixed sum imposed upon a yard, pound, gallon, etc., without regard to the intrinsic value of the article or to its cost.

AD VALOREM DUTY is a percentage on the cost of goods in the country from which they are brought.

ART. 271. In goods brought from abroad there will necessarily be more or less waste and damage. In consequence of this waste, etc., the government makes certain allowances or deductions before computing the duties.

ART. 272. DRAFT is an allowance made for waste, which is to be deducted from the gross weight.

The rules which govern the deduction for DRAFT are as follows :

On 112 pounds or less the draft is.....						1 pound.
Above 112 pounds, and not exceeding 224 pounds,						2 pounds.
"	224	"	"	336	"	3 "
"	336	"	"	1120	"	4 "
"	1120	"	"	2016	"	7 "
"	2016	"				9 "

ART. 273. TARE is an allowance for the weight of the box, bag, cask, etc. containing the goods.

ART. 274. LEAKAGE is an allowance for waste of liquids imported in casks.

ART. 275. BREAKAGE is a deduction for loss of liquids imported in bottles.

ART. 276. GROSS WEIGHT is the entire weight of the

* *Ad valorem* is a Latin phrase, and means, *according to the value*.

goods, together with that of the box, bag or chest, etc. containing them.

NET WEIGHT is that which remains after all deductions have been made.

DUTIES are always collected on the net weight or net value.

ART. 277. On some goods specific duties exclusively are collected; on others the duty is *ad valorem*; whilst on others there is a combination of *ad valorem* and specific duties.

In estimating duties, first deduct the *draft*, if any; then deduct the *tare*, and compute the duty on the remainder.

All custom-house duties are payable in **GOLD**.

EXAMPLES.

1. What is the duty on 25 bales of cinnamon, invoiced at 90 pounds each, tare being 6 per cent., and the rate of duty 30 cents per pound? *Ans.* \$634.50.

There being no draft, deduct the tare, and compute the duty on the remainder.

2. A merchant in Baltimore imported a case of Leghorn bonnets, invoiced at \$2573; what is the duty, the rate being 40% *ad valorem*? *Ans.* \$1029.20.

3. A merchant in New York imported 45 ceroon of cocoa, each ceroon containing 45 pounds. Tare being 8%, what would be the duty in gold, the rate being 3 cents per pound? *Ans.* \$55.89.

4. What would be the duty on a case of silks, invoiced in Paris at 24675 francs, the custom-house value of the franc being $18\frac{6}{10}$ cents, and the rate of duty 60% *ad valorem*? *Ans.* \$2753.73.

5. An importer in Boston paid duty on 45 casks of Zante currants, each cask invoiced at 110 pounds. How much was the duty, tare being 10%, and the rate 5 cents per pound? *Ans.* \$222.75.

EQUATION OF PAYMENTS.

ART. 278. EQUATION OF PAYMENTS is the method of finding the *mean* or *average* time of paying debts due at different times, without loss either to the debtor or to the creditor.

The date at which the several debts may be paid is called the EQUATED TIME.

Ex. 1. A miller owes a farmer \$90, of which \$45 is to be paid in 4 months; \$30 is due in 7 months; and the remaining \$15 in 8 months; at what time may these debts be paid in one single payment, so that no loss may be sustained by the miller or the farmer?

Analysis. If the miller can keep \$45 for 4 months, he can keep \$1 for 45 times as many months = 180 months. Again, if he can keep \$30 for 7 months, he can keep \$1 for 30 times as many months = 210 months; and if he can keep \$15 for 8 months, he can keep \$1 for 15 times as many months = 120 months. By adding these several sums, we see that the miller can keep \$1 for 510 months; and if he can delay the payment of \$1 for 510 months, he can delay the payment of \$90 (the whole debt) only $\frac{1}{90}$ as many months = $\frac{510}{90}$ months = $5\frac{2}{3}$ months = $5\frac{2}{3}$ months, or 5mo. 20da.

Ans. 5mo. 20da.

From the *Analysis* we derive the following

RULE.

Multiply each payment by the time before it is due, and divide the sum of these products by the whole debt; the quotient will be the equated time.

EXAMPLES.

2. A debt of \$975 is due as follows: viz., \$150 in 5 months, \$375 in 4 months, and \$450 in 8 months; at what time should the whole sum be paid?

Ans. 6mo.

3. A merchant of Savannah bought a bill of goods in Boston amounting to \$3600, of which \$1500 was to be paid in 3mo., \$1200 in 4mo., and \$900 in 5 months; what is the average time for the payment of the whole? *Ans.* 3mo. 25da.

4. A gentleman owes the following sums, due as follows: viz, \$175 in 20 days, \$150 in 1mo. 15da., and \$200 in 60 days; at what time should the whole debt be paid?

Ans. 42 $\frac{3}{4}$ da.

5. A drover bought 80 horses and 110 mules, giving \$120 apiece for the horses and \$100 for the mules. He agreed to pay $\frac{1}{2}$ of the money in 4 months, $\frac{2}{3}$ in 6 months, and the balance in 10 months; how long a credit should he have on the whole sum if he pays it at one time? *Ans.* 5mo. 12da.

6. R owes S \$750; of which \$200 is to be paid down, \$250 in 3 months, and the balance in 4 months; if he agrees to pay the whole at one payment, what is the equated time?

Ans. 2mo. 18da.

NOTE.—In examples like the last the amount paid down must be regarded as a payment due in 0 days or months; and while the product of this first payment by 0 is *nothing*, yet, in dividing the sum of the products by the whole debt, the cash payment must be reckoned as a part of the divisor.

7. A farmer purchases \$300 worth of fertilizers, agreeing to pay $\frac{1}{4}$ down, $\frac{1}{4}$ in 2 months, $\frac{1}{4}$ in 4 months, and the balance in 6 months; what credit should be allowed to pay the whole at one time? *Ans.* 3 months.

Observation.—The foregoing Rule, though sanctioned by custom, is not *perfectly* accurate. It proceeds upon the supposition that a sum of money paid a certain length of time *before* it becomes due balances an equal sum paid the same length of time *after* it becomes due; or, in other words, that the *discount* on a sum of money is the same as the *interest* on the same sum for an equal time; which is not true. For

short periods and for small sums of money the difference is so slight that it is practically disregarded.

ART. 279. The TRUE EQUATED TIME can be found when desired. We will illustrate by an example:

8. H owes B \$1339, of which \$663 is due in 4 months, and \$676 in 8 months; what is the *true* equated time of paying the whole at one payment?

Analysis. If money is worth 6%, the *present worth* of \$663 due in 4mo. is \$650 (Art. 258); and the *present worth* of \$676 due in 8mo. is also \$650. The sum of the two present worths is \$1300. If H pays B \$1300 down, *exact justice* will be done. But the sum to be paid at *some time* is not \$1300, but \$1339. The question then is: "In how long a time will \$1300 amount to \$1339 at 6 per cent.?"—i. e., how long a time will it require for \$1300 *principal* to gain \$39 *interest*?

PROBLEM II., Art. 250, answers this question. We say: the interest of \$1300 for 1 year at 6% is \$78. Then, if \$78 interest requires 1 year, \$1 will require $\frac{1}{78}$ of a year, and \$39 will require 39 times as long = $\frac{39}{78}$ of a year = .5 of a year, or 6 months. The *true* time is therefore 6 months. If we find the average time according to the Rule, it will be found to give 6.019 months = 6mo. 0da. 13.68hr., making a difference of a little more than *half* a day.

What we have said in regard to finding the *true equated time* may be summed up in the following

RULE.

Find the present worth of each payment due; then find the TIME in which the sum of all these present worths will amount to the sum of all the payments.

EXAMPLES.

9. P owes W \$785, of which \$255 is due in 3mo., and \$530

in 9mo.; if money is worth 8%, what would be the *true* equated time to cancel the whole debt by one payment?

Ans. 7mo.

10. A farmer owes a commission merchant the following sums: viz., \$306 payable in 4mo., \$412 in 6mo., and \$530 in a year; what would be the *true* equated time of payment, money being worth 6%?

Ans. 8mo.

RATIO.

ARTICLE 280. When we compare two quantities of the same kind, we frequently say that one is *twice* as great, or *three* times or *four* times as great, as another. Thus, we say 8 pounds is twice as great as 4 pounds, because 4 is contained twice in 8; 12 shillings is three times as great as 4 shillings, because 4 is contained 3 times in 12. This is expressed concisely by saying that the RATIO of 8 to 4 is 2; of 12 to 4 is 3, etc. Hence,

ART. 281. RATIO is the relation of one quantity to another of the same kind. The RATIO of two numbers is indicated by two dots, placed one above the other, between the two numbers; thus $9 : 3$ expresses the RATIO of 9 to 3.

The two quantities compared are the *terms* of the ratio, the first term being called the *antecedent*,* the second the consequent,† and the two terms taken together are called a *couplet*.

ART. 282. RATIO is also expressed in the form of a fraction by making the antecedent the *numerator*, and the consequent the *denominator*; thus

$$\begin{aligned} 12 : 3 &= \frac{12}{3} = 4, \\ \text{and } 9 : 15 &= \frac{9}{15} = \frac{3}{5}. \end{aligned}$$

* *Antecedent*, from the Latin *antecedo*, signifying to go before.

† *Consequent*, from the Latin *consequor*, to follow after.

EXAMPLES.

1. What is the ratio of 5 to 9? Ans. $\frac{5}{9}$.
2. What is the ratio of 18 to 24? Ans. $\frac{3}{4} = \frac{3}{4}$
3. What is the ratio of 15 to 17? Ans. $\frac{15}{17}$.
4. What is the ratio of 24 to 16? Ans. $\frac{3}{2} = \frac{3}{2}$.
5. What is the ratio of 36 to 12? Ans. —.
6. What is the ratio of 9 to 27? Ans. —.
7. What is the ratio of 45 to 15? Ans. —.
8. Express 18 as a fractional part of 54.

Analysis. The question simply means: "What part of 54 is 18?" We say 1 unit is $\frac{1}{3}$ part of 54 units, and 18 units will be 18 times as much, which will give $\frac{18}{3} = 6$. Ans. $\frac{1}{3}$.

9. What part of 27 is 15? Ans. $\frac{5}{9}$.
10. Express 24 as a fractional part of 56. Ans. $\frac{3}{7}$.

ART. 283. To find the ratio of two quantities, they must be of the same *denomination*. Thus, if we were required to find the ratio of 5 gallons to 7 quarts, we could not get it directly, but by reducing the 5 gallons to 20 quarts, the ratio would be that of 20 quarts to 7 quarts, or $\frac{20}{7}$.

11. What is the ratio of 2ft. 3in. to 10in.? Ans. $\frac{27}{10}$.
12. What is the ratio of 3s. 6d. to 7d.? Ans. 6.
13. What is the ratio of 3pk. 6qt. to 2bu. 2pk. 4qt.? Ans. $\frac{6}{14}$.
14. What is the ratio of 528yd. to 1mi.? Ans. $\frac{3}{10}$.
15. What is the ratio of 6hr. 24min. to 2da.? Ans. $\frac{13}{25}$.
16. What is the ratio of 45ct. to \$3? Ans. $\frac{45}{300} = \frac{3}{20}$.
17. What is the ratio of 75ct. to \$5? Ans. $\frac{75}{500} = \frac{3}{20}$.
18. What is the ratio of \$7 to 85ct.? Ans. $\frac{700}{85} = \frac{140}{17}$.
19. What is the ratio of \$2.25 to \$9? Ans. $\frac{1}{4}$.
20. What is the ratio of \$3.75 to \$11.25? Ans. $\frac{1}{3}$.
21. What is the ratio of $\frac{9}{10}$ to $\frac{3}{10}$?

Analysis. It is obvious that $\frac{9}{10}$ is 3 times as much as $\frac{3}{10}$; hence the ratio of $\frac{9}{10}$ to $\frac{3}{10}$ is 3. Ans. 3.

22. What is the ratio of $\frac{1}{2}$ to $\frac{3}{4}$?

Analysis. $\frac{1}{2}$ is evidently 6 times as much as $\frac{3}{4}$; hence the ratio of $\frac{1}{2}$ to $\frac{3}{4}$ is 6. *Ans.* 6.

From the above *Analysis* we learn the truth that *the RATIO of two fractions which have a common denominator is the same as the ratio of their NUMERATORS.*

23. What is the ratio of $\frac{7}{8}$ to $\frac{3}{8}$? *Ans.* $\frac{7}{8} = \frac{1}{1}$

24. What is the ratio of $\frac{2}{3}$ to $\frac{1}{3}$? *Ans.* $\frac{2}{1}$.

25. What is the ratio of $\frac{1}{2}$ to $\frac{3}{4}$? *Ans.* $\frac{2}{3}$.

26. What is the ratio of $\frac{2}{3}$ to $\frac{3}{4}$?

Analysis. As these fractions have not a common denominator, we cannot get the ratio directly; but by reducing them to a *common denominator*, we have $\frac{8}{12}$ and $\frac{9}{12}$; and, from what we have seen above, the ratio is the same as that of 8 to 9, that is $\frac{8}{9}$. *Ans.* $\frac{8}{9}$.

27. What is the ratio of $\frac{5}{6}$ to $\frac{3}{6}$? *Ans.* $\frac{5}{3}$.

28. What is the ratio of $\frac{2}{3}$ to $\frac{1}{3}$? *Ans.* $\frac{2}{1}$.

29. What is the ratio of $\frac{9}{10}$ to $\frac{3}{10}$? *Ans.* $\frac{3}{1}$.

30. What is the ratio of $\frac{3}{4}$ to $\frac{1}{4}$? *Ans.* $\frac{3}{1}$.

31. What is the ratio of $\frac{3}{4}$ to $3\frac{3}{4}$? *Ans.* $\frac{1}{4}$.

32. What is the ratio of $3\frac{3}{4}$ to $5\frac{3}{4}$? *Ans.* $\frac{3}{5}$.

ART. 284. Since a RATIO is expressed by a fraction of which the *antecedent* is the numerator and the *consequent* the denominator, it follows that the terms of a ratio may be *multiplied* or *divided* just as the terms of a fraction.

PROPORTION.

ART. 285. PROPORTION is an equality of ratios.

The numbers 15, 5, 12 and 4 are *proportional*, for the ratio of 15 to 5 is equal to the ratio of 12 to 4.

ART. 286. PROPORTION is indicated by placing two dots between the first and second terms; four between the second and third; and two between the third and fourth; thus,

$$15 : 5 :: 12 : 4, \text{ or } 15 : 5 = 12 : 4.$$

This proportion is read thus: 15 is to 5 as 12 is to 4; or it may be read thus: the ratio of 15 to 5 is equal to the ratio of 12 to 4.

ART. 287. The first and last terms of a proportion are called the *extremes*, and the second and third terms, the *means*.

Thus, in the proportion $3 : 6 :: 4 : 8$, 3 and 8 are the *extremes*, and 6 and 4 the *means*.

ART. 288. Since in any proportion the ratio of the first term to the second is equal to the ratio of the third term to the fourth, in the proportion $3 : 6 :: 5 : 10$ we have $\frac{3}{6} = \frac{5}{10}$. Reducing these fractions to a common denominator we have $\frac{3 \times 10}{6 \times 10} = \frac{5 \times 6}{6 \times 10}$; and since the denominators are the same, and the values of the fractions equal, we have $3 \times 10 = 5 \times 6$; which illustrates a fundamental truth in Proportion, viz.:

In any proportion the product of the extremes is always equal to the product of the means.

ART. 289. Since in any proportion the *product of the extremes is equal to the product of the means*, this result naturally follows, viz.:

If the product of the extremes is divided by one of the means, the quotient is the other mean; or if the product of the means is divided by one of the extremes, the quotient is the other extreme.

Thus, in the proportion $12 : 4 :: 9 : 3$,

$$12 \times 3 \div 4 = 9, \text{ one of the means.}$$

$$12 \times 3 \div 9 = 4, \text{ the other mean.}$$

$$4 \times 9 \div 3 = 12, \text{ one of the extremes.}$$

$$4 \times 9 \div 12 = 3, \text{ the other extreme. Hence,}$$

ART. 290. *In any proportion, if we have the first, the second and the third terms given, we can find the fourth term by multiplying together the second and the third, and dividing the product by the first.*

SIMPLE PROPORTION.

ART. 291. SIMPLE PROPORTION is an equality of two *simple* ratios. By most writers on Arithmetic SIMPLE PROPORTION is called "RULE OF THREE," because *three* terms are always given to find a *fourth*.

Ex. 1. If 6 yards of velvet cost 24 dollars, what would be the cost of 18 yards?

Analysis. The answer is required in *dollars*. It is obvious that the cost of 18 yards will be as many times greater than 24 dollars (the cost of 6 yards) as 18 yards is greater than 6 yards; or, in other words, there must be the same ratio between 24 dollars and the answer as there is between 6 yards and 18 yards. Our proportion will now read thus:

$$6\text{yd.} : 18\text{yd.} :: \$24 : \text{fourth term.}$$

Multiplying the two means together, and dividing the product by the first term (Art. 290), we shall have \$72 as the fourth term, or answer. Our proportion will then read thus:

$$6\text{yd.} : 18\text{yd.} :: \$24 : \$72.$$

Here we see that the fourth term is 3 times as great as the third, just as the second is 3 times as great as the first. We see also that the first and second terms are of the same denomination, viz., *yards*; and the third and fourth terms are of the same denomination, viz., *dollars*.

From the foregoing explanations we derive the

RULE.

I. *Make that number which is of the same name with the answer the third term.*

II. *If from the nature of the question the answer must be greater than the third term, make the greater of the two remaining quantities the second term; but if the answer ought to be less than the third term, make the least of the two quantities the second term.*

III. *Multiply the second and third terms together ; divide the product by the first, and the quotient will be the fourth term , or answer, in the same denomination as the third term*

EXAMPLES.

2. If a locomotive can run 75 miles in 5 hours, how far could it run in 12 hours at the same rate?

The answer is required in *miles* ; therefore we make 75 *miles* the third term ; and from the nature of the question we know that the locomotive can run farther in 12 hours than in 5 hours ; hence we make 12 hours the second term, and 5 hours the first term. Multiplying the second and third terms together, and dividing by the first, we have 180 miles as the fourth term, or answer.

OPERATION.

$$\begin{array}{r} \text{hr. hr. mi.} \\ 5 : 12 :: 75 : \text{Ans.} \\ \quad \quad \quad 12 \\ 5 \overline{)900} \\ \quad \underline{180} \text{ miles.} \end{array}$$

Ans. 180 miles

Analysis. If in 5 hours it runs 75 miles, then in 1 hour it runs $\frac{1}{5}$ of 75 miles = 15 miles ; and in 12 hours it would run 12 times as many miles = 180 miles ; the same as by Proportion.

3. If 5 men can build a wall in 21 days, how long would 7 men be occupied in building the same?

OPERATION.

$$\begin{array}{r} \text{men men da.} \\ 7 : 5 :: 21 : \text{Ans.} \\ \quad \quad \quad 5 \\ 7 \overline{)105} \\ \quad \underline{15} \text{ days.} \end{array}$$

Ans. 15 days.

As the answer is required in *days*, we make 21 days the third term ; and because 7 men can do the work in less time than 5 men, the second term must be less than the first.

Analysis. If 5 men can do the work in 21 days, it would occupy one man 5 times as long = 105 days ; and it would occupy 7 men only *one seventh* as many days, which will be $\frac{105}{7}$ days = 15 days ; the same as by Proportion.

Ans. 15 days

4. If 15 bushels of wheat cost \$18.75, how much would 45 bushels cost? *Ans.* \$56.25.

5. If a man earn \$225 in 9 months, how much could he earn in 7 months? *Ans.* \$175.

6. If 12 yards of cassimere cost \$36, how many yards could you buy for \$75? *Ans.* 25 yards.

7. If 12 men can reap a field of wheat in 15 days, how many men could reap it in 20 days? *Ans.* 9 men.

8. If 9 sacks of corn cost \$20.25, what would 33 sacks cost at the same rate? *Ans.* \$74.25.

9. If 13 bushels of wheat weigh 806 pounds, what would be the weight of 56 bushels? *Ans.* 3472 pounds.

10. If an upright staff 10 feet long cast a shadow of 12 feet, what must be the height of a tree that casts a shadow of 108 feet? *Ans.* 90 feet.

11. If 9 silk hats cost \$45, how many hats could be bought for \$175? *Ans.* 35 hats.

12. If a man walk 324 miles in 12 days, how far could he walk in 17 days? *Ans.* 459 miles.

13. If 7 horses eat a load of hay in 15 days, how many days would it last 5 horses? *Ans.* 21 days.

14. If 8 tons of coal cost \$72, how many tons could be purchased for \$171? *Ans.* 19 tons.

15. If $\frac{7}{8}$ of a yard of velvet cost \$5.25, what would be the cost of 18 yards? *Ans.* \$108.

16. If 12 persons consume a barrel of flour in 18 days, how many persons would consume it in 8 days? *Ans.* 27 persons.

17. If the freight on 3cwt. 2qr. 16lb. of cheese for a certain distance be \$7.32, what would be the freight on 2T. 6cwt.? *Ans.* \$92.

NOTE.—First reduce the first and second terms to the same denomination.

18. If 3A. 2R. 9r. of land be worth \$68.28, what would be the value of 33 acres? *Ans.* \$633.60.

19. If 7gal. 1qt. of wine cost \$17.40, what would be the value of 3qt. 1pt.? *Ans.* \$2.10.

20. If 16 bushels of wheat be worth \$19.84, what would 56bu. 2pk. 4qt. be worth? *Ans.* \$70.21 $\frac{1}{2}$.

21. If 15 pounds of rice be worth \$1.35, how much would 225 pounds cost? *Ans.* \$20.25.

22. If a locomotive run 28 miles in 56 minutes, how long would it occupy in running 256 miles? *Ans.* 8hr. 32min.

23. If 7 bales of cotton can be carried 100 miles for \$10, how far ought 12 bales to be carried for the same money? *Ans.* 58 $\frac{1}{2}$ miles.

24. If 15 pounds of almonds cost \$3.75, how many pounds could be bought for \$18.75? *Ans.* 75 pounds.

25. If a pasture sustain 16 cows for 18 days, how many cows could be sustained 24 days? *Ans.* 12 cows.

26. If 35 yards of carpeting one yard wide be required to cover a floor, how many yards would be required when the carpeting is $\frac{7}{8}$ of a yard wide? *Ans.* 40 yards.

27. If the interest on a certain sum of money is \$252 for 9 months, how much would it be for 11 months? *Ans.* \$308.

28. If 40 rolls of paper 16 inches wide be required to paper the walls of a room, how many rolls would be required when the paper is 20 inches wide? *Ans.* 32 rolls.

29. If 13 yards of calico cost \$1.95, how many yards could you buy for \$11.25? *Ans.* 75 yards.

30. If the interest of \$225 for one year be \$15.75, what would be the interest of \$3625 for the same time? *Ans.* \$253.75.

31. If 6 $\frac{1}{2}$ yards of cassimere cost \$16.20, what would be the cost of 37 yards? *Ans.* \$88.80.

32. A ship's crew of 450 men have provisions enough for

12 months; how long will the provisions last if 150 more men be taken on board? *Ans.* 9 months.

33. If 9 yards of calico cost \$1.26, what would be the cost of 16 pieces, each measuring 34 yards? *Ans.* \$76.16.

34. If $\frac{5}{8}$ of a yard of cassimere cost \$2.50, what would $\frac{7}{8}$ of a yard cost? *Ans.* \$2.62 $\frac{1}{2}$.

35. If 12 men can build a house in 45 days, how many men could build the same in 30 days? *Ans.* 18 men.

36. If 42 acres of land produce 924 bushels of corn, how many bushels could 126 acres produce? *Ans.* 2772 bushels.

37. If 4 $\frac{1}{2}$ yards of velvet cost \$26.78, how much would 24 $\frac{1}{2}$ yards cost? *Ans.* \$151.41.

38. If 7.5 yards of silk cost \$24.30, what would be the cost of 145.25 yards? *Ans.* \$470.61.

COMPOUND PROPORTION, OR DOUBLE RULE OF THREE.

ART. 292. Suppose we have the two simple proportions,

$$8 : 4 :: 6 : 3, \text{ and}$$

$$10 : 5 :: 4 : 2.$$

Now let us multiply together the corresponding terms of these two proportions, and we shall have this proportion :

$$80 : 20 :: 24 : 6,$$

in which the ratio of the first and the second terms is the same as that of the third and the fourth terms.

This latter proportion is called a COMPOUND PROPORTION.

ART. 293. COMPOUND PROPORTION is used in solving questions the answers to which depend upon two or more different circumstances, and which therefore require two or more simple proportions.

Ex. 1. If 9 horses eat 36 bushels of oats in 18 days, how many bushels would suffice 15 horses for 12 days?

Here the answer is to be in *bushels*. The number of bushels will depend upon two things—viz., upon the number of *horses* and the number of *days*. If we consider the question only with reference to the number of horses, we shall have this proportion :

$$9 \text{ horses} : 15 \text{ horses} :: 36 \text{ bushels} : \text{Ans.}$$

Again, if we consider it only in reference to the length of time, we obtain this proportion :

$$18 \text{ days} : 12 \text{ days} :: 36 \text{ bushels} : \text{Ans.}$$

Writing these two proportions together, we shall have

$$\begin{array}{l} 9 : 15 :: \\ 18 : 12 :: \end{array} \left. \vphantom{\begin{array}{l} 9 : 15 :: \\ 18 : 12 :: \end{array}} \right\} 36 : \text{Ans.}$$

Multiplying together the corresponding terms, we have

$$162 : 180 :: 36 : \text{Ans.}$$

Proceeding with this last proportion according to the method given in Art. 290, we find the answer to be 40 bushels.

By observing this example we see that the numbers are given in pairs, with the exception of one. This one is always of the same name with the answer, and is usually called the *odd term*. Thus, in the example given, 9 horses and 15 horses constitute the first pair of terms; 18 days and 12 days the second pair, and 36 bushels is the odd term.

The foregoing explanations are briefly summed up in the following

RULE.

- I. *Make the ODD TERM the third term.*
- II. *Arrange the several pairs of terms as in SIMPLE PROPORTION. (Art. 291.)*
- III. *Multiply the third term by the product of all the second terms, and divide the result by the product of all the first terms; the quotient will be the fourth term, or answer.*

EXAMPLES.

2. If 8 men in 6 weeks can earn \$576, how much could 9 men earn in 5 weeks?

Here the *odd term*, or answer, is to be in dollars; hence \$576 must be the odd term. To arrange the first pair of terms, we make 8 men the first term and 9 men the second, because 9 men would earn *more* in the same time than 8 men. To arrange the second pair, we make 6 weeks the first term and 5 weeks the second term, because the same number of men could earn *less* in 5 weeks than in 6 weeks; we shall then have the proportions:

$$\begin{array}{l} 8 : 9 \\ 6 : 5 \end{array} \} :: 576 : \text{Ans.}$$

Now multiplying 576 (the third term) by the product of 9 and 5 (the second terms), and dividing the result by the product of 8 and 6 (the first terms), we have \$540 as the answer.

Ans. \$540.

By *Analysis*:

If 8 men in 6 weeks can earn \$576 dollars,

1 man in 6	"	"	72	"	
1	"	in 1 week	"	12	"
9 men in 1	"	"	108	"	
9	"	in 5 weeks	"	540	"

Ans. \$540.

NOTE.—Examples in Compound Proportion can be more readily and more satisfactorily solved by Analysis than by any system of Rules.

3. If 6 persons consume 240 pounds of sugar in 8 months, how many pounds would 9 persons consume in 11 months?

Ans. 495 pounds.

4. If 36 acres of meadow-land keep 60 sheep 30 days, how many sheep can be kept on 48 acres 20 days?

Ans. 120 sheep.

5. If \$375 yield an interest of \$20 in 8 months, in how many months would \$450 yield an interest of \$33?

Ans. 11 months.

6. If the freight on 36 boxes of tobacco for 120 miles is \$54, how many miles could 32 boxes be carried for \$84?

Ans. 210 miles.

7. If a man can walk 270 miles in 10 days, walking 9 hours a day, in how many days could he walk 432 miles, walking 8 hours a day?

Ans. 18 days.

8. If 15 persons consume 6 barrels of flour in 9 months, how many barrels would 20 persons consume in 12 months?

Ans. $10\frac{2}{3}$ barrels.

9. If \$350 gain \$14 in 8 months, in how many months would \$1500 gain \$75?

Ans. 10 months.

10. How much would it cost to carry 72 bales of cotton 120 miles, if it costs \$60 to carry 48 bales 96 miles?

Ans. \$112.50.

11. If 6 men in 24 days, working 10 hours a day, can build a stone wall 48 yards long, how many men can build a wall 72 yards in length, in 18 days, working 8 hours a day?

Ans. 15 men.

PARTNERSHIP.

ART. 294. When two or more persons combine to carry on any particular branch of business the connection so formed is called PARTNERSHIP. The persons composing the partnership are called a *company*, *firm* or *house*. The money invested is called the *capital* or *stock*, and the gain or loss to be shared among the partners is called the DIVIDEND.

Ex. 1. R and S formed a partnership to carry on business. R furnished \$2475, and S put in \$3150; they gained \$1875. What was each one's share of the gain?

Analysis. The whole capital is \$5625. We say: If \$5625

gain \$1875, then \$1 gains $\frac{1}{8825}$ of \$1875 = $\frac{1875}{8825}$, or $\frac{1}{8}$ of a dollar; and \$2475 (R's capital) will gain 2475 times as much = $\frac{2475}{8}$, or \$825; and since \$1 gains $\frac{1}{8}$, then \$3150 will gain 3150 times as much = $\frac{3150}{8}$, or \$1050.

Ans. R, \$825; S, \$1050.

Proof. R's share, \$825, added to S's share, \$1050, will give \$1875.

By noticing the above *Analysis* we see the reason of the following

RULE.

Make the whole gain or loss the numerator, and the whole stock employed the denominator, of a fraction; then multiply this fraction successively by each man's stock.

Another rule frequently given is this: say,

As the whole stock is to each man's stock, so is the whole gain or loss to each man's share of the gain or loss.

EXAMPLES.

2. A and B jointly rented a pasture for \$24; A put in 36 cows and B 24 cows; how much of the rent ought each to pay?

Ans. A, \$14.40; B, \$9.60.

3. Thomas and Smith traded in company. Thomas invested \$840 and Smith \$480. At the end of two years they had lost \$264; how ought this loss to be shared?

Ans. Thomas, \$168; Smith, \$96.

4. Three men, A, B and C, jointly own a house costing \$9000; A furnishing \$4500, B, \$2500 and C, \$2000; the house rents for \$810; what share of the rent ought each to have?

Ans. A, \$405; B, \$225; C, \$180.

5. P, Q and R hired a carriage for \$15.75, each agreeing to pay in proportion to the number of miles he rode. P rode 90 miles, Q, 75, and R, 60 miles; what part of the hire ought each to pay?

Ans. P, \$6.30; Q, \$5.25; R, \$4.20.

6. D, E and F jointly erected a cotton-factory, D furnishing \$23450 of the capital, E, \$27975, and F, \$32425. After a certain time they found their profits were \$13975; how ought this gain to be divided?

Ans. D, \$3908 $\frac{1}{2}$; E, \$4662 $\frac{1}{2}$; F, \$5404 $\frac{1}{2}$.

ART. 295. In the Examples thus far given the capital of each partner has been employed for an equal length of time, but in practical business persons frequently enter into partnerships where their capital is employed for unequal lengths of time.

1. H and R engaged in trade; H put in \$540 for 6 months, and R, \$450 for 8 months. They gained \$513; what was each man's share of the gain?

Analysis. \$540 for 6 months is the same as 6 *times* as many dollars for 1 month = \$3240 for 1 month; and \$450 for 8 months is the same as 8 *times* as many dollars for 1 month = \$3600 for 1 month. Hence, \$3240 (H's money) for 1 month, added to \$3600 (R's capital) for 1 month, is equivalent to \$6840 for 1 month. Then, since the whole capital of \$6840 gains \$513, \$1 gains $\frac{1}{6840}$ of \$513 = $\frac{513}{6840}$, or $\frac{3}{40}$; and \$3240 will gain 3240 times as much = $\frac{3240 \times 3}{40}$, or \$243; and \$3600 will gain 3600 times as much as \$1, which will give $\frac{3600 \times 3}{40}$, or \$270.

Ans. H, \$243; R, \$270.

From the *Analysis* the reason is obvious for the

RULE.

I. *Multiply each man's stock by the time it was continued in trade.*

II. *Make the sum of these products the denominator of a fraction, and the whole gain or loss the numerator; then multiply this fraction successively by the product of each man's stock into its time.*

Those who prefer it may employ the following

RULE.

Multiply each man's stock by the time it was continued in trade; then say: As the sum of these products is to each separate product, so is the whole gain or loss to each man's share of the gain or the loss.

Solving the above Example by this latter Rule, we have the following proportions:

$$\$6840 : \$3240 :: \$513 : \text{H's share} = \$243.$$

$$\$6840 : \$3600 :: \$513 : \text{R's} \quad " \quad = \$270.$$

For the mental *training* of the pupil we recommend the method by *Analysis* as preferable to any rules.

EXAMPLES.

2. D and E formed a partnership in trade. D put in \$750 for 12 months, and E, \$600 for 18 months. They gained \$594; How should the gain be divided? *Ans.* D, \$270; E, \$324.

3. Z and W engaged in speculation. Z employed \$945 for 6 months, and W \$810 for 8 months. They lost \$729; how should this loss be divided? *Ans.* Z, \$340.20; W, \$388.80.

4. Three men, A, B and C, rented a pasture for \$70.56. A put in 36 cows for 5 months; B, 48 cows for 4 months; and C, 72 cows for 3 months; what part of the rent ought each to pay? *Ans.* A, \$21.60; B, \$23.04; C, \$25.92.

5. A gentleman employed three mowers to cut hay. The first mowed 9 days, at 10 hours a day; the second worked 12 days, at 9 hours a day; and the third mowed 15 days, at 12 hours a day. Their joint wages amounted to \$45.36. What share of the wages ought each to receive?

Ans. First, \$10.80; second, \$12.96; third, \$21.60.

6. C and H engaged in trade. C put in \$924 for 10 months, and H \$855 for 12 months. They gained \$585; how should the gain be divided? *Ans.* C, \$277.20; H, \$307.80.

ALLIGATION.

ARTICLE 296. ALLIGATION MEDIAL explains how to find the average price of a mixture of different articles when the quantity of each article and its price are given.

Ex. 1. A farmer combines 9 bushels of oats worth 90 cents a bushel, with 9 bushels of rye worth \$1.20, and 12 bushels of barley worth \$1.25; what is a bushel of the mixture worth?

Analysis.

9bu. of oats	at	90ct. per bushel	are worth	\$8.10
9bu. of rye	at	\$1.20	" "	10.80
12bu. of barley	at	\$1.25	" "	15.00
30bu. = whole compound				= \$33.90

Hence, if 30 bushels = \$33.90, 1 bushel is worth $\frac{1}{30}$ of \$33.90 = \$1.13.

Ans. \$1.13 per bushel.

The *Analysis* shows the *reason* of the following

RULE.

Find the value of each article, then divide the total value of the articles mixed by the number of articles.

NOTE.—An ingredient having no commercial value, such as water, may enter into a compound. Although its value is nothing, still its quantity must be added as constituting a part of the number or sum of the articles

EXAMPLES

2. A grocer mixes 20lb. of clarified sugar worth 15 cents per pound, with 24lb. of A sugar at 10 cents, and 28lb. of brown sugar at 9 cents; what is a pound of the mixture worth?

Ans. 11 cents.

3. A whisky-seller mixes 24 gallons of corn whisky worth

64 cents per gallon, with 8 gallons of water, which costs nothing; what is a gallon of the mixture worth?

Ans. 48 cents.

4. A farmer bought 12 pigs; for 3 of them he gave \$4 apiece; for 5 of them \$8 apiece, and for the remainder \$5 each; what was the average price of each? *Ans.* \$6.

5. A grocer mixes 15lb. of tea worth 80 cents a pound, with 20lb. worth 60 cents, and 25lb. worth 90 cents; what is a pound of the mixture worth? *Ans.* 77½ cents.

6. A locomotive ran as follows: for 4 hours at 20 miles an hour, 5 hours at 18 miles per hour, 3 hours at 24 miles, and 1 hour at 31 miles; what was the average hourly speed?

Ans. 21 miles.

7. A thermometer stood for 4 days at 76°, 3 days at 78°, 5 days at 80°, and 6 days at 84°; what was the average temperature? *Ans.* 80½°.

ALLIGATION ALTERNATE.

ART. 297. **ALLIGATION ALTERNATE** is the method of finding *how much* of several articles whose values are given must be taken to form a compound of a particular value.

Ex. 1. A grocer has sugar worth 7 cents, 8 cents, 13 cents and 14 cents a pound, which he wishes to mix together, so that he may be able to sell the mixture at 11 cents per pound; how much of each kind must he take?

Analysis. It is plain that if he puts in 1 pound of the 7-cent sugar and sells it for 11 cents, he *gains* 4 cents; then, to gain 1 cent, he must put in only $\frac{1}{4}$ of a pound. Again, if he puts in 1 pound of the 14-cent sugar, he *loses* 3 cents; then, to lose 1 cent, he must put in $\frac{1}{3}$ of a pound. Now, the 1 cent *gained* on the $\frac{1}{4}$ of a pound of the 7-cent sugar exactly balances the 1 cent *lost* on the $\frac{1}{3}$ of a pound of the 14-cent sugar. Hence, the quantities of each kind of sugar must be to each

other as $\frac{1}{4}$ is to $\frac{1}{3}$. But (according to Art. 283) $\frac{1}{4}$ is to $\frac{1}{3}$ as $\frac{3}{4}$ is to $\frac{1}{3}$, or as 3 is to 4. Hence, there must be 3 pounds of the 7-cent sugar and 4 pounds of the 14-cent sugar.

Again, if he puts in 1 pound of the 8-cent sugar, he *gains* 3 cents; then, to gain 1 cent, he must put in $\frac{1}{3}$ of a pound. Again, if he puts in 1 pound of the 13-cent sugar, he *loses* 2 cents; then, to lose 1 cent, will require $\frac{1}{2}$ of a pound. But the 1 cent *gained* on the $\frac{1}{3}$ of a pound of the 8-cent sugar will balance the 1 cent *lost* on the $\frac{1}{2}$ of a pound of the 13-cent sugar. Hence, the quantity of these two must be to each other as $\frac{1}{3}$ is to $\frac{1}{2}$, or (Art. 283) as $\frac{2}{3}$ is to $\frac{1}{2}$, or as 2 to 3. Therefore, for every 2 pounds of the 8-cent there must be 3 pounds of the 13-cent sugar, and we have already shown that there must be 3 pounds of the 7-cent and 4 pounds of the 14-cent sugar. The relative quantities, therefore, are 3lb. at 7c., 2lb. at 8c., 3lb. at 13c., and 4lb. at 14c.

Proof.

3lb. at 7c. = 21c.

2lb. at 8c. = 16c.

3lb. at 13c. = 39c.

4lb. at 14c. = 56c.

12lb. = mixture = \$1.32. Therefore 1lb. = $\$1.10$ = 11c.

The usual method of solving questions like the foregoing is to link the values in pairs, as indicated in the operation on the right, the **RULE** for which would be about as follows :

OPERATION.

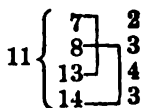
11 $\left\{ \begin{array}{l} 7 \\ 8 \\ 13 \\ 14 \end{array} \right\} \begin{array}{l} 3 \\ 2 \\ 5 \\ 4 \end{array}$

Write the values of the articles in a column, with the mean or average value on the left. Link each value less than the average with one greater, and each greater with one less. Write the difference between the average and each value opposite to the value with which it is linked. The numbers thus placed opposite the given values will be the relative quantities required.

This RULE is simply an ingenious *mechanical* device. The *numerical* results arrived at are precisely the same with those reached by *analysis*, but it should not be allowed to supersede *analysis*. After the student has become familiar with the analytic method he may then avail himself of the "RULE," as being *short, easy* and *practical*.

In solving questions by the RULE a variety of answers may be obtained, corresponding to the different ways in which the values may be linked. Thus, taking the example already given, and linking the values, as in the margin, we have as the result : 2lb. at 7ct., 3lb. at 8c., 4lb. at 13ct., and 3lb. at 14ct.

OPERATION.



Proof.

$$2\text{lb. at } 7\text{c.} = 14\text{ct.}$$

$$3\text{lb. at } 8\text{c.} = 24\text{ct.}$$

$$4\text{lb. at } 13\text{c.} = 52\text{ct.}$$

$$3\text{lb. at } 14\text{c.} = 42\text{ct.}$$

$$\underline{12\text{lb.}} = \text{mixture} = \$1.32. \quad \text{Therefore } 1\text{lb.} = \$\frac{1.32}{12} = 11\text{ct.}$$

If we *double* each of the two answers already found, we will then have *four* different answers; if we multiply each of them by *three*, we will then have *six* correct answers, and so on. From which we see that to all questions in Alligation the number of answers is unlimited.

It sometimes happens that one of the articles is limited to a particular or definite quantity. In the example already given, suppose the grocer had had 24 pounds of the 7-cent sugar, and wished to find the corresponding quantities of other kinds to mix with it. Taking our first answer, we found the relative quantities to be 3, 2, 3, 4. But as 24 pounds of 7-cent sugar, instead of 3 pounds, are required to enter into the mixture, we must take $\frac{24}{3} = 8$ times each of the other kinds, in order that the gain and the loss may be equal. Our answer will then be : 24lb. at 7ct., 16lb. at 8ct., 24lb. at 13ct., and 32lb. at 14ct.

Proof.

24lb. at 7ct. = \$1.68

16lb. at 8ct. = 1.28

24lb. at 13ct. = 3.12

12lb. at 14ct. = 4.48

96lb. = mixture = \$10.56. Therefore 1lb. = $\frac{\$10.56}{96} = 11\text{ct.}$

It may sometimes happen that the *whole* mixture is limited to a definite quantity. In the example given above, suppose the grocer had wished to fill a barrel to contain 240 pounds. Taking the first answer to our example, we found the *whole* mixture to be 12 pounds, and the *relative* quantities were 3, 2, 3 and 4. But the required mixture is to contain 240 pounds instead of 12. Hence, $\frac{240}{12} = 20 =$ the number of times that the *relative* quantities must be increased, in order to give the required amount of the mixture. Our answer will then be: 60lb. at 7ct., 40lb. at 8ct., 60lb. at 13ct., and 80lb. at 14ct.

EXAMPLES.

2. In what ratio must sugars worth respectively 7 cents and 12 cents per pound be mixed, so that the compound may sell for 10 cents a pound? *Ans.* 2lb. at 7ct.; 3lb. at 12ct.

3. What quantities of coffee worth respectively 23 and 30 cents per pound must be mixed so as to sell the mixture for 27 cents a pound? *Ans.* 3lb. of first; 4lb. of second.

4. A grocer has three different qualities of soda, worth respectively 12 cents, 13 cents and 18 cents per pound; how much of each quality must he take so as to sell the compound at 16 cents per pound?

Ans. 2lb. at 12ct.; 2lb. at 13ct.; 7lb. at 18ct.

NOTE.—Two of the values in this example being *below* the average, and one *above*, the two below must be compared in succession with the one above.

5. A merchant has vinegar worth 30 cents, 37 cents and 40

cents a gallon; what quantity of each kind must he take so as to sell the mixture at 35 cents a gallon?

Ans. 7gal. at 30ct.; 5gal. at 37ct.; 5gal. at 40ct.

Suppose the merchant had wished to mix 35 gallons of the 30ct. vinegar with the others; how much must he have taken?

Ans. 25gal. of each.

Suppose he had wished to fill a cask to contain 136 gallons; what would have been the respective quantities?

Ans. 56gal. at 30ct.; 40gal. at 37ct.; 40gal. at 40ct.

6. How many gallons of molasses, worth respectively 36 cents, 42 cents and 45 cents per gallon, must be mixed together, so as to sell the mixture at 40 cents a gallon?

Ans. 7gal. at 36ct.; 4gal. at 42ct.; 4gal. at 45ct.

Suppose there had been 40 gallons of the 45-cent molasses; how many gallons of the other kinds must have been taken?

Ans. 70gal. at 36ct.; 40gal. at 42ct.

Suppose it had been required to fill a cask to hold 135 gallons; how much of each sort would have been needed?

Ans. 63gal. at 36ct.; 36gal. at 42ct.; 36gal. at 45ct.

7. A merchant has four kinds of pepper, worth 23, 24, 27 and 30 cents per pound; how much of each kind must be taken, so that he may sell the mixture at 25 cents a pound?

Ans. { 5lb. at 23ct.; 2lb. at 24ct.; 1lb. at 27ct.; 2lb. at 30ct.
Or, 2lb. at 23ct.; 5lb. at 24ct.; 2lb. at 27ct.; 1lb. at 30ct.

Suppose there had been 30 pounds of the 23-cent pepper, how much of the other kinds would have been required?

Ans. { 12lb. at 24ct.; 6lb. at 27ct.; 12lb. at 30ct.
Or, 75lb. at 24ct.; 30lb. at 27ct.; 15lb. at 30ct.

Suppose the merchant had wished to fill a bag to contain 180 pounds; how much of each kind must have been taken?

90lb. at 23ct.; 36lb. at 24ct.; 18lb. at 27ct.; 36lb. at 30ct.
Or, 36lb. at 23ct.; 90lb. at 24ct.; 36lb. at 27ct.; 18lb. at 30ct.

Ans

EXCHANGE.

ARTICLE 298. EXCHANGE is the process of receiving or of paying money in one place for an equivalent sum in another place, by *order* or *Bill of Exchange*.

ART. 299. A BILL OF EXCHANGE is a written order drawn on a person in a place more or less distant, requesting him to pay a specified sum of money to another person or to his order. The person who draws or signs the bill is called the DRAWER or MAKER; the person on whom it is drawn is called the DRAWEE; and the person to whom the money is to be paid is called the PAYEE.

ART. 300. Exchange is said to be *at par* when a given sum at the place from which it is remitted will pay an equal amount in the place to which it is remitted. It is said to be *above par* when the sum to be sent will pay a less amount at the place to which it is sent; and *below par* when the sum to be sent will pay a greater amount at the place to which it is sent.

DOMESTIC BILLS OF EXCHANGE.

ART. 301. Bills of exchange are called DOMESTIC when the *drawer* and *drawee* both reside in the same country. They are commonly called DRAFTS or CHECKS.

FORM OF A DOMESTIC BILL OF EXCHANGE.

No. 18. STAMP.	\$3000. MACON, GA., Aug. 25, 1872. <i>Ten days after sight, pay to the order of J. B. Lippincott & Co. Three Thousand Dollars. Value received, and charge to our account.</i> <i>Cubbedge & Hazlehurst.</i> <i>To N. B. Browne, Philadelphia.</i>
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In the preceding bill of exchange Messrs. Cubbedge & Hazlehurst are the *drawers*, N. B. Browne is the *drawee*, and J. B. Lippincott & Co. are the *payees*.

EXAMPLES.

1. A manufacturer in Providence wishes to send \$7857 to Mobile, exchange being at a premium of $1\frac{1}{2}\%$; what must he pay for a check?

Analysis. $1\frac{1}{2}$ per cent. of \$7857 is \$117.85 $\frac{1}{2}$. Adding \$117.85 $\frac{1}{2}$, the premium, to \$7857, we have \$7974.85 $\frac{1}{2}$ as the price of the check.

Ans. \$7974.85 $\frac{1}{2}$.

2. What would be the cost of a draft at St. Paul, Minn., on Boston for \$6472, at $1\frac{5}{8}$ per cent. premium?

Ans. \$6577.17.

3. Exchange being at a discount of $1\frac{3}{4}$ per cent., what would be the cost in Savannah, Ga., of a check for \$1256, on Rutland, Vt.?

Ans. \$1234.02.

4. A merchant in New Orleans wishes to pay \$4528 in Philadelphia, exchange being $2\frac{1}{8}$ per cent. premium; what must he pay for a check?

Ans. \$4624.22.

5. What would be the cost of a check at Newark, N. J., on Vicksburg, Miss., for \$4856.40, at $\frac{7}{8}$ of 1 per cent. discount?

Ans. \$4813.9065.

FOREIGN BILLS OF EXCHANGE.

ART. 302. Bills of exchange are called FOREIGN when the drawer and drawee live in different countries; for example, when one lives in the United States and the other in England.

ART. 303. In order that debts between foreign countries may be paid, it is necessary to know how much money of one country is equivalent to a specified sum in another country. If a citizen of New York wished to pay a debt of 450 pounds sterling in England, he must be able to tell how many *dollars*

and cents in New York would be required to make 450 pounds sterling in England. The process used is called REDUCTION OF CURRENCY.

EXCHANGE BETWEEN THE UNITED STATES AND ENGLAND.

ART. 304. Bills of exchange on England are drawn in pounds sterling, shillings, pence and farthings. The pound sterling is represented by a gold coin called a *sovereign*, and its custom-house value in the United States is \$4.84. A law of Congress in 1789 fixed the value of the pound sterling at \$4.44 $\frac{2}{3}$, and this continued to be the value up to 1834. It will be seen that \$4.84, the custom-house value of the pound sterling, is 9 per cent. above the old par value of \$4.44 $\frac{2}{3}$. Hence, when it is said that bills of exchange on England are at a premium of 9 per cent., it must be borne in mind that the pound sterling is reckoned at \$4.44 $\frac{2}{3}$, and that exchange is really at par, taking the custom-house value as the standard.

Ex. 1. A merchant in Baltimore purchased a bill of exchange on Liverpool for £456 12s. at 9 per cent. premium; how much in gold did he pay for the bill?

Analysis. £1 = \$4.44 $\frac{2}{3}$ at the old par value; and £456 12s. = £456.6. Since £1 = \$4.44 $\frac{2}{3}$, then £456.6 will be worth 456.6 times as much = \$2029.33 $\frac{1}{3}$; taking 9 per cent. off \$2029.33 $\frac{1}{3}$, we have \$182.64, which added to \$2029.33 $\frac{1}{3}$ gives \$2211.97 $\frac{1}{3}$ as the price of the bill.

The operation might have been shortened thus:

$$£456\ 12s = £456.6$$

$$\$4.44\frac{2}{3} = \$4\frac{2}{3} = \$4\frac{4}{3}$$

Then $\$4\frac{4}{3} \times 456.6 = \$2029.33\frac{1}{3}$ at the old par value,
and $\$2029.33\frac{1}{3} \times .09 = 182.64$, the premium at 9 per cent.

Ans. \$2211.97 $\frac{1}{3}$ = price of the bill.

2. A merchant in Philadelphia owes \$3630.24 for cutlery in Sheffield, England; for how many pounds sterling must he

purchase a bill to pay the debt, exchange being at a premium of 8 per cent.?

Analysis. £1 = \$4.44 $\frac{1}{2}$; and taking 8% of \$4.44 $\frac{1}{2}$, we have \$0.35 $\frac{1}{2}$; adding this to \$4.44 $\frac{1}{2}$, we get \$4.80 as the commercial value of £1. Now, as it requires \$4.80 to purchase £1, it follows that \$3630.24 will purchase as many pounds as \$4.80 is contained times in \$3630.24, which gives £756.3, or £756 6s.

Ans. £756 6s.

3. Exchange being at 9% premium, what would be the cost of a bill purchased in Cincinnati to pay £648 16s. in Birmingham?

Ans. \$3143.07 $\frac{1}{2}$.

4. When exchange is at 10 $\frac{1}{4}$ per cent. premium what would be the face of a bill on Manchester which could be purchased in Mobile for \$4040.05?

Ans. £824 10s.

5. A broker in Richmond, Va., sold a bill of exchange on London, the face of which was for £864 4s. 6d.; how much did he receive for the bill, exchange being at 10% premium?

Ans. \$4225.10.

EXCHANGE ON FRANCE.

ART. 305. In France the currency is reckoned in *francs* and *centimes*. Centimes are hundredths of a franc. Thus, 5.24 francs may be read, 5 francs 24 centimes. The custom-house value of the franc is 18 $\frac{6}{10}$ cents. The commercial value is not uniform, but, like the pound sterling, it fluctuates with the state of trade. Bills of exchange on France are bought and sold at a certain number of francs and centimes to the dollar.

Ex. 1. A merchant of St. Louis, Mo., buys a bill of exchange on Paris for \$4275, exchange being 5.25 francs to the dollar; what is the face of his bill in francs and centimes?

Analysis. Since \$1 = 5.25 francs, \$4275 must be equal to 4275 times as many francs = $5.25 \times 4275 = 22443.75$ francs

Ans. 224 3.75fr.

2. A silk merchant in New York owes 12969 francs in Lyons, and buys a bill of exchange, the rate being 5 francs 24 centimes to the dollar; what must he pay for his bill?

Analysis. Since 5.24 francs = \$1, there will be as many dollars as 5.24 is contained times in 12969, which gives \$2475.

Ans. \$2475.

3. A wine merchant in New Orleans wishes to remit a bill of \$4273 to Bordeaux, exchange being 5 francs 31 centimes to the dollar; what must be the face of his bill of exchange?

Ans. 22689.63fr.

4. Exchange on Rouen being 5 francs 35 centimes to the dollar, how many dollars would be required to buy a bill of exchange covering 13096.80 francs?

Ans. \$2448.

5. A merchant in Louisville, Ky., owes \$3256 in Marseilles; exchange being 5.32 francs to the dollar, what must be the face of his bill of exchange to pay the debt?

Ans. 17321.92fr.

NOTE.—For more minute information respecting Exchange consult "*Sanford's Higher Analytical Arithmetic.*"

DUODECIMALS.

ARTICLE 306. DUODECIMALS are a peculiar kind of compound quantities which *decrease* from left to right uniformly in a *twelvefold* ratio. They derive their name from the Latin word *duodecim*, meaning *twelve*. They are used in the measurement of *surfaces* and *solids*, as plastering, painting and masonry.

ART. 307. The denominations of Duodecimals are *feet*, *inches* or *primes*, *seconds*, *thirds*, *fourths*, etc., which are distinguished from each other by certain marks called INDICES.

Thus, inches or primes are marked ($'$), seconds are marked ($''$), thirds ($'''$), fourths ($''''$), etc.

ART. 308. The foot is always taken as the *unit*, whether it be linear, square, or cubic measure, and the several denominations have the relations indicated below.

$$1' = \frac{1}{12} \text{ of a foot.}$$

$$1'' = \frac{1}{12} \text{ of } 1' = \frac{1}{12} \text{ of } \frac{1}{12} \text{ of 1 ft.} = \frac{1}{144} \text{ of a foot.}$$

$$1''' = \frac{1}{12} \text{ of } 1'' = \frac{1}{12} \text{ of } \frac{1}{144} \text{ of 1 ft.} = \frac{1}{1728} \text{ of a foot.}$$

$$1'''' = \frac{1}{12} \text{ of } 1''' = \frac{1}{12} \text{ of } \frac{1}{1728} \text{ of 1 ft.} = \frac{1}{20736} \text{ of a foot.}$$

ART. 309. In Duodecimals, 12 of any lower denomination make one of the next higher, or one of a higher is equivalent to 12 of the next lower. Hence,

12 fourths ($''''$) make 1 third, ($'''$).

12 thirds ($'''$) " 1 second, ($''$).

12 seconds ($''$) " 1 prime, ($'$).

12 primes ($'$) " 1 foot, ft.

ART. 310. Duodecimals may be *added* or *subtracted* in the same manner as compound quantities.

EXAMPLES.

1. What is the sum of 35ft. 8' 10'', 27ft. 6' 9'' and 32ft. 5' 8'' ?
Ans. 95ft. 9' 3''.

2. Add together 213ft. 7' 9'' 8''', 124ft. 8' 10'' 7''' and 185ft. 9' 7'' 6'''.
Ans. 524ft. 2' 3'' 9'''.

3. From 186ft. 5' 8'' 10''' subtract 128ft. 9' 6'' 5'''.

Ans. 57ft. 8' 2'' 5'''.

4. From 323ft. 5' 4'' 7''' 6'''' take 185ft. 8' 2'' 9''' 4''''.

Ans. 137ft. 9' 1'' 10''' 2''''

MULTIPLICATION OF DUODECIMALS.

ART. 311. Plasterers, painters, carpenters and rock-masons employ duodecimals in estimating their work. One of the most common applications is in measuring lumber.

ART. 312. The multiplication of duodecimals is the same in principle as the multiplication of common fractions, with this difference, that by the artifice of INDICES we are saved the necessity of writing the denominators.

Thus $5\text{ft.} \times 7\text{ft.} = 35$ (square) feet.

$$4\text{ft.} \times 6' = 4\text{ft.} \times \frac{6}{12}\text{ft.} = \frac{24}{12}\text{ft.} = 2'.$$

$$5' \times 9' = \frac{5}{12} \times \frac{9}{12} = \frac{45}{144} = 45''.$$

$$7' \times 3'' = \frac{7}{12} \times \frac{3}{144} = \frac{21}{1728} = 21'''.$$

$$4'' \times 8'' = \frac{4}{144} \times \frac{8}{144} = \frac{32}{20736} = 32''''.$$

We learn from the foregoing illustrations that feet multiplied by feet give (square) feet; feet multiplied by primes give primes; primes multiplied by primes give seconds; primes multiplied by seconds give thirds; seconds multiplied by seconds give fourths, etc. Hence, to determine the denomination of the product of any two factors,

Add the INDICES of the two factors.

Ex. 1. Multiply 4ft. 5in. by 3ft. 6in.

Analysis. We begin on the right and multiply first by the *primes*, and then by the feet of the multiplier. We proceed thus: $5' \times 6' = 30'' = 2'$ and $6''$. We set down the $6''$, and carry the $2'$ to the next product. Again, $4 \times 6' = 24'$, and $2'$, which we carry, make $26' = 2\text{ft.}$ and $2'$, which we write in their proper columns.

OPERATION.

$$\begin{array}{r} 4\text{ft. } 5 \\ 3\text{ft. } 6' \\ \hline 2\text{ft. } 2' 6'' \\ 13\text{ft. } 3' \\ \hline 15\text{ft. } 5' 6'' \text{ Ans.} \end{array}$$

Next multiply by 3ft. We say: $5' \times 3 = 15' = 1\text{ft.}$ and $3'$. We write the $3'$ under the *primes* of the line above, and carry the 1ft. to the next product. Lastly, $4\text{ft.} \times 3\text{ft.} = 12\text{ft.}$, to which, adding the 1ft. from the last product, we have 13ft. , which we set under the feet in the first line. Adding the separate products, we get $15\text{ft. } 5' 6''$ for the answer.

Ans. 15ft. 5' 6''

Hence, we have the following

RULE.

I. *Place the several terms of the multiplier under the corresponding terms of the multiplicand.*

II. *Begin at the right, and multiply each term of the multiplicand by each term of the multiplier in succession, reducing and carrying as in Compound Multiplication, and placing numbers of the same denomination in the same column. Add the partial products, and their sum will be the product required.*

EXAMPLES.

2. Multiply 7ft. 8' by 5ft. 7'. *Ans.* 42 sq. ft. 9' 8".

3. How many sq. feet in a plank 13ft. 5' in length, and 2ft. 3' in breadth? *Ans.* 30 sq. ft. 2' 3".

4. A dining-table is 8ft. 6' 9" long, and 5ft. 7' broad; how many sq. feet does it contain? *Ans.* 47 sq. ft. 9' 8" 3".

5. How many cubic feet in a block of granite 10ft. 4' long, 4ft. 6' wide and 2ft. 8' deep? *Ans.* 124 cu. ft.

6. How many square feet in the floor, ceiling and four walls of a room 18ft. 6' long, 15ft. 9' wide, and 8ft. 4' high?

Ans. 1153 sq. ft. 7'.

INVOLUTION.

ARTICLE 313. When a number is multiplied *into itself* the product is called a **POWER**.

ART. 314. **POWERS** are called *first, second, third, fourth, etc.*, according to the number of times that the given number is used as a factor.

ART. 315. The *first power* of a number is the number itself. The first power is also called the **ROOT** of the other powers. Thus the *first power* of 5 is 5; the *first power* of 2 is 2.

ART. 316. The *second power*, or *square*, of a number, is the product of the number used *twice* as a factor. Thus, the *second power* or *square* of 5 is $5 \times 5 = 25$.

What is the second power of 3? of 4? of 6? of 7? of 8? of 9? of 10?

ART. 317. The *third power* or *cube* of a number is the product of the number used *three times* as a factor. Thus, the *third power* or *cube* of 4 is $4 \times 4 \times 4 = 64$.

What is the third power or cube of 3? of 5? of 2? of 6? of 7? of 9?

ART. 318. The *fourth power* of a number is the product of the number used *four times* as a factor. Thus, the fourth power of 3 is $3 \times 3 \times 3 \times 3 = 81$.

What is the fourth power of 2? of 4? of 5? of 6?

On the same principle we define the fifth, sixth, seventh or any other power.

ART. 319. The process of raising a number to any required power is called INVOLUTION.

ART. 320. To indicate the power to which a number is to be raised we place a small figure, called an *index* or *exponent*, at the right of the number and a little above it. Thus,

$3^2 = 3 + 3 = 9$, and is read, the second power or square of 3.

$3^3 = 3 \times 3 \times 3 = 27$, and is read, the cube or third power of 3.

$3^4 = 3 \times 3 \times 3 \times 3 = 81$, the fourth power of 3.

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$, the fifth power of 3, etc.

From the foregoing explanations we derive the

RULE FOR INVOLUTION.

Multiply the number by itself until it is used as a factor as many times as there are units in the index of the required power.

NOTE.—In raising a number to any power the number of multiplications is always *one* less than the number of units in the index of the power; thus, *one* multiplication gives the *second* power; *two* multiplications, the *third* power; and so on.

EXAMPLES.

1. What is the second power of 18? Ans. 324
2. What is the second power of 27? Ans. 729
3. What is the third power of 15? Ans. 3375
4. What is the fourth power of 12? Ans. 20736
5. What is the fifth power of 15? Ans. 759375
6. What is the square of 56? Ans. 3136
7. What is the cube of 45? Ans. 91125
8. What is the value of 5^4 ? Ans. 625
9. What is the value of 12^3 ? Ans. 1728
10. What is 95^2 equal to? Ans. 9025

ART. 321. *Decimals* are involved in the same manner as whole numbers, care being taken to point off the product.

11. What is the second power of 2.3? Ans. 5.29
12. What is the second power of .25? Ans. .0625
13. What is the third power of .21? Ans. .009261
14. What is the square of 4.5? Ans. 20.25
15. What is the third power of .25? Ans. .015625

ART. 322. A *common fraction* is raised to a power by involving the *numerator* and the *denominator* separately.

NOTE.—Before involving, the fraction should be reduced to its simplest form. Mixed numbers should be changed to proper fractions, and compound and complex fractions to simple ones. Thus,

$$\text{The square of } \frac{15}{20} = \left(\frac{15}{20}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}.$$

$$\text{The square of } \left(4\frac{2}{3}\right) = \left(4\frac{2}{3}\right)^2 = \left(\frac{14}{3}\right)^2 = \frac{196}{9} = 21\frac{7}{9}.$$

16. What is the second power of $\frac{5}{7}$? Ans. $\frac{25}{49}$
17. What is the third power of $\frac{3}{4}$? Ans. $\frac{27}{64}$

18. Involve $\frac{9}{12}$ to the third power. Ans. $\frac{27}{4}$

19. Involve $2\frac{3}{4}$ to the third power. Ans. $5\frac{1}{2}^3 = 18\frac{3}{4}$

20. Raise $4\frac{1}{2}$ to the second power. Ans. $20\frac{1}{4}$

21. Involve $1\frac{3}{4}$ to the fourth power. Ans. $9\frac{9}{16}$

22. Involve $\frac{3}{4}$ of $\frac{8}{9}$ to the third power. Ans. $1\frac{8}{27}$

23. What is the third power of $\frac{5}{12}$? Ans. $\frac{125}{1728}$

24. Involve $2\frac{1}{4}$ to the fourth power. Ans. $25\frac{1}{16}$

ART. 323. All the powers of 1 are 1; for if we multiply 1 into itself any number of times, the product will be 1.

EVOLUTION.

ARTICLE 324. EVOLUTION is the *reverse* of INVOLUTION.

In INVOLUTION the *root* is given to find a *power*.

In EVOLUTION the *power* is given to find a *root*.

ART. 325. A *ROOT* of a number is a factor, which, being multiplied into itself a certain number of times, will produce the given number. Thus, 6 is the *square* root of 36, because $6 \times 6 = 36$; 4 is the *cube* root of 64, because $4 \times 4 \times 4 = 64$.

ART. 326. A *perfect square* is a number whose *square root* can be *exactly* found; 1, 4, 9, 16, 25 and 36 are *perfect squares*.

A *perfect cube* is a number whose *cube root* can be *exactly* found; thus, 1, 8, 27, 64 and 125 are *perfect cubes*.

ART. 327. The symbol $\sqrt{}$, called the *radical* sign, is used to denote the square root of a number; as $\sqrt{36} = 6$; $\sqrt{49} = 7$.

The *cube* root is indicated by the figure 3 written over the radical sign; thus $\sqrt[3]{8} = 2$; $\sqrt[3]{27} = 3$.

ART. 328. Every root, as well as every power of 1, is 1. *No other number has this property.*

EXTRACTION OF THE SQUARE ROOT.

ART. 329. To EXTRACT THE SQUARE ROOT of a number is to find a number which, multiplied by itself, will produce the given number.

ART. 330. If we take the largest and the smallest numbers that can be expressed by *one* figure, viz., 9 and 1, we find their squares are 81 and 1. Taking the largest and smallest expressed by *two* figures, viz., 99 and 10, their squares are 9801 and 100. Again, if we take the largest and the smallest expressed by *three* figures, 999 and 100, their squares are 998001 and 10000.

We see from these examples that for every *two* figures in the square there will always be *one* figure in the square root; and if there is an odd figure in the square, there will be another figure in the root corresponding to the odd figure in the square.

Hence, if we separate a square into groups or periods of two figures each, by placing a dot over units, another over hundreds, and so on, the number of dots will indicate the number of figures in the square root. Thus, the square root of 5625 will consist of 2 figures; the square root of 18225 will have 3 figures.

ART. 331. If we analyze the process of *squaring* a number, we shall discover a regular law which will enable us to extract the *square root* of any number.

Suppose we are required to find the *square* of 23.

By the usual method we simply multiply 23 into itself, thus:

$$\begin{array}{r}
 23 \\
 23 \\
 \hline
 69 \\
 46 \\
 \hline
 529 = \text{square of } 23.
 \end{array}$$

We may proceed in a different manner, by regarding 23 as composed of 2 tens (20) and 3 units, and multiply as follows :

$$\begin{array}{r}
 23 = 20 + 3 \\
 \quad 20 + 3 \\
 \quad \hline
 \quad 60 + 9 \\
 400 + 60 \\
 \hline
 400 + 120 + 9 = 529 = (23)^2.
 \end{array}$$

In squaring $20 + 3$ we first multiply $20 + 3$ by 3; this gives $60 + 9$. We then multiply $20 + 3$ by 20, which gives $400 + 60$. Adding these two products, we have $400 + 120 + 9 = 529 = (23)^2$.

By analyzing the above, we see that the first term, 400, is the square of 20, or 2 tens; the second term, 120, is twice 20 multiplied by 3; and the third term, 9, is the square of 3. Hence, *the square of a number composed of TENS and UNITS is equal to the square of the TENS, plus twice the TENS multiplied by the UNITS, plus the square of the UNITS.*

ART. 332. Now, let us extract the square root of 529.

Analysis. This will be to reverse the operation we have just performed. From what was said in Art. 330, we see that the root will consist of two figures—i. e., of tens and units.

According to Art. 331, 529 contains the square of the tens, plus twice the tens multiplied by the units, plus the square of the units. The square of the tens must be found in the hundreds of the number. Now, the square of 2 tens is 4 hundreds; and the square of 3 tens is 9 hundreds; hence, the greatest number of tens whose square can be found in 5 (hundreds) is 2 (tens), which we write in the quotient as the tens' figure of the root. Subtracting the square of the 2 (tens) = 4 (hundreds), we find the remainder, 129. This remainder, 129, must evidently be equal to twice the tens multiplied by the units,

OPERATION.

$$\begin{array}{r}
 529(23 \\
 \underline{4} \\
 43)129 \\
 \underline{129}
 \end{array}$$

plus the square of the units, or (which is the same thing) equal to *twice* the tens plus the units, multiplied by the units. Now, our object is to find the units' figure of the root. By way of trial, we divide 129 by *twice* the tens (40); the quotient is 3. Completing the divisor (40) by annexing these 3 units to twice the tens (40), we have 43, which, multiplied by 3 (the units), gives *twice* the product of the tens by the units, plus the square of the units—a product that exactly equals 129. The root, therefore, is 23. *Ans.* 23.

Proof. Square the root, and if the result is equal to the given number, the root is correct; thus, $(23)^2 = 23 \times 23 = 529$.

ART. 333. The explanation above given will apply generally, without regard to the number of figures in the root. Hence, to extract the square root we have the following

RULE.

I. *Separate the given number into periods of two figures each, beginning at the units' place, by placing a dot over units, then over every second figure toward the left in whole numbers, and toward the right in decimals.*

II. *Find the greatest square number in the left-hand period, and write its root for the first figure in the root; subtract the square number from the left-hand period, and to the remainder bring down the next period for a dividend.*

III. *Double the first figure of the root, and write it at the left of the dividend for a TRIAL DIVISOR; divide the dividend, exclusive of its right-hand figure, by the trial divisor, and write the quotient as the second figure of the root, and also place it at the right of the trial divisor, to form the TRUE DIVISOR.*

IV. *Multiply the TRUE DIVISOR by the second figure of the root, subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

V. *Double the root already found for a NEW TRIAL DIVISOR,*

and continue the operation as before, until all the periods have been brought down.

EXAMPLES.

1 What is the square root of 5625?

OPERATION.

Analysis. The greatest square in the left hand period is 49, and its root is 7, which we set in the quotient. Subtracting 49 from the left-hand period, 56, we have the remainder 7, to which we bring down the next period, 25; making 725. Doubling the first figure 7, we have 14 as our *trial divisor*; then dividing 14 into 72 (leaving off the figure 5), we find 5 to be the next root-figure; placing 5 at the right of 14, we have 145 as the *true divisor*. Multiplying 145 by 5, the product 725 equals the dividend, making the root 75.

$$\begin{array}{r} 5625(75 \text{ Ans} \\ 49 \\ \hline 145)725 \\ 725 \\ \hline \end{array}$$

Ans. 75

2. Find the square root of 2025, and of 2704.

Ans. 45; and 52.

3. Find the square root of 1225, and of 1764.

Ans. 35; and 42.

4. What is the square root of 3136, and of 4096?

Ans. 56; and 64.

5. Find the square root of 784, and of 1024.

Ans. 28; and 32.

6. Find the square root of 7225, and of 9216.

Ans. 85; and 96.

7. What is the square root of 65536?

OPERATION.

Analysis. The greatest square in the left-hand period is 4, and its root is 2, which we set in the quotient. Subtracting 4 from 6, we have the remainder 2, to which we bring down the next period, 55, making 255. Doubling the first root-figure 2, we have 4 as our *trial divisor*;

$$\begin{array}{r} 65536(256 \text{ Ans} \\ 4 \\ \hline 45)255 \\ 225 \\ \hline 506)3036 \\ 3036 \\ \hline \end{array}$$

then dividing 4 into 25 (leaving off the right-hand figure 5), the next root-figure would *seem* to be 6, but is, in *reality*, 5;* placing 5 at the right of 4, we have 45 as the *true divisor*. Multiplying 45 by 5, and subtracting the product, 225, from 255, we have the remainder, 30, to which we bring down the next period, 36; making 3036. Doubling the root already found, we have 50 as our next *trial divisor*; dividing 50 into 303 (leaving off the 6), we find 6 to be the next root-figure; placing 6 at the right of 50, we have 506 as the *true divisor*. Multiplying 506 by 6, we have the product, 3036, equal to the dividend, making the root 256. *Ans.* 256.

8. Find the square root of 50625, and of 60025.

Ans. 225; and 245.

9. Find the square root of 104976, and 186624.

Ans. 324; and 432

10. Find the square root of 297025, and 404496.

Ans. 545; and 636.

11. Find the square root of 555025, and 746496.

Ans. 745; and 864.

12. What is the square root of 27112849?

OPERATION.

When the *trial divisor* is not contained in the dividend, as is the case in this example, the corresponding root-figure will be 0, and we also annex 0 to the trial divisor, and bring down the next period to complete the new dividend

$$\begin{array}{r}
 27112849(5207 \text{ Ans.} \\
 \underline{25} \\
 102)211 \\
 \underline{204} \\
 10407)72849 \\
 \underline{72849}
 \end{array}$$

13. What is the square root of 94864?

Ans. 308.

* The *trial divisor* being usually *less* than the *true divisor*, some allowance must be made for this fact in getting the *true* root-figure. The young student is more liable to make the root-figure *too large* than *too small*. *Practice* alone will teach him the proper allowance.

14. What is the square root of 4227136? *Ans.* 2056.
 15. What is the square root of 64096036? *Ans.* 8006
 16. What is the square root of 2560764816?
Ans. 50604.
 17. What is the square root of .6889, and of 592.9225?

OPERATION.

$$\begin{array}{r} .6889(.83 \text{ Ans.}) \\ 64 \\ \hline 163 \overline{)489} \\ 489 \\ \hline \end{array}$$

OPERATION.

$$\begin{array}{r} 592.9225(24.35 \text{ Ans.}) \\ 4 \\ \hline 44 \overline{)192} \\ 176 \\ \hline 483 \overline{)1692} \\ 1449 \\ \hline 4865 \overline{)24325} \\ 24355 \\ \hline \end{array}$$

These two examples show that if the given number is a *decimal*, or a whole number and a decimal, we may extract the root as in whole numbers, care being taken to put the first dot over *units*, and to point off toward the *left* for whole numbers, and toward the *right* for decimals. If the last period in decimals is incomplete, we *annex* a cipher, since the cipher will fill out the period, but will not alter the value of the decimal.

There will be a *decimal* figure in the root for every *decimal* period in the given power.

18. What is the square root of 12.96? *Ans.* 3.6.
 19. Find the square root of 76.5625. *Ans.* 8.75.
 20. What is the square root of .180625? *Ans.* .425.
 21. What is the square root of .069696? *Ans.* .264.
 22. What is the square root of 10.24? *Ans.* 3.2.
 23. Find the square root of 20.25. *Ans.* 4.5.
 24. What is the square root of 53.29? *Ans.* 7.3.
 25. Find the square root of 27.5625. *Ans.* 5.25.
 26. What is the square root of .097344? *Ans.* .312.
 27. What is the square root of .075625? *Ans.* .275.

28. What is the square root of 638?

OPERATION.

From this example we learn that if there is a remainder after all the periods in the given number have been brought down, the operation may still be continued to any degree of exactness, by annexing successive periods of ciphers, and regarding these ciphers as decimals.

$$\begin{array}{r}
 638(25.25 + \text{Ans} \\
 \underline{4} \\
 45)238 \\
 \underline{225} \\
 502)1300 \\
 \underline{1004} \\
 5045)29600 \\
 \underline{25225} \\
 4375
 \end{array}$$

29. What is the square root of 536? *Ans.* 23.1516 +.

30. Find the square root of 12. *Ans.* 3.4641 +.

31. What is the square root of 1048? *Ans.* 32.3728 +.

32. What is the square root of 7? *Ans.* 2.6457 +.

33. What is the square root of $\frac{48}{5}$?

OPERATION.

$$\sqrt{\frac{48}{5}} = \sqrt{\frac{16}{\frac{5}{3}}} = \frac{4}{\sqrt{\frac{5}{3}}} \quad \text{Ans. } \frac{4}{\sqrt{\frac{5}{3}}}.$$

34. What is the square root of $7\frac{1}{2}$?

OPERATION.

$$\sqrt{7\frac{1}{2}} = \sqrt{\frac{15}{2}} = \frac{\sqrt{15}}{\sqrt{2}} = 2\frac{1}{2} \quad \text{Ans. } 2\frac{1}{2}.$$

Hence, to extract the square root of a common fraction, we reduce the fraction to its simplest form, then extract the square root of the numerator and denominator separately, provided they are perfect squares; if they are not, reduce the fraction to a decimal, and extract the root.

35. What is the square root of $\frac{36}{100}$, and of $\frac{98}{162}$?

Ans. $\frac{6}{10}$; and $\frac{7}{9}$.

36. Find the square root of $\frac{128}{25}$, and of $\frac{108}{2}$.

Ans. $\frac{16}{5}$; and $\frac{6}{\sqrt{2}}$.

37. Find the square root of $30\frac{1}{2}$, and of $42\frac{1}{2}$.

Ans. $5\frac{1}{2}$; and $6\frac{1}{2}$.

38. What is the square root of $\frac{3}{4}$?

Ans. .61237 +.

APPLICATION OF THE SQUARE ROOT.

ART. 334. The *area* of any figure having its sides perpendicular or at right angles to each other is found by *multiplying the length by the breadth*; but since a *square* has all its sides equal, the *area of a square* is found by *squaring one of its sides*.

Knowing the area of a square, we can find one of the sides by extracting the square root of the area.

EXAMPLES.

1. A farmer has a barley lot in the shape of a square containing 24336 square feet; what is the length of each side?

Ans. 156 feet.

2. A number of gentlemen built a church for 5184 dollars, each contributing as many dollars as there were gentlemen; how many gentlemen were there?

Ans. 72 gentlemen.

3. A farmer has a field containing 55696 stalks of corn, and there are as many stalks in each row as there are rows; how many stalks in each row?

Ans. 236 stalks.

4. A piece of land 756 feet long and 84 feet broad is to be exchanged for an equivalent square piece; what is the length of each side of the square piece?

Ans. 252 feet.

5. How many more palings, each 1 foot wide, would be required to enclose a garden 168 feet in length and 42 feet in breadth than would be sufficient to enclose the same garden in a square form?

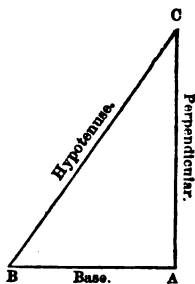
Ans. 84 palings.

6. How many yards long will the side of a square lot measure, to contain just one acre of land?

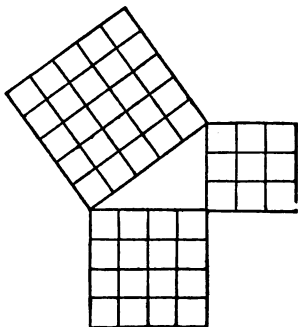
Ans. 69.57 yards.

ART. 335. A TRIANGLE is a figure bounded by three straight lines. When one of the angles is formed by two lines perpendicular to each other, the triangle is a RIGHT-ANGLED TRIANGLE. The side opposite the right angle is

called the *hypotenuse*; the other two sides the *base* and *perpendicular*. Thus, ABC is a right-angled triangle; AB is the base, AC the perpendicular, and BC the hypotenuse.



ART. 336. It is an established principle of Geometry that the *square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides*. This is illustrated by the diagram on the right. By counting the small squares of the hypotenuse, we find that they are equal to the sum of the small squares on the base and perpendicular.



ART. 337. The *base* and *perpendicular* being given, to find the *hypotenuse*.

Add the square of the base to the square of the perpendicular, and extract the square root of their sum.

ART. 338. The *hypotenuse* and either side being given, to find the other side.

Subtract the square of the given side from the square of the hypotenuse, and extract the square root of the remainder.

7. The base of a right-angled triangle is 21 inches, and the perpendicular 28 inches; what is the hypotenuse?

Ans. 35 inches.

8. The hypotenuse of a right-angled triangle is 65, and the perpendicular 39; what is the base?

Ans. 52.

9. The hypotenuse of a right-angled triangle is 115, and the base 69; what is the perpendicular?

Ans. 92.

10. A ladder 55 feet long reaches to the top of a house when the foot of the ladder rests on the ground 33 feet from the house; what is the height of the house? *Ans.* 44 feet.

11 Two persons start to travel from the same point; one goes directly east 135 miles, and the other directly north 180 miles How far apart are they in a straight line?

Ans. 225 miles.

12. Suppose a room to be 20 feet square; what is the distance between two opposite corners? *Ans.* 28.28 feet.

13. Two rafters, each 25 feet long, meet at the ridge of a roof 15 feet above the garret floor; what is the width of the house? *Ans.* 40 feet.

EXTRACTION OF THE CUBE ROOT.

ART. 339. To EXTRACT THE CUBE ROOT of a number, is to resolve it into three equal factors; or, it is to find a number which, multiplied into itself twice, will produce the given number.

Thus, 5 is the cube root of 125; for $5 \times 5 \times 5 = 125$.

ART. 340. The cubes of the first nine numbers are as follows:

Roots—1, 2, 3, 4, 5, 6, 7, 8, 9,
Cubes—1, 8, 27, 64, 125, 216, 343, 512, 729.

Now let us add a cipher to each of the numbers, and let us notice the effect on the cubes.

Roots— 10, 20, 30, 40, 50, 60, 70,
Cubes—1000, 8000, 27000, 64000, 125000, 216000, 343000

A little examination of the above will show two facts:

I. That a root, consisting of one figure, may have one, two, or even three, figures in its cube.

II. That the addition of 1 place to the root will add 3 places to the cube. Therefore, there will always be one figure

in the *root* for each period of *three* figures in the *cube*, and an extra figure in the root if there are *two* figures, or even if there is only *one* figure, over complete periods of *three* figures in the *cube*. Hence,

If we place a dot (.) over every third figure of any number, beginning with UNITS, the number of dots will indicate the number of figures in the cube root. Thus, $\dot{3}37\dot{5}$, $1\dot{3}82\dot{4}$ and $42\dot{1}87\dot{5}$ have each *two* figures in the cube root.

ART. 341. We will better understand the extraction of the cube root if we analyze what takes place in forming a cube by the multiplication of its factors.

Suppose we wish to find the *cube* of 23.

$23 = 20 + 3$, and the cube of $23 =$ the cube of $(20 + 3)$.

To ~~the~~ $(20 + 3)$ we must multiply $(20 + 3)$ into itself ~~to~~ *in*.

OPERATION.

$$\begin{array}{r}
 20 + 3 \\
 20 + 3 \\
 \hline
 60 + 9 \\
 400 + 60 \\
 \hline
 400 + 120 + 9 = (23)^2 = 529 \\
 20 + 3 \\
 \hline
 1200 + 360 + 27 \\
 8000 + 2400 + 180 \\
 \hline
 8000 + 3600 + 540 + 27 = (23)^3 = 12167 \\
 8000 = (20)^3; 3600 = 3 \times (20)^2 \times 3; 540 = 3 \times 20 \\
 \times (3)^2; \text{ and } 27 = (3)^3.
 \end{array}$$

We see, then, that the *first* term, 8000, is the cube of 20; the *second* term, 3600, is 3 times the square of 20 multiplied by 3; the *third* term, 540, is 3 times 20 multiplied by the square of 3; and the *last* term, 27, is the cube of 3. Hence, we derive this general truth, viz.:

The cube of a number composed of TENS and UNITS is

equal to the cube of the TENS + 3 times the square of the TENS multiplied by the UNITS + 3 times the TENS multiplied by the square of the UNITS + the cube of the UNITS.

ART. 342. Let it be required to find the cube root of 12167.

OPERATION.

$$\begin{array}{r}
 \text{12167(23 Ans} \\
 \underline{8} \\
 \text{Trial divisor} = 3 \times (20)^2 = 3 \times 400 = 1200 \quad \text{4167} \\
 \quad \quad \quad 3 \times 20 \times 3 = 180 \\
 \quad \quad \quad (3)^3 = 9 \\
 \text{True divisor} = 1389 \quad \underline{4167}
 \end{array}$$

Analysis. According to Art. 340, we see that the number of figures in the root must be *two*—i. e., the root must consist of *tens* and *units*. Now, according to the preceding article, the number 12167 = the cube of the *tens* + 3 times the square of the *tens* multiplied by the *units* + 3 times the *tens* multiplied by the square of the *units* + the cube of the *units*. The *cube* of the *tens* must be found in the first period, 12 (thousand). The greatest *perfect* cube in the first period is 8 (thousand), the cube root of which is 2 (*tens*), which we place on the right, as the first root-figure.

Subtracting the cube of 2 (*tens*), which is 8 (thousand), from the first period, 12 (thousands), and bringing down the next period, we have 4167.

After we subtract the cube of the *tens*, the remainder, 4167, must evidently = 3 times the square of the *tens* multiplied by the *units* + 3 times the *tens* multiplied by the square of the *units* + the cube of the *units*. We already have the *tens* of the root, and our object now is to find the *units*. By way of trial, we divide 4167 by 3 times the square of the *tens*—i. e., by $3 \times (20)^2 = 3 \times 400 = 1200$. The quotient is 3 (*units*), which we write as the *units'* figure in the root. The

dividend, 4167, besides containing 3 times the square of the tens multiplied by the units, also contains 3 times the product of the tens by the square of the units + the cube of the units. We therefore complete the divisor by adding to the *trial divisor* 3 times the product of the tens by the units—i. e., $3 \times 20 \times 3 = 180$; and to this we add the square of the units—i. e., $(3)^2 = 9$. Then, $1200 + 180 + 9 = 1389$, the true divisor. Multiplying the true divisor by the units' figure, 3, and subtracting, we have no remainder. *Ans.* 23.

The same mode of explanation will apply if there are more than *two* figures in the root. Hence we deduce the following

RULE.

I. *Separate the given number into periods of three figures each, beginning at the units' place.*

II. *Find the greatest cube in the left-hand period, and place its root on the right as the first root-figure. Subtract its cube from the left-hand period, and to the remainder annex the next period for a dividend.*

III. *Square the first root-figure, annex two ciphers, multiply the result by 3, and place it on the left as a TRIAL DIVISOR; divide the dividend by the trial divisor, and write the quotient as the second figure of the root. To obtain the TRUE DIVISOR, add to the trial divisor 3 times the product of the last root-figure by the root previously found, annexing one cipher, and also add the square of the last root-figure.*

IV. *Multiply the true divisor by the last root-figure, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.*

V. *Square the whole root now found, annex two ciphers, multiply the result by 3 for a new trial divisor, and continue the operation as before until all the periods have been brought down.*

PROOF. *Cube the root, and if the result is equal to the given number, the work may be presumed to be right.*

NOTE 1.—In paragraph III. of the Rule just given the direction for finding the trial divisor is, to “square the first root-figure, annex two ciphers,” etc. The *reason* for “annexing two ciphers” to the square of the first root-figure is briefly this: When the root consists of two figures, the first figure will be a *ten* or some number of *tens*. When we square the first root-figure without reference to its *local* value, we simply square it as so many *units*; when we square it as so many *tens*, the square will be 100 times as large as the square of the same number of *units*. Now, *in practice*, we *do* square the first figure as simple units, and accordingly make the result 100 times *too small*; hence, to obtain the true square, we must multiply the result by 100, or *annex two ciphers*.

The same principle explains why we annex “one cipher to the result” in forming the true divisor. When we multiply the second root-figure by the first, we *practically* multiply them as though they were each *units*; but one of them being *tens*, the multiplication of them as *units* gives a product 10 times *too small*; hence, to make the product of the right value, we must multiply it by 10, or *annex one cipher*.

The same reasoning will apply when the root consists of more than two figures, for practically we regard the first figure in the root as standing in the *tens*’ place, and we apply the rule accordingly.

NOTE 2.—The trial divisor, being always less than the true divisor, will sometimes give too large a quotient figure. When the product of the true divisor by the last quotient figure is larger than the corresponding dividend, the quotient figure must be diminished.

NOTE 3.—If the divisor is not contained in its appropriate dividend, write a cipher in the root, annex two ciphers to the trial divisor, and bring down the next period for a new dividend.

NOTE 4. If there is a remainder after all the periods have been brought down, we may annex periods of *three ciphers each*. For each period of ciphers there will be a decimal figure in the root.

EXAMPLES.

1. What is the cube root of 34012224?

OPERATION.

34012224(324 <i>Ans.</i>	27
1st trial divisor= 900×3= 2700	7012 =dividend.
30×2×3= 180	
(2) ² = 4	
1st true divisor= 2884	5768 =subtrahend.
2d trial divisor=102400×3=307200	1244224=dividend.
32×4×3= 3840	
(4) ² = 16	
2d true divisor=311056	1244224=subtrahend

Proof. $(324)^3 = 324 \times 324 \times 324 = 34012224.$

2. What is the cube root of 256047875?

OPERATION.

256047875(635 <i>Ans.</i>	216
1st trial divisor= 3600×3= 10800	40047 = 1st div'd
60×3×3= 540	
(3) ² = 9	
1st true divisor= 11349	34047 = subtrah'd.
2d trial divisor=396900×3=1190700	6000875 = 2d div'd
630×5×3= 9450	
(5) ² = 25	
2d true divisor=1200175	6000875=subtrah'd

Proof. $(635)^3 = 635 \times 635 \times 635 = 256047875.$

Find the cube roots of the following:

3. 13824.	<i>Ans.</i> 24.	8. 48228544.	<i>Ans.</i> 364.
4. 32768.	<i>Ans.</i> 32.	9. 88121125.	<i>Ans.</i> 445.
5. 157464.	<i>Ans.</i> 54.	10. 244140625.	<i>Ans.</i> 625.
6. 421875.	<i>Ans.</i> 75.	11. 523606616.	<i>Ans.</i> 806.
7. 12812904.	<i>Ans.</i> 234.	12. 1749690125	<i>Ans.</i> 1205.

13. 34169931125.	Ans. 3245.	17. 34.328125.	Ans. 3.25.
14. 1108.717875.	Ans. 10.35.	18. 50.	Ans. 3.684 +.
15. .015625.	Ans. .25.	19. 15.	Ans. 2.466 +
16. .019683.	Ans. .27.	20. 23.	Ans. 2.84 +.

ART. 343. To extract the cube root of a common fraction

Reduce the fraction to its simplest form, and extract the cube root of the numerator and denominator separately, if they are perfect cubes; if not, reduce the fraction to a decimal, and then extract the root.

Find the cube roots of the following :

1. $\frac{250}{488}$.	Ans. $\frac{5}{8}$.	4. $\frac{343}{8}$.	Ans. $3\frac{1}{2}$.
2. $\frac{216}{848}$.	Ans. $\frac{6}{9}$.	5. $81\frac{1}{2}$.	Ans. $4\frac{1}{8}$.
3. $\frac{375}{1588}$.	Ans. $\frac{5}{8}$.	6. $20\frac{5}{8}$.	Ans. $2\frac{5}{8}$.

PRACTICAL QUESTIONS.

1. A carpenter wishes to make a cubical cistern that will contain 13824 cubic feet of water; what must be the length of each of its edges? Ans. 24 feet.

2. A man has a pile of wood containing 864 cords. Suppose the wood to be piled in the shape of a regular cube, what would be the length of each edge? Ans. 48 feet.

3. A gallon of wine contains 231 cubic inches. What must be the depth of a cubical cistern to hold 2500 gallons of wine? Ans. 83.27 inches

ARITHMETICAL PROGRESSION.

ARTICLE 344. An ARITHMETICAL PROGRESSION is a series of numbers which *increase* or *decrease* by a *common difference*. When they *increase* they form an ASCENDING SERIES; when they *decrease*, a DESCENDING SERIES.

Thus, 2, 4, 6, 8, 10, 12, etc., is an **ASCENDING** series;
16, 14, 12, 10, 8, 6, etc., is a **DESCENDING** series.

ART. 345. The numbers which form the series are called **TERMS**. The first and last terms are called the **EXTREMES**, and the other terms, the **MEANS**.

ART. 346. In Arithmetical Progression five particulars are to be considered: 1st, the *first* term; 2d, the *last* term; 3d, the *common* difference; 4th, the *number* of terms; 5th, the *sum* of the series.

Any *three* of these quantities being known, the other *two* can be found.

CASE I.

ART. 347. *The first term, the common difference and the number of terms being given, to find the LAST TERM.*

Analysis. Let us take the *ascending* series:

3, 5, 7, 9, 11, 13, 15, 17.

Here 3 is the first term, 17 the last term, 2 the common difference and 8 the number of terms.

In this series we see that the *second* term equals the first, plus *once* the common difference; the *third* term equals the first, plus *twice* the common difference; the *fourth* term equals the first, plus *three* times the common difference; the *fifth* term equals the first, plus *four* times the common difference; and, in general, *any* term equals the first term, plus the common difference multiplied by the number of terms less 1. Hence, we have this

RULE.

Multiply the common difference by the number of terms less 1, and add the product to the first term.

NOTE.—When the series is descending the product must be *subtracted* from the first term.

Ex. 1. The first term of an ascending series is 5, the common difference 4 and the number of terms 10; what is the last term?

OPERATION.

$$5 + (10 - 1) \times 4 = 5 + 9 \times 4 = 5 + 36 = 41. \text{ Ans.}$$

2. A boy bought 9 melons, paying 20 cents for the first, 24 cents for the second, 28 cents for the third, and so on; how much did he pay for the last? *Ans.* 52 cents.

3. A man discharged a debt by 7 different payments. The first payment was 25 dollars, and each succeeding payment was 10 dollars more than the preceding; what was the last payment? *Ans.* 85 dollars.

4. The first term of a descending series is 124, and the common difference 6; what is the 20th term? *Ans.* 10.

5. What is the 35th term of an ascending progression whose first term is 7 and common difference 5? *Ans.* 177.

CASE II.

ART. 348. *The first term, the last term and number of terms being given, to find the COMMON DIFFERENCE.*

From the last article it is evident that the difference between the *two extremes* is always equal to the *common difference multiplied by the number of terms less 1*; from which it follows that if we divide the difference between the extremes by the number of terms *less 1*, we shall have the common difference. Hence, the

RULE.

Divide the difference between the extremes by the number of terms less 1.

Ex. 1. If the first term is 4, the last term 60 and the number of terms 9, what is the common difference?

OPERATION.

$$\frac{60 - 4}{9 - 1} = \frac{56}{8} = 7, \text{ common difference.}$$

2. If the last term is 76, the first term 6 and the number of terms 15, what is the common difference? *Ans.* 5.

3. The extremes of a series are 5 and 89, and the number of terms is 13; what is the common difference? *Ans.* 7.

4. A man travels 13 days on a railroad, going 117 miles the first day, and 357 the last, increasing each day's travel by an equal difference; what is the daily increase? *Ans.* 20 miles.

CASE III.

ART. 349. *The extremes and common difference being given, to find the NUMBER of terms.*

Since the difference between the two extremes is equal to the common difference multiplied by the number of terms less 1, it follows that if we divide the difference between the two extremes by the common difference, the quotient will give the *number of terms less 1*; and if we *add* one to this quotient, the result will be the number of terms. Hence, we derive the

RULE.

Divide the difference between the two extremes by the common difference, and add 1 to the quotient.

Ex. 1. If the extremes of a series are 102 and 6, and the common difference 8, what is the number of terms?

OPERATION.

$$\frac{102 - 6}{8} + 1 = \frac{96}{8} + 1 = 12 + 1 = 13, \text{ number of term.}$$

2. The extremes of a series are 118 and 10, and the common difference 12; what is the number of terms? *Ans.* 10.

3. A man on a journey traveled the first day 15 miles, and

the last day 39 miles, each day increasing his distance 3 miles; how many days did he travel? *Ans.* 9 days.

4. The ages of the children in a certain family are in arithmetical progression, the common difference being $2\frac{1}{2}$ years. The oldest is 22, and the youngest 4 years of age; how many children in the family? *Ans.* 9 children.

CASE IV.

ART. 350. *The extremes and number of terms being given, to find the SUM of the series.*

Ex. 1. What is the sum of the series 2, 5, 8, 11, 14, 17?

Analysis. Let us take the series given in our example, and let us write under it the same series in an *inverted* order, and let us add the several terms in the same column :

$$\begin{array}{r} 2 + 5 + 8 + 11 + 14 + 17 = 57 = \text{sum of first series.} \\ 17 + 14 + 11 + 8 + 5 + 2 = 57 = \text{sum of inverted series.} \\ \hline 19 + 19 + 19 + 19 + 19 + 19 = 114 = \text{sum of both series.} \end{array}$$

We at once see that the sum of the terms in the two series is equal to 19 multiplied by 6 = 114. But 19 is the *sum* of the *extremes*, and 6 is the *number of terms* in each series.

Since the sum of the extremes, multiplied by the number of terms, gives *twice* the series, it is obvious that if we divide this product by 2 we shall have the sum of the single series = 57. Hence the

RULE.

Multiply half the sum of the extremes by the number of terms.

2. The extremes of a series are 15 and 75, and the number of terms is 12; what is the sum of the series? *Ans.* 540.

OPERATION.

$$\frac{15 + 75}{2} \times 12 = \frac{90}{2} \times 12 = 45 \times 12 = 540 = \text{sum of series.}$$

3. The extremes of a series are 12 and 56, and the number of terms 18; what is the sum of the series? *Ans.* 612.

4. A man discharged a debt in 9 months, paying \$15 the first month and \$127 the last month; what was the whole debt? *Ans.* \$639.

5. How much would a person contribute who should put into a charity-box a cent the first day, 2 cents the second day, 3 cents the third day, and so on through the whole of December? *Ans.* \$4.96.

6. The first term of a series is 6, common difference 4 and number of terms 25; what is the sum of the series?

Ans. 1350.

NOTE.—First find the *last* term, by CASE I.

7. A workman served his employer 12 years. For the first year he received \$75, and every successive year his salary was raised \$25; what was the whole amount of his wages?

Ans. \$2550.

GEOMETRICAL PROGRESSION.

ARTICLE 351. Any series of numbers *increasing* by a *common multiplier* or *decreasing* by a *common divisor* is called a GEOMETRICAL PROGRESSION. When the numbers *increase* they form an ASCENDING series; when they DECREASE they form a DESCENDING series.

Thus, 5, 15, 45, 135, 405 is an *ascending* series.

405, 135, 45, 15, 5 is a *descending* series.

ART. 352. The common multiplier or common divisor is called the RATIO. The *ratio* may always be regarded as a *multiplier*, either integral or fractional. Thus, in the first series above the ratio is 3, in the second series it is $\frac{1}{3}$.

ART. 353. The numbers forming the series are called **TERMS**. The first and last terms are called **EXTREMES**, and the other terms, the **MEANS**.

ART. 354. In Geometrical Progression five particulars enter into calculation—viz., 1st, the *first* term; 2d, the *last* term; 3d, the *ratio*; 4th, the *number* of terms; 5th, the *sum* of the series.

Any *three* of these being given, the other *two* can be found.

CASE I.

ART. 355. *The first term, the ratio and the number of terms being given, to find the LAST TERM.*

Let us take the following series, having 5 for the first term, and ratio 3:

5, 15, 45, 135, 405.

It will be noticed that each term is deduced from the preceding one by multiplying it by the ratio, and the series might be exhibited under this form:

5, 5×3 , $5 \times 3 \times 3$, $5 \times 3 \times 3 \times 3$, $5 \times 3 \times 3 \times 3 \times 3$,
or, 5, 5×3 , 5×3^2 , 5×3^3 , 5×3^4 .

We see that the *second* term is equal to the first term multiplied by the ratio; the *third* term equals the first multiplied by the *second* power of the ratio; the *fourth* equals the first multiplied by the *third* power of the ratio; and so *any* term will always equal the first multiplied by that power of the ratio whose index is 1 less than the number of terms. Hence we have the

RULE.

Raise the ratio to a power whose index is 1 less than the number of terms, then multiply this power by the first term

EXAMPLES.

1. The first term of a geometrical series is 4, the ratio 3 and number of terms 6; what is the last term?

OPERATION.

Last term $= 4 \times 3^5 = 4 \times 243 = 972$. *Ans.*

The series will be 4, 12, 36, 108, 324, 972.

2. The first term of a series is 512, the ratio $\frac{1}{2}$ and the number of terms 7; what is the last term?

OPERATION.

Last term $= 512 \times (\frac{1}{2})^6 = 512 \times \frac{1}{64} = \frac{512}{64} = 8$. *Ans.*

The series will be 512, 256, 128, 64, 32, 16, 8.

3. The first term of a series is 5, the ratio 3 and number of terms 7; what is the last term? *Ans.* 3645.

4. The first term of a series is 1458, the ratio $\frac{1}{3}$ and number of terms 6; what is the last term? *Ans.* 6.

5. A man travels 9 days, going $1\frac{1}{2}$ miles the first day, 3 miles the second day, 6 miles the third day, and so on. What is the last day's travel? *Ans.* 384 miles

CASE II.

ART. 356. *The first term, the last term, and the ratio being given, to find the SUM of the series.*

Ex. 1. What is the sum of the series 3, 15, 75, 375, and 1875?

Analysis. If we multiply each term of our given series by the ratio 5, we will have the following series:

15, 75, 375, 1875, 9375.

It is evident that the sum of this last series is *five* times the sum of the given series. Writing the given series under the latter, we have as follows:

$15 + 75 + 375 + 1875 + 9375 = 5 \text{ times first series.}$

$3 + 15 + 75 + 375 + 1875 = 1 \text{ time first series.}$

If we subtract the lower series from the upper, we see that all the terms will cancel except the first and the last, and there

will remain $9375 - 3 = 9372$; this remainder evidently equals four times the given series; and dividing 9372 by 4, the quotient, 2343, will be the sum of the required series. In this operation 9375 is the *last* term multiplied by the ratio—i. e., 1875×5 ; 3 is the *first* term, and 4, by which we divided, is the *ratio* less 1—i. e., $(5 - 1)$. Hence, we derive the

RULE.

Multiply the last term by the ratio, subtract the first term, and divide the remainder by the difference between the ratio and 1; the result will be the sum of the series.

NOTE.—To apply this rule when the series is *descending*, let the student *reverse* the series after finding the last term.

EXAMPLES.

2. The first term of a geometric series is 5, the ratio 4 and number of terms 7; what is the sum of the series?

Ans. 27305.

OPERATION.

$$\begin{aligned} \text{Last term} &= 4^6 \times 5 = 4096 \times 5 = 20480, \\ \text{Sum of series} &= \frac{(20480 \times 4) - 5}{4 - 1} = \frac{81920 - 5}{3} = \frac{81915}{3} = 27305. \end{aligned}$$

3. The first term of a series is 8, the ratio 3, and number of terms 6; what is the sum of the series? *Ans.* 2912.

4. How large a debt may be discharged in 7 months by paying \$6 the first month, \$18 the second month, \$54 the third month, and so on in geometrical progression?

Ans. \$6558.

5. The first term of a series is 3, the ratio 4 and number of terms 7; what is the sum of the series? *Ans.* 16383.

6. How large a sum could be raised in a year by monthly contributions, giving one cent the first month, and three times as much each succeeding month? *Ans.* \$2657.20

MISCELLANEOUS QUESTIONS SOLVED BY ANALYSIS.

ARTICLE 357. ANALYSIS literally means a *taking to pieces*. If we wish to understand the construction of a complicated piece of mechanism, we *take it to pieces*, and thus learn the relation that the parts bear to each other. The same principle holds good in Arithmetic. To solve each question as it comes up, we must take it to pieces; in other words, we must *analyze* it. Analysis is nothing more than *common sense* applied to the solution of problems.

Innumerable questions which do not legitimately come under any of the ordinary "RULES" of Arithmetic can be solved readily and satisfactorily by *Analysis*.

EXAMPLES.

1. If 6 pounds of soda cost 90 cents, how much would 13 pounds cost at the same rate?

Analysis. If 6 pounds cost 90 cents, 1 pound will cost $\frac{1}{6}$ of 90 cents = 15 cents, or 15 cents; and if 1 pound cost 15 cents, then 13 pounds would cost 13 times as many cents = \$1.95.

Ans. \$1.95.

NOTE TO TEACHERS.—It will greatly facilitate the progress of the pupil to make him *write* each step on the slate or blackboard while explaining the Analysis. It prepares him for the study of Algebra by making him familiar with the idea of "Equations."

The example just given might be written thus:

OPERATION.

$$\begin{array}{rcl} 6\text{lb.} & = & 90\text{ct.} \\ 1\text{lb.} & = & 15\text{ct.} \\ 13\text{lb.} & = & \$1.95. \end{array}$$

2. If 5 yards of calico cost 65 cents, what would be the cost of 18 yards? *Ans.* \$2.34.

3. If 7 acres of land produce 168 bushels of wheat, how many bushels would 36 acres produce? *Ans.* 864 bushels.

4. If a locomotive can run 115 miles in 5 hours, how far could it run in 15 hours? *Ans.* 345 miles.

5. If 12 bushels of peas be worth \$9.96, how much would 5 bushels be worth at the same rate? *Ans.* \$4.15.

6. How many pounds of raisins could be bought for \$23, at the rate of \$3 for 15 pounds?

NOTE.—The pupil should never forget one fundamental principle of *Analysis*, which is this:

NEVER begin the analysis of a question with that term which is of the same name with the answer. To illustrate, we give the *Analysis* of the last example.

Analysis. Here the answer is required in *pounds*, and we therefore begin with *dollars*. We say: If \$3 buy 15 pounds, \$1 will buy $\frac{1}{3}$ of 15 pounds = 5 pounds; and if \$1 buy 5 pounds, then \$23 would buy 23 times as many pounds = 115 pounds. *Ans.* 115 pounds.

OPERATION.

$$\text{\$3} = 15\text{lb.}$$

$$\text{\$1} = 5\text{lb.}$$

$$\text{\$23} = 115\text{lb}$$

7. How many barrels of flour could be bought for \$135 if 3 barrels cost \$27? *Ans.* 15 barrels.

8. If 6 yards of cassimere cost \$18, how many yards could you buy for \$72? *Ans.* 24 yards.

9. If 7 acres of ground cost \$35, how many acres would \$140 buy? *Ans.* 28 acres.

10. If $\frac{1}{5}$ of a pound of honey cost 24 cents, what would a pound be worth?

Analysis If four fifths of a pound cost 24 cents, one fifth

would cost $\frac{1}{4}$ of 24 cents = $2\frac{1}{4}$ cents, or 6 cents; and if *one fifth* cost 6 cents, *five fifths*, or a *whole* pound, would cost 5 times as many cents = 30 cents. *Ans.* 30 cents

OPERATION.

$$\frac{1}{4}\text{lb.} = 24\text{ct.}$$

$$\frac{1}{5}\text{lb.} = 6\text{ct.}$$

$$\frac{1}{5} = 1\text{lb.} = 30\text{ct.}$$

11. If $\frac{7}{8}$ of a yard of muslin cost 56 cents, what is the price per yard? *Ans.* 72 cents.

12. If $\frac{2}{3}$ of a man's age is 40 years, how old is he?

Ans. 64 years.

13. If $\frac{7}{12}$ of a bale of cotton sell for \$63, what would be the price of a whole bale? *Ans.* \$108.

14. If $\frac{1}{5}$ of a grist-mill be worth \$6375, what would be the value of the whole mill? *Ans.* \$8075.

15. A gentleman owning $\frac{1}{4}$ of a factory sold $\frac{3}{8}$ of his share for \$33264; what was the value of the whole factory?

Ans. \$87318.

16. A man owning $\frac{3}{8}$ of a steamboat sold $\frac{1}{4}$ of what he owned for \$29475; what was the value of the steamboat?

Ans. \$65500.

17. If $\frac{7}{12}$ of a yard of alpaca cost 49 cents, what would $\frac{1}{3}$ of a yard be worth? *Ans.* 70 cents.

OPERATION.

$$\frac{7}{12}\text{yd.} = 49\text{ct.}$$

$$\frac{1}{12} \text{ " } = 7\text{ct.}$$

$$\frac{1}{3}\text{yd.} = 1 \text{ " } = 84\text{ct.}$$

$$\frac{1}{8} \text{ " } = 14\text{ct.}$$

$$\frac{1}{4} \text{ " } = 70\text{ct.}$$

18. If $\frac{1}{4}$ of a bushel of peas cost 60 cents, what would be the value of $\frac{3}{4}$ of a bushel? *Ans.* 63 cents.

19. If $\frac{1}{3}$ of a cask of wine contain 20 gallons, how many gallons in $\frac{2}{3}$ of the cask? *Ans.* 27 gallons.

20. If a workman can earn \$22.50 in $\frac{3}{4}$ of a month, how much could he earn in 5 months? *Ans.* \$150.

21. If $2\frac{2}{3}$ tons of coal cost \$24, what would be the cost of 7 tons? *Ans.* \$63.

22. If 3 quarts of Madeira wine cost \$4.50, what would be the cost of 33 gallons?

Analysis. If 3 quarts cost \$4.50, then 1 quart will cost $\frac{1}{3}$ of \$4.50 = \$1.50; and 4 quarts, or 1 gallon, will cost 4 times as much = \$6; and if 1 gallon cost \$6, then 33 gallons would cost 33 times as much = \$198. *Ans.* \$198.

23. If 3 pecks of wheat cost \$1.20, how much would 72 bushels cost? *Ans.* \$115.20.

24. If 2qt. 1pt. of Teneriffe wine be worth \$1.80, what would be the cost of 27 gallons? *Ans.* \$77.76.

25. If 12cwt. 2qr. 12lb. of iron cost \$37.86, what would be the value of 3 tons? *Ans.* \$180.

26. If 6 men can build a house in 12 weeks, how many men could build the same in 8 weeks?

Analysis. To build the house in 12 weeks requires 6 men; then to build it in *one* week will require 12 *times* as many men = 72 men; and to build it in 8 weeks would require only $\frac{1}{8}$ as many men = $\frac{72}{8}$ men, or 9 men. *Ans.* 9 men.

27. If 9 men can reap a field of wheat in 8 days, how many men could reap the same field in 12 days? *Ans.* 6 men.

28. If a barrel of flour last a family of 10 persons 6 weeks how long would it last 12 persons? *Ans.* 5 weeks.

29. If a pasture serve 24 sheep 32 days, how long would the same pasture serve 16 sheep? *Ans.* 48 days.

30. If 42 yards of carpeting 30 inches wide are required to cover a floor, how many yards will it take when the carpeting is only 24 inches wide? *Ans.* $52\frac{1}{2}$ yards.

31. A gentleman had a farm consisting of a certain number of acres. After giving $\frac{1}{4}$ of his land to his daughter and $\frac{1}{4}$ to

his son, he had 315 acres left; how many acres had he at first?

Ans. 756 acres.

Analysis. Reducing $\frac{1}{2}$ and $\frac{1}{4}$ to a common denominator, we have $\frac{2}{4}$ and $\frac{1}{4}$; the sum of these is $\frac{3}{4}$; the gentleman had given away $\frac{3}{4}$ of his land, and of course had $\frac{1}{4}$ of it remaining. Now, if $\frac{1}{4}$ of his land is equal to 315 acres, then $\frac{1}{4}$ of it would be $\frac{1}{4}$ of 315 acres = $\frac{315}{4}$ acres = 63 acres; and if $\frac{1}{4}$ is 63 acres, then $\frac{3}{4}$, or all his land, would be 12 times as much = 756 acres.

32. A gentleman after spending $\frac{1}{2}$ of his money for railroad fare, and $\frac{1}{4}$ for board, found that he had \$175 remaining; how much had he at first?

Ans. \$375.

33. A market-woman bought a certain number of apples, and after selling one third, one fifth and one sixth of them, found she had 45 apples left; how many had she at first?

Ans. 150 apples.

34. What number is that, the sum of whose third part, fourth part and fifth part is 564?

Ans. 720.

35. Henry can do a piece of work in 6 days, and Charles can do the same work in 8 days. In how many days could they finish it by working together?

Ans. $3\frac{2}{3}$ days.

Analysis. If Henry can do the work in 6 days, then in 1 day he can do $\frac{1}{6}$ of it; and if Charles can do it in 8 days, in 1 day he can do $\frac{1}{8}$ of it. The two together can do $\frac{1}{6} + \frac{1}{8}$ of it in a day, or, reducing these fractions to a common denominator, they can do $\frac{4}{24} + \frac{3}{24} = \frac{7}{24}$ in a day. Then, if $\frac{7}{24}$ of the work is done in 1 day, $\frac{1}{24}$ of the work will require $\frac{1}{7}$ of a day, and $\frac{24}{7}$, or the whole work, will require 24 times as long = $\frac{24}{7}$ of a day, or $3\frac{2}{3}$ days.

36. One man can plough a field of corn in 5 days, and another can plough it in 9 days; in what time can the two together plough it?

Ans. $3\frac{3}{4}$ days.

37. If B can mow a meadow in 10 days, C in 12 days, and

340 MISCELLANEOUS QUESTIONS BY ANALYSIS.

D in 15 days, in what time can they mow it by working together?
Ans. 4 days.

38. A man divided \$7875 between his two sons in the ratio of 6 to 9; what was the share of each?

Ans. \$3150 and \$4725.

Analysis. It is evident that one receives \$6 as often as the other receives \$9; hence, if one son has 6 parts of the whole, the other must have 9 parts. The two together will have 15 parts; then, if 15 parts are worth \$7875, 1 part will be worth $\frac{1}{15}$ of \$7875 = $\$525$; and if 1 part is worth \$525, then 6 parts will be equal to 6 times \$525 = \$3150, and 9 parts will be 9 times \$525 = \$4725.

39. Henry and James had 1500 chestnuts, which they divided in the ratio of 5 to 7; how many chestnuts had each?

Ans. Henry, 625, and James, 875.

40. Three gentlemen built a mill for \$8928, and the money furnished by each was in the ratio of the numbers 3 and 4 and 5. What amount did each furnish?

Ans. \$2232, \$2976 and \$3720.

41. A gentleman divided \$4200 between his son and his daughter in the ratio of $\frac{1}{2}$ to $\frac{1}{4}$; what sum did each receive?

Ans. Son, \$2400; daughter, \$1800.

Observe, that the ratio of $\frac{1}{2}$ to $\frac{1}{4}$ is the same as $\frac{2}{1}$ to $\frac{1}{2}$, or as 4 to 3.

42. A, B and C formed a partnership in trade, and gained \$2835, which they divided in the ratio of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$; what was the share of each?

Ans. A's, \$1350; B's, \$810; C's, \$675.

43. P and S hired a pasture for \$26.25. P put 9 cows and S 12 cows in the pasture; what part of the rent ought each to pay?

Ans. P, \$11.25; S, \$15.

44. X and W rented a pasture jointly, for which they were

to pay \$40.95. X put in 75 sheep for 3 months, and W had 90 sheep for 4 months; what part of the rent ought each to pay?
Ans. X, \$15.75; W, \$25.20.

45. A gentleman owned $\frac{5}{8}$ of a tract of land, $\frac{1}{8}$ of which he sold at \$7 an acre, and received \$945; of how many acres did the tract consist?

Analysis. $\frac{1}{8}$ of $\frac{5}{8}$ is $\frac{5}{64}$; he therefore sold $\frac{5}{64}$ of the tract. If $\frac{5}{64} = \$945$, then $\frac{1}{64}$ will equal $\frac{1}{5}$ of $\$945 = \$214 = \$189$. Then, if $\frac{1}{64} = \$189$, $\frac{1}{8}$, or the whole tract, will be equal to 18 times as many dollars = \$3402. Now, since \$7 will buy 1 acre, \$1 will buy $\frac{1}{7}$ of an acre, and \$3402 will buy 3402 times as much = ~~3402~~ acres = 486 acres.
Ans. 486 acres.

46. A shepherd owning $\frac{7}{8}$ of a flock of sheep, sold $\frac{1}{4}$ of his share at \$4 per sheep, and thereby realized \$504; how many sheep were in the whole flock?
Ans. 576 sheep.

47. A market-woman had a certain number of oranges, and sold $\frac{3}{4}$ of them at 6 cents apiece, for \$3.60; how many oranges did she have at first?
Ans. 84 oranges

48. If $\frac{3}{4}$ of a certain number exceeds $\frac{2}{5}$ of the same by 27, what is the number?

Analysis. The question means that the difference between $\frac{3}{4}$ of the number and $\frac{2}{5}$ of it is 27. Reducing these fractions to a common denominator, we have $\frac{15}{20}$ and $\frac{8}{20}$. The difference between $\frac{15}{20}$ and $\frac{8}{20}$ is $\frac{7}{20}$. Hence, $\frac{7}{20}$ of the number = 27, and $\frac{1}{20} = \frac{1}{7}$ of 27, which is 9; and as $\frac{1}{20}$ of the number is 9, then $\frac{7}{20}$, or the entire number, will be 20 times as much = 180.
Ans. 180.

49. Four fifths of a certain number exceeds $\frac{3}{4}$ of it by 42. what is the number?
Ans. 315.

50. A gentleman being asked his age, replied: "If you add $\frac{1}{4}$ of my age to $\frac{1}{8}$ of it, and increase the result by $\frac{1}{8}$ of my age, the sum will be 47 years;" what was his age?

Ans. 60 years

51. One half, one third, one fourth and one fifth of a certain number, added together, make 308; what is the number?

Ans. 240.

52. What number is that, which, being multiplied by 5, and the product divided by 6, the quotient will be 20?

Ans. 24.

Analysis. To divide a number by 6, is the same as to take $\frac{1}{6}$ of it. Hence, $\frac{1}{6}$ of the number multiplied by 5 = 20; then, the number itself multiplied by 5 will be equal to 6 times as much = 120. Then, if the number multiplied by 5 be 120, the number itself will be $\frac{1}{5}$ of this = $\frac{120}{5} = 24$.

Proof. $24 \times 5 = \frac{120}{6} = 20$.

53. What is that number which, being multiplied by 4, and the product divided by 5, the quotient will be 36?

Ans. 45.

54. A gentleman having been asked the value of his watch, replied: "If you multiply the price by 7, and divide the product by 10, the quotient will be \$84;" what was the value of the watch?

Ans. \$120.

55. A man spent $\frac{1}{4}$ of his life in New York, $\frac{2}{5}$ in Virginia, and the remainder, which was 28 years, in Georgia; to what age did he live?

Analysis. Adding $\frac{1}{4}$ and $\frac{2}{5}$ together, we have $\frac{9}{20}$. The gentleman spent $\frac{9}{20}$ of his life in New York and Virginia; therefore $\frac{11}{20}$ of it was spent in Georgia. But 28 years was spent in Georgia. Hence, $\frac{11}{20}$ of his life = 28 years, and $\frac{1}{20} = \frac{1}{11}$ of 28 years = $\frac{28}{11}$ years = 4 years; and since $\frac{1}{20} = 4$ years, then $\frac{20}{1} = 80$, or his whole life = 20 times as many years = 80 years.

Ans. 80 years.

56. In an orchard $\frac{1}{4}$ of the trees bear peaches, $\frac{1}{5}$ of them apples, $\frac{1}{6}$ pears, $\frac{1}{8}$ cherries, and the remaining 36 bear plums. how many trees in the orchard?

Ans. 288 trees.

57. In a pasture $\frac{1}{3}$ of the animals are mules, $\frac{1}{4}$ are cows.

$\frac{2}{5}$ are goats, $\frac{1}{3}$ are sheep, and the remaining 5 are horses; how many animals in the pasture? *Ans.* 180 animals

58. There is a fish whose head is $\frac{1}{4}$ of his whole length, his body is $\frac{2}{3}$ of his length, and his tail is 5 inches; what is the length of the fish? *Ans.* 30 inches

59. A shepherd, after selling $\frac{1}{5}$ of his flock of sheep, losing $\frac{1}{4}$ of the remainder by disease, had 224 sheep left; how many had he at first?

Analysis. After selling $\frac{1}{5}$ of his flock he had $\frac{4}{5}$ of it left, and after losing $\frac{1}{4}$ of the remainder he must have had $\frac{3}{4}$ of the remainder left—i. e., he had $\frac{3}{4}$ of $\frac{4}{5}$ of the original number left. But $\frac{3}{4}$ of $\frac{4}{5}$ is $\frac{3}{5}$. Therefore he had $\frac{3}{5}$ of his flock left. But he also had 224 sheep left; hence $\frac{3}{5}$ of the flock = 224 sheep, and $\frac{1}{5}$ of the flock = $\frac{1}{3}$ of 224 sheep = $74\frac{2}{3}$ sheep = 28 sheep; and $\frac{4}{5}$, or the whole flock, equals 15 times as much as sheep = 420 sheep. *Ans.* 420 sheep

60. A merchant, after investing $\frac{1}{4}$ of his money in groceries and spending $\frac{1}{5}$ of the remainder for crockery, had \$1245 left; how much had he at first? *Ans.* \$1992

61. After spending $\frac{2}{3}$ of my money, and $\frac{1}{3}$ of what was left, I had \$48 remaining; how much had I at first? *Ans.* \$180

62. A gentleman bought a horse and saddle for \$175, the horse cost 6 times as much as the saddle; what was the cost of the saddle? *Ans.* \$25

63. A, B and C engaged in trade and gained \$2025. In dividing the profits, B had twice as much as C, and A received 3 times as much as B; what was the share of each?

Ans. A, \$1350; B, \$450; C, \$225

64. Some fishermen went on an excursion and caught 100 fish, consisting of bream, trout and mullet. There were 3 times as many bream as trout, and twice as many mullet as bream; how many of each kind did they catch?

Ans. Bream, 72; trout, 24; mullet, 144

65. When flour is worth \$9 a barrel, and a 5-cent loaf of bread weighs 8 ounces, how much ought the loaf to weigh when flour is worth \$12 per barrel?

Analysis. If it weighs 8 ounces when flour is worth \$9, then when flour is \$1 per barrel it ought to weigh 9 times as much, or 72 ounces; and when flour is \$12 it ought to weigh only $\frac{1}{12}$ as much as at \$1 per barrel = $7\frac{1}{2}$ ounces, or 6 ounces.

Ans. 6 ounces.

66. The 5-cent loaf weighing 8 ounces when flour is \$10 per barrel, how much should it weigh when flour is \$7?

Ans. $11\frac{1}{2}$ ounces.

67. If I lend a friend \$450 for 4 months, how long ought he to lend me \$300 to requite the favor? *Ans.* 6 months.

68. If a ship of war with a crew of 525 men allows each man 42 ounces of biscuit per day, what would be the daily allowance if 75 men were taken off? *Ans.* 49 ounces.

What would be the allowance if 75 more men were put on the ship? *Ans.* $36\frac{1}{2}$ ounces.

69. If a miller grinds grain, taking $\frac{1}{8}$ for toll, how many bushels of corn must a farmer carry to mill, in order to get 28 bushels of meal?

Analysis. It is evident that to get $\frac{7}{8}$ of a bushel he must carry 1 bushel, and to get $\frac{1}{8}$ he must carry $\frac{1}{7}$ of a bushel, and to get $\frac{8}{8}$, or a *whole* bushel, he must carry 8 times as much = $\frac{8}{7}$ of a bushel, and to get 28 bushels he must carry 8 times as many bushels = $23\frac{1}{7}$ bushels, or 32 bushels.

Ans. 32 bushels.

70. If the toll for carding wool be $\frac{1}{4}$, how many pounds must be carried to the factory in order to receive 16 pounds of rolls? *Ans.* $21\frac{1}{4}$ pounds.

71. The rate of toll being $\frac{1}{8}$, how many bushels of wheat must a provision-dealer send to mill in order to receive 800 bushels of wheat ground up? *Ans.* $914\frac{2}{3}$ bushels.

72. If a train of cars move 44 feet per second, what is its rate in miles per hour? *Ans.* 30 miles.

73. A merchant in selling goods deducts 8% from the marked price for cash; what is the marked price of a trunk which he sells for cash for \$9.66?

Analysis. 100% may represent the marked price; then, as he deducts 8% for cash, $100\% - 8\% = 92\%$ will equal the selling price. But \$9.66 is the selling price; therefore $92\% = \$9.66$, and 1% will equal $\frac{1}{92}$ of \$9.66, $= \$\frac{9.66}{92}$; and 100% will equal 100 times as much, $= \$\frac{9.66}{92} \times 100 = \$10.50$. *Ans.* \$10.50.

The operation on the blackboard would stand thus:

OPERATION.

$100\% = \text{Marked price.}$

$100\% - 8\% = 92\% = \$9.66 = \text{Selling price.}$

$1\% = \$\frac{9.66}{92}$

$100\% = \$\frac{9.66}{92} \times 100 = \$10.50 = \text{M. price.}$

74. Suppose a grocer deducts 10% for cash, what must be the marked price for a barrel of flour for which he receives \$6.75 cash? *Ans.* \$7.50.

75. If a jeweler deducts $6\frac{1}{4}\%$ for cash, what is his marked price for a pin which he sells for cash for \$22.50?

Ans. \$24.

76. In $6\frac{1}{4}$ hours a wheel makes 10854 revolutions; how many are made per hour? *Ans.* 1608.

77. If $\frac{3}{4}$ of a number is $347\frac{3}{4}$, what is the number?

Ans. $810\frac{3}{4}$.

78. A man spent $\frac{1}{4}$ of $\frac{3}{4}$ of his money and had \$488 $\frac{3}{4}$ left; how much had he at first? *Ans.* \$743 $\frac{3}{4}$.

79. A is $6\frac{1}{4}$ feet high; B's height is $\frac{5}{8}$ of A's; C's height is $\frac{7}{8}$ of B's; and D's is $\frac{9}{10}$ of C's; what is D's height?

Ans. $5\frac{5}{14}$ feet.

80. The difference between $\frac{1}{4}$ and $\frac{1}{5}$ of a number is $43\frac{3}{4}$; what is the number? *Ans.* 875.

81. A has $\frac{4}{5}$ of \$393; B has $2\frac{3}{4}$ as much as A, and C has $2\frac{1}{2}$ times as much as A and B together; how much has C?

Ans. \$2593 $\frac{3}{4}$.

82. A sum of money was divided between 3 persons, A, B and C. A received $\frac{1}{4}$ of it, B $\frac{1}{5}$ of it, and C received \$148 $\frac{1}{2}$; what was the sum divided?

Ans. \$270.

83. A man spent $\frac{1}{4}$ of his money, then $\frac{1}{8}$ of what remained, and finally $\frac{1}{2}$ of what yet remained; what part was left?

Ans. $\frac{1}{4}$.

84. What fractional part of 75 is $31\frac{1}{4}$?

Ans. $\frac{5}{12}$.

85. B can reap a field of wheat in 5 days, and C can reap the same in 7 days; what part of the work can B do in a day? What part can C do in a day? How much can they both do in a day? How long will it take the two together to reap the field?

Last ans. $2\frac{1}{2}$ days.

86. If a grocer clears $3\frac{1}{2}$ cents on every quart of strawberries he sells, how many quarts must he sell to clear 87 $\frac{1}{2}$ cents?

Ans. 28 quarts.

87. A has \$105; B has $\frac{5}{8}$ as much as A, lacking \$5, and C has $\frac{1}{8}$ more than A and B together, lacking \$7 $\frac{1}{2}$; how much has C?

Ans. \$242 $\frac{3}{4}$.

88. How many revolutions will a wheel $4\frac{3}{4}$ feet in circumference make during a journey of 36 miles?

Ans. 40731 $\frac{3}{4}$.

89. If the driving-wheel of a locomotive is $16\frac{1}{2}$ feet in circumference, and if it travels $56\frac{1}{4}$ miles an hour, how often must it revolve in a second?

Ans. 5 times.

90. A boy invested 12 cents in apples; the one-third, one-fourth and one-sixth of what he had amounted to 54 apples; how many did he buy for half a cent?

Ans. 3 apples.

91. A clock gains $4\frac{1}{2}$ minutes per day; how should its hands be placed at noon so as to point exactly to the true time at 8 o'clock P. M.?

Ans. $1\frac{1}{2}$ min. before 12.

92. A received \$756, which was $\frac{3}{4}$ of B's money, but 3 times B's money was $\frac{1}{2}$ of C's; what sum did C have?

Ans. \$6048.

93. Three persons, A, B and C, divide \$500 among themselves in such a way that as often as A gets \$2 $\frac{1}{4}$, B receives \$3 $\frac{3}{8}$ and C gets \$4 $\frac{1}{2}$. What is the share of each?

Ans. A, \$108; B, \$176; C, \$216.

94. A, B and C do a piece of work, for which they receive \$61.71. A works 9 days of 12 hours each, B works 12 days of 10 hours each, and C works 15 days of 9 hours each; how ought the \$61.71 to be divided among them?

Ans. A, \$18.36; B, \$20.40; C, \$22.95.

95. If 12 men can complete a job of work in 12 $\frac{1}{2}$ days, how many men would be required to do the same work in 18 $\frac{3}{4}$ days?

Ans. 8 men.

96. During a voyage at sea, 15% of the provisions was consumed during the first week, and 20% of the remainder was used during the second week; what % of the provisions still remained?

Ans. 68%.

97. If we add 40% of 40 to $\frac{5}{8}$ of 40, what will be the sum?

Ans. 44 $\frac{1}{2}$.

98. A has \$896; B's money is 56 $\frac{1}{4}$ % of A's, and C's is 66 $\frac{2}{3}$ % of B's; how much has C?

Ans. \$336.

99. A certain number increased by 20% of 30% of itself is 477; what is the number?

Analysis. 20% of 30% = $.20 \times .30 = .0600 = .06 = 6\%$. Therefore our question might be so constructed as to read thus: "What number increased by 6% of itself will make 477?"

Let 100% = The number.

Then, 100% + 6% = 106% = 477,

and 1% = 4 $\frac{1}{2}$.

100% = 100 $\times$ 4 $\frac{1}{2}$ = 450.

100. A certain sum of money increased by $16\frac{2}{3}\%$ of 24% of itself amounts to \$998.40; what is the sum? *Ans.* \$960.

101. A farmer had a flock of sheep, and during a certain period he lost 50% of 32% of his flock, and then had 189 left; of how many did his flock consist at first?

Ans. 225 sheep.

102. If $\frac{7}{9}$ be increased by 25% of itself, what fraction will represent the sum?

Analysis. $\frac{7}{9} \times 25\% = \frac{7}{9} \times .25 = 1\frac{7}{5}$. Then, $\frac{7}{9} + 1\frac{7}{5} = 2\frac{7}{5} = \frac{17}{5} = \frac{34}{10} = \frac{17}{5}$. *Ans.* $\frac{17}{5}$.

Or we may explain it thus:

$25\% = \frac{1}{4}$; therefore $\frac{1}{4}$ of $\frac{7}{9} = \frac{7}{36}$. Then, $\frac{7}{9} + \frac{7}{36} = \frac{28}{36} + \frac{7}{36} = \frac{35}{36}$.

103. If $\frac{3}{4}$ be increased by 20% of itself, what fraction will represent the result? *Ans.* $\frac{9}{10}$.

104. If $\frac{8}{9}$ be diminished by 25% of itself, what fraction will express the result? *Ans.* $\frac{2}{3}$.

105. A grocer sold a lot of flour for \$201.25, and thereby made a profit of 15% ; what was the cost of the flour?

Analysis.

$100\% = \text{Cost.}$

$15\% = \text{Gain.}$

Then, $100\% + 15\% = 115\% = \$201.25 = \text{Selling price.}$

$1\% = \$1.75.$

$100\% = 100 \times \$1.75 = \$175.$

Ans. \$175.

106. A gentleman sold a horse for \$188.80, and thereby gained 18% ; what was the cost of the horse? *Ans.* \$160.

107. A merchant sold a piece of velvet for \$151.20, which was 12% more than the cost; what was the cost?

Ans. \$135.

108. A provision-dealer sold a cask of bacon for \$106.25, thereby sustaining a loss of 15% ; what was the cost of the bacon?

Analysis.

$$100\% = \text{Cost.}$$

$$15\% = \text{Loss.}$$

$$\text{Then, } 100\% - 15\% = 85\% = \$106.25 = \text{Selling price}$$

$$1\% = \$1.25.$$

$$100\% = 100 \times \$1.25 = \$125 = \text{cost.}$$

109. A grocer sells a quantity of molasses for \$162, and thereby loses 10%; what was the cost of the molasses?

Ans. \$180.

110. By selling cassimere at \$1.80 per yard, a merchant sustains a loss of $16\frac{2}{3}\%$; what was the cost of the cassimere?

Ans. \$2.16.

111. B and C rented a pasture for \$43.20. B put in 9 cows and 6 calves for $4\frac{1}{2}$ months, 2 calves being considered equivalent to 1 cow; C put in 10 cows and 16 sheep for 3 months, 4 sheep being regarded as equal to 1 cow; how much of the rent ought each to pay? *Ans.* B, \$24.30; C, \$18.90.

112. A rabbit has 50 yards the start of a hound. Now, if the hound runs 7 yards while the rabbit runs 5 yards, how far must the hound run before he overtakes the rabbit?

Ans. 175 yards.

113. If $\frac{2}{3}$ of a piece of velvet be worth \$33.75, what would be the value of $\frac{3\frac{1}{2}}{5\frac{1}{4}}$ part of it?

Ans. \$40.50.

114. A father left to his wife $\frac{1}{3}$ of his estate, to his daughter $\frac{2}{3}$ of the remainder, and to his son what was left; what was the share of each, if the shares of the wife and the daughter differed by \$600?

Ans. Wife, \$3600; daughter, \$3000; son, \$1500.

115. A, B, C, D and E own shares in a railroad. A owns $6\frac{1}{2}$ shares; B's stock is $\frac{3}{4}$ of A's; C's is $\frac{2}{3}$ of B's; D's is $\frac{1}{2}$ of C's; and E's is $\frac{1}{3}$ of D's; how many shares has E?

Ans. $5\frac{7}{8}$ shares.

116. A man spent $\frac{2}{3}$ of his money, and afterward $\frac{1}{4}$ of the

remainder; then $\frac{2}{3}$ of what was left was \$25; what sum had he at first? *Ans.* \$375.

117. A gentleman bought a horse and buggy for \$360. The value of the horse was $\frac{5}{7}$ of the buggy; what was the value of each?

Analysis.

Let $7 =$ Price of the buggy.

$5 =$ Price of the horse.

Then, $7 + 5 = 12 =$ Price of both.

Therefore $12 = \$360$.

$7 = \$360 \div 12 = \$30$.

$5 = \$150 =$ Price of horse.

$7 = \$210 =$ Price of buggy.

118. A's age is $\frac{7}{8}$ of B's, and the sum of the ages of the two is 112 years; what is the age of each?

Ans. A's, 49; B's, 63.

119. A gentleman bought a rockaway, a buggy and a wagon for \$608. The cost of the buggy was $\frac{2}{3}$ of that of the rockaway, and the wagon $\frac{1}{10}$ that of the rockaway; what was the cost of each?

Ans. Rockaway, \$320; buggy, \$192; wagon, \$96.

120. Eighteen persons unite to build a church, each agreeing to pay \$80. Now, if 6 more persons unite in the undertaking, what must be the subscription of each? *Ans.* \$60.

121. If \$52.50 be the wages of 7 men for 6 days, what would be the wages of 12 men for 15 days? *Ans.* \$225.

122. A grocer bought a barrel of syrup for \$21.60, supposing it to contain 48 gallons, but on measuring the cask it was found to be deficient $16\frac{2}{3}\%$; at what price per gallon must he sell the remainder so as to lose nothing? *Ans.* 54 cents.

123. A merchant bought a piece of cassimere, containing 45 yards, for \$81. By accident, 20% of it was so injured as to be worthless. At what price per yard must he sell the remainder so as to gain 20% on the cost? *Ans.* \$2.70.

124. A man lost 45 sheep, which was 20% of what he then had; how many had he at first?

Analysis. $100\% = \text{Flock after losing 45.}$

$20\% = 45 = \text{Number lost.}$

$1\% = \frac{45}{20} = \frac{9}{4}.$

$100\% = \frac{9}{4} \times 100 = 225.$

Then, $225 + 45 = 270.$

Ans. 270 sheep.

125. A gentleman spent \$81, which was 18% of what he had left; how much had he at first?

Ans. \$531.

126. A farmer sold 36 bushels of wheat, which was 15% of what he had remaining; how many bushels had he at first?

Ans. 276 bushels.

127. $\frac{2}{3}$ of $\frac{3}{4}$ of 315 is $16\frac{2}{3}\%$ of what number? *Ans.* 324.

128. Five companies guard a fort, in which the daily duty requires 152 men. The first company has 108 men, the second 102 men, the third 96 men, the fourth 78 men, and the fifth 72 men; how many men must each company furnish for daily duty?

Ans. 1st, 36; 2d, 34; 3d, 32; 4th, 26; 5th, 24 men.

129. A grocer and a farmer barter together. The grocer has coffee which he sells for 22 cents for cash per pound, and the farmer has wheat which he sells at \$1.10 per bushel. Now, if the grocer raises the price of his coffee $2\frac{1}{4}$ cents, what ought the farmer to ask for his wheat?

Ans. \$1.21 $\frac{1}{4}$.

130. Smith and Gray barter together. Smith has 3200 pounds of flour worth \$4.25 per hundred; Gray has sugars worth respectively 8 and 12 cents per pound. Smith agrees to take for his flour $\frac{2}{3}$ of the amount in cash and the balance in sugar, of which he is to receive the same number of pounds of each sort. How much cash, and how many pounds of each sort of sugar, must Smith receive? *Ans.* \$51; 425 pounds.

131. What fraction increased by 12% of itself will be equal to $\frac{3}{4}$?

Analysis. Let 100% = The fraction.

12% = The increase.

Then, $100\% + 12\% = 112\% = \frac{28}{25}$.

$1\% = \frac{25}{2800}$.

$100\% = \frac{2500}{28} = \frac{625}{7}$. *Ans.* $\frac{625}{7}$.

132. What fraction increased by 24% of itself will give $\frac{11}{8}$? *Ans.* $\frac{5}{7}$.

133. What fraction increased by $33\frac{1}{3}\%$ of itself will give $\frac{5}{3}$? *Ans.* $\frac{2}{3}$.

134. What fraction diminished by 40% of itself will give $\frac{8}{15}$?

Analysis. Let 100% = The fraction.

40% = The decrease.

Then, $100\% - 40\% = 60\% = \frac{3}{5}$.

$1\% = \frac{5}{1500}$.

$100\% = \frac{500}{15} = \frac{100}{3}$. *Ans.* $\frac{100}{3}$.

135. What fraction diminished by 20% of itself will give $\frac{7}{8}$? *Ans.* $\frac{35}{32}$.

136. What fraction diminished by $16\frac{2}{3}\%$ of itself will give $\frac{1}{2}$? *Ans.* $\frac{3}{4}$.

137. A boy bought 48 apples. For $\frac{2}{3}$ of them he paid at the rate of 5 cents for 4 apples, and for the remainder 4 cents for 5 apples; at what rate must he sell them each so as to gain 25% ? *Ans.* $1\frac{1}{2}$ cents.

138. A watch, which goes 3 minutes too slow every day, is set right at 12 o'clock. What will be the true time when the hands of the watch point at 24 minutes after 10 P.M.?

Ans. 25min. 18sec. after 10.

139. A cistern is supplied by two pipes; the first fills it in 2 hours, the other in 3 hours. The water is drawn from the cistern by a third pipe, which empties it in $1\frac{1}{2}$ hours. If the cistern is empty, and the three pipes act at the same time, how long will it take to fill the cistern? *Ans.* 6 hours.

MENSURATION.

ARTICLE 358. MENSURATION literally means the art of *measuring*. It teaches rules for measuring lines, surfaces and solids. These rules are derived from Geometry. As the pupil is not presumed to be acquainted with Geometry, the definitions and rules here given are presented in a brief and simple manner, without any attempt to *prove* their correctness.

DEFINITIONS.

1. A **LINE** has *length*, without breadth or thickness.
2. A **SURFACE** has *length* and *breadth*, without thickness.
3. A **SOLID** or **BODY** has *length*, *breadth* and *thickness*.
4. The **AREA** of a figure is the surface which it contains.

The **MEASURING UNIT** of surfaces is a square surface, as a square inch, square foot, square yard, etc.

5. A **TRIANGLE** is a figure with three sides and three angles.



6. The **BASE** of a triangle is the side on which it stands

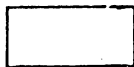
7. The **ALTITUDE** of a triangle is a line drawn from one of the angles perpendicular to its base.

8. A **PARALLELOGRAM** is a four-sided figure whose opposite sides are parallel.



9. The **ALTITUDE** of a parallelogram is the perpendicular distance between its opposite sides.

10. A **RECTANGLE** is a parallelogram whose opposite sides are equal, and its angles are right angles.

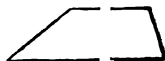


11. A **SQUARE** is a parallelogram having all its sides equal and all its angles right angles.



12. A **TRAPEZOID** is a four-sided figure having only two of its sides parallel.

13. The **PERIMETER** of a figure is the sum of the lines that bound it.



ART. 359. TO FIND THE AREA OF A PARALLELOGRAM.

Multiply the base by the altitude.

Ex. 1. How many sq. ft. in a floor 22ft. long and 18ft. wide? *Ans.* 396 sq. ft.

2. How many sq. yd. in a garden 55yd. long and 45yd. wide? *Ans.* 2475 sq. yd.

3. What is the area of a table 12ft. 6in. in length and 3ft. 6in. in width? *Ans.* 43.75 sq. ft.

4. What are the contents of a plank 24ft. long and 2ft. 9in. wide? *Ans.* 66 sq. ft.

5. A farmer has a square barley-lot measuring 45 rods on each side; how many acres does it contain?

Ans. 12A. 2R. 25r.

6. How many sq. ft. in a board 18ft. 3in. long and 15in. wide? *Ans.* 22.8125, or $22\frac{13}{16}$ sq. ft.

7. How many acres in a field in the shape of a parallelogram, the length being 96 rods and the breadth 36 rods?

Ans. 21A. 2R. 16r.

8. What are the contents of a plank 18ft. 6in. in length and 21in. broad? *Ans.* 32.375, or $32\frac{3}{8}$ sq. ft.

9. How many yards of carpeting, 1yd. wide, would cover a floor 18ft. long and 16ft. wide? *Ans.* 32yd.

10. How many yards of matting, $\frac{7}{8}$ of a yd. wide, would cover a floor 18ft. square? *Ans.* $41\frac{1}{2}$ yd.

ART. 360. TO FIND THE AREA OF A TRIANGLE.

In Geometry it is demonstrated that the area of a triangle is *half* that of a parallelogram having the same base and the same altitude: therefore to find the area of a triangle,

Multiply half the base by the altitude.

Ex. 1. The base of a triangle is 24ft. and its altitude 16ft. ; what is the area? *Ans.* 192 sq. ft.

2. What are the contents of a triangle whose base is 32ft. 6in. and altitude 18.5ft. ? *Ans.* 300.625, or $300\frac{5}{8}$ sq. ft.

When the three sides are given we use this method :

From HALF THE SUM of the three sides subtract each side separately ; multiply together the half sum and the three remainders, and extract the square root of the product.

3. What are the contents of a triangular garden whose sides are 40, 50 and 60 yards? *Ans.* 992.15 sq. yd.

75 = half sum ; 35 = 1st. rem. ; 25 = 2d. rem. ; 15 = 3d. rem.

Then, $\sqrt{75 \times 35 \times 25 \times 15} = \sqrt{984375} = 992.15$ sq. yd.

4. What are the contents of a triangular lot whose sides are 56, 72 and 84 feet? *Ans.* 221.23 sq. yd.

ART. 361. TO FIND THE AREA OF A TRAPEZOID.

Multiply half the sum of the parallel sides by the perpendicular distance between them.

1. The parallel sides of a trapezoid are 36 and 24 feet, and its breadth 16 feet ; what is the area? *Ans.* 480 sq. ft.

2. A garden is in the shape of a trapezoid. One of the parallel sides is 65yd., the other 49yd., and the width 42yd. ; what is the area? *Ans.* 2394 sq. yd.

3. How many acres in a lot of land 40 rods long, 30 rods wide at one end, and 20 rods wide at the other?

Ans. $6\frac{1}{4}$ acres.

4. What are the contents of a plank 16ft. long, 2ft. 3in. wide at one end, and 1ft. 9in. wide at the other?

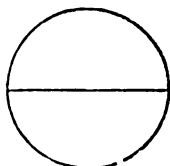
Ans. 32 sq. ft.

THE CIRCLE.

ART. 362. A CIRCLE is a figure bounded by a curved line, every part of which is equally distant from a point within called the *centre*.

The *circumference* of a circle is the line which bounds it.

The *diameter* is a straight line passing through the centre, and terminated on both sides by the circumference.



The *radius* is a line extending from the centre to the circumference. It is half the diameter.

ART. 363. TO FIND THE CIRCUMFERENCE OF A CIRCLE, THE DIAMETER BEING GIVEN.

Multiply the diameter by 3.1416.

Ex. 1. The moon's diameter being 2160 miles, what is its circumference? *Ans.* 6785.856 miles.

2. Over what space would the driving-wheel of a locomotive, 5ft. 3in. in diameter, pass in making 25 revolutions?

Ans. 412.335 feet.

3. The radius of the earth's equator being 3963 miles, at what rate per hour is a man on the equator carried round by its rotation in 24 hours?

Ans. 1037.5134 miles.

ART. 364. TO FIND THE DIAMETER, THE CIRCUMFERENCE BEING GIVEN.

Divide the circumference by 3.1416.

Ex. 1. The circumference of a cannon-ball being 37.6992in., what is the diameter? *Ans.* 12 inches.

2. If a wheel at each revolution describes 235.62 inches, what is the diameter of the wheel? *Ans.* 75 inches.

3. What is the diameter of a water-wheel 96 feet in circumference? *Ans.* 30.557 + feet

ART. 365. TO FIND THE AREA OF A CIRCLE.

Multiply the square of the diameter by .7854;

Or, multiply the square of the radius by 3.1416.

Ex. 1. A circular flower-garden is 36 feet in diameter; what is its area? *Ans.* 113.0976 sq. yd.

2. How many yards of carpeting, 1yd. wide, would be required to cover a circular hall 45ft. in diameter?

Ans. 176.715yd.

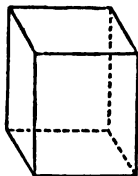
3. If a horse be fastened to a post in a pasture by a halter 54ft long, over how much space may he graze?

Ans. 1017.8784 sq. yd.

MENSURATION OF SOLIDS

ART. 366. A **PRISM** is a solid whose two ends or *bases* are parallel and equal, and its sides are parallelograms.

The **ALTITUDE** of a prism is the perpendicular distance between its upper and lower bases. Ordinary boxes are instances of prisms.



ART. 367. TO FIND THE SURFACE OF A PRISM.

Multiply the perimeter of the base by the altitude, and add the area of the two ends to the product.

Ex. 1. What is the surface of a prism whose length is 9in., breadth 7in. and height 8in.?

Ans. 382 sq. in.

Perimeter of base = 32in. Then, $32 \times 8 = 256$ sq. in. = surface of sides; $9 \times 7 = 63$ sq. in. = area of one end; $63 \times 2 = 126$ sq. in. = area of two ends.

Then, 256 sq. in. + 126 sq. in. = 382 sq. in. = entire surface.

2. How many square feet of plank would make a box 12ft. 5in. long, 6ft. wide and 5ft. 6in. deep, with a cover?

Ans. $353\frac{1}{2}$ sq. ft

3. A merchant has a paper box 24in. long, 16in. wide and 12in. deep. How many square feet of pasteboard would be required to make this box and a cover to fit it?

Ans. 12 sq. ft

ART. 368. TO FIND THE SOLIDITY OF A PRISM.

Multiply the area of the base by the altitude.

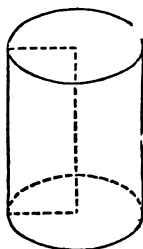
Ex. 1. How many gallons of wine could be contained in a vat 5ft. 10in. long, 3ft. 4in. wide and 2ft. 10in. deep; 231 cubic inches being a gallon?

Ans. $412\frac{2}{3}$ gal.

2. How many cubic feet in a pile of wood 42ft. long, 8ft wide and 7ft. high?

Ans. 2352 cu. ft.

ART. 369. A CYLINDER is a solid of uniform diameter whose ends or bases are equal circles, parallel to each other. A common roller is an example of a cylinder.



ART. 370. TO FIND THE SURFACE OF A CYLINDER.

If a cylinder be covered with a thin substance, like paper, which can be spread out on a plane, it is evident that the figure will be a rectangle whose length is equal to the length of the cylinder, and its breadth equal to the circumference of the cylinder; therefore, to find the surface of a cylinder,

Multiply the circumference of the base by the altitude, and to the product add the areas of the two bases.

Ex. 1. What is the entire surface of a cylinder whose altitude is 22in. and the radius of whose base is 8in.?

$3.1416 \times 16 = 50.2656 =$ circumference of base. Then,

$50.2656 \times 22 = 1105.8432$ sq. in. = convex surface.

$8^2 \times 3.1416 = 64 \times 3.1416 = 201.0624$ sq. in. = area of one base; $201.0624 \times 2 = 402.1248 =$ area of both bases.

And $1105.8432 + 402.1248 = 1507.968$ sq. in. = entire surface.

2. How many sheets of tin, 12 by 15 inches, would be required to make a lard-can 20 inches in diameter and 34 inches high?

Ans. 15.358 + sheets.

3. How many sq. ft. in a cylindrical roller 7ft. long and 5ft. in diameter?

Ans. 149.226 sq. ft.

ART. 371. TO FIND THE SOLID CONTENTS OF A CYLINDER.

Multiply the area of the base by the altitude.

Ex. 1. What are the solid contents of a cylinder whose altitude is 12ft. and the radius of whose base is 3ft.?

Ans. 339.2928 cu. ft.

2. If 231 cu. in. make a gallon, how many gallons of oil could be contained in a can whose altitude is 34in. and the radius of the base 10in.?

Ans. 46.24gal.

ARTIFICERS' ESTIMATES.

ART. 372. Rules for Mensuration similar to those presented in the preceding pages furnish various classes of workmen with methods for estimating the cost of their work.

Plasterers, painters, paper-hangers and paviors generally estimate their work by the square yard.

House-carpenters usually reckon weather-boarding, flooring, ceiling and shingling by the SQUARE of 100 feet.

Ex. 1. What would be the cost of paving a sidewalk 540ft. long and 10ft. 4in. wide, at 24 cents per sq. yd.?

Ans. \$148.80.

2. How many bricks 8in. long and 4in. wide would be required to pave a walk 280yd. in length and 9ft. wide?

Ans. 34020 bricks.

3. How much would it cost to plaster the ceiling of a room 20ft. 3in. long and 16ft. wide, at 30 cents per sq. yd.?

Ans. \$10.80.

4. A church is 90ft. long and 66ft. wide; what would it cost to lay the floor at \$2.25 per SQUARE of 100ft.?

Ans. \$133.65.

5. How many shingles would be required to cover a roof 42ft. long, the rafters on each side being 18ft. 4in. long, and the shingles to be 4in. wide, and to show 6in.?

Ans. 9240 shingles

NOTE.—The pupil will notice that each side of the roof is a *rectangle*, and 6 shingles cover a square foot.

BRICK AND STONE WORK.

ART. 373. Bricklayers usually charge for their work at so much per thousand bricks. Ordinarily, 20 bricks are counted as a cubic foot.

Stone-masons sometimes estimate their work by the *perch*, which is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide and 1 ft. thick. The stone-mason's *perch* therefore contains $24\frac{3}{4}$ cu. ft., but for convenience of calculation it is frequently called 25 cu. ft.

Ex. 1. How many bricks would be required to build a wall 84 ft. long, 12 ft. 3 in. high and 16 in. thick, allowing 20 bricks to the cubic foot? *Ans.* 27440 bricks.

2. How many perches in a stone wall 75 ft. long, 12 ft. high and 2 ft. 6 in. thick? *Ans.* 90 perches.

NOTE.—First get the solid contents in feet, and divide the result by 25 to reduce to perches.

TIMBER AND PLANK MEASURE.

ART. 374. All lumber with square edges, as planks, joists and scantling, is usually estimated by plank measure, in which the thickness is considered to be one inch. Thus, a plank 18 feet long, 1 foot wide and 1 inch thick is said to contain 18 square feet of lumber.

ART. 375. TO FIND THE CONTENTS OF A PLANK.

Multiply the length of the plank, taken in inches, by the breadth, taken also in inches, and divide the product by 144; the quotient will give the contents in square feet.

Or, multiply the length, taken in feet, by the breadth in inches, and divide the product by 12.

Ex. 1. What are the contents of a plank 15 in. wide and 16 ft. long? *Ans.* 20 sq. feet.

2. What are the contents of a plank 16in. wide and 18ft. 9in. long? *Ans.* 25 sq. feet.

3. How many feet in a plank 21in. wide and 24ft. long? *Ans.* 42 sq. feet.

4. How many feet in a plank 17in. wide at one end and 13in. at the other, and 20ft. long? *Ans.* 25 sq. feet.

See Rule for Trapezoid.

5. What are the contents of a plank 24ft. long, 15in. wide at one end, and tapering to a point at the other? *Ans.* 15 sq. feet.

See Rule for finding area of Triangle.

ART. 376. TO FIND THE CONTENTS OF JOISTS, SCANTLING, ETC.

Multiply the breadth in inches by the thickness, and this product by the length in feet; divide the product by 12, and the quotient will be the contents in feet.

Ex. 1. What are the contents of a scantling 6in. wide, 3in. thick and 15ft. long? *Ans.* $22\frac{1}{2}$ sq. feet.

2. What are the superficial contents of a joist 18ft. long, 4in. wide and 3in. thick? *Ans.* 18 sq. feet.

3. How many feet of lumber in a beam 22ft. long, 8in. wide and 3in. deep? *Ans.* 44 sq. feet.

ART. 377. *Practical* measurers of lumber employ methods much *shorter* than those just given. We will illustrate by an example:

Suppose a plank to be 24 feet long, 1 foot wide and 1 inch thick. Its contents are 24 sq. feet.

The length remaining the same, suppose it had been 15 inches wide? The contents would evidently be 30 feet, because the width has been increased 3 inches $= \frac{3}{12} = \frac{1}{4}$ of a foot, and therefore the contents would also be increased *one fourth* of what they were. If it had been 16 inches wide, the contents would have been increased *one third*, and there-

fore would have been 32 feet. If the width had been 18 inches, the area would have been increased *one half*, and would have been 36 feet. Hence, we have this general rule:

When the plank is 1 foot wide and 1 inch thick, the contents will be just as many square feet as there are feet in the length; and for every additional inch in the width we must add to the contents $\frac{1}{12}$ of the number of square feet.

The same principle holds good when the width is *less* than 1 foot. In such cases we *subtract* $\frac{1}{12}$ of the contents for each inch *less* than 1 foot.

If the thickness should be more than 1 inch, we must increase the contents by an additional amount; for example, if the thickness be $1\frac{1}{4}$ inches, we must increase the area *one fourth* of the previous amount; if $1\frac{1}{2}$ inches, we must add *one half*, and so on.

Ex. 1. What are the contents of a board 18ft. long and 16in. wide? Ans. 24 sq. feet.

2. Suppose the plank of the same length, but 20in. wide? Ans. 30 sq. feet.

3. Suppose it to be 16in. wide and $1\frac{1}{4}$ in. thick? Ans. 30 sq. feet.

4. Suppose it to be 18in. wide and $1\frac{1}{2}$ in. thick? Ans. $40\frac{1}{2}$ sq. feet.

5. Suppose it to be 10in. wide and 1in. thick? Ans. 15 sq. feet.

6. Suppose it to be 8in. wide and $1\frac{1}{4}$ in. thick? Ans. 15 sq. feet.

CONTENTS OF BINS AND CRIBS.

ART. 378. It is sometimes desirable to know how many bushels of corn, wheat or other grain may be contained in a box or bin of given dimensions.

For this purpose we employ the following

RULE.

Measure the length, breadth and depth of the box in inches ; multiply these three dimensions together, and divide the product by 2150.42 ; the result will be the number of bushels.

Ex. 1. How many bushels of wheat can be contained in a box 6ft. 8in. long, 4ft. 2in. wide and 3ft. 6in. deep ?

Ans. 78.12 + bushels.

2. How many bushels of corn in a crib 14ft. long, 9ft. 2in. wide and 8ft. 4in. deep ?

Ans. 859.36 + bushels.

Another very easy method is this :

Multiply together the length, breadth and depth of the crib, taken in feet and decimals of a foot, and then multiply this product by $\frac{8}{10}$; the result will give the contents in bushels, very nearly.

The reason of this Rule is, that a cubic foot is very nearly $\frac{8}{10}$ of a bushel.

Ex. 1. How many bushels of barley could be contained in a box 8ft. long, 5ft. wide and 4ft. 6in. deep ?

Ans. 144 bushels.

2. If a bin be 10ft. long, 6ft. 3in. wide and 5ft. 6in. deep, how many bushels of potatoes would it contain ?

Ans. 275 bushels.

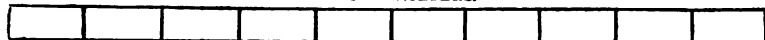
3. How many bushels of corn could be contained in a crib 16ft. long, 12ft. 6in. wide and 8ft. 3in. deep ?

Ans. 1320 bushels.

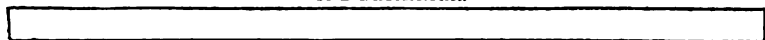
ONE HUNDRED MILLIMETERS.



TEN CENTIMETERS.



ONE DECIMETER.



APPENDIX.

THE METRIC SYSTEM OF WEIGHTS AND MEASURES.

ARTICLE 1. The METRIC SYSTEM of Weights and Measures is based upon the decimal plan of Notation.

ART. 2. It is called METRIC because the METRE* is taken as the standard or unit of measure. The METRE is the one-ten-millionth part of the distance on the earth from the equator to the pole; it is very nearly 39.37 inches, or 3.28 feet, in length.

ART. 3. This system originated in France about the year 1795, and was intended for universal adoption in order to introduce a *uniform* system of weights and measures in place of the confused and irregular plan then in use.

ART. 4. It has been adopted, wholly or in part, by several of the nations of Europe; in Great Britain and the United States its use is permitted and recommended, though not compulsory.

ART. 5. One of the principal recommendations of the Metric System is the ease with which the TABLES can be learned. When we have fixed upon the unit or standard of measure, all denominations ascend and descend regularly by multiplying or by dividing by 10, and the *names* of these denominations

* *Mètre*, from the Greek word *Metron*, meaning "measure"

also have a corresponding symmetry. Thus, if we take the Metre as the unit, the several denominations *above* the Metre are as follows, viz.: the *Decametre*, or 10 metres; the *Hectometre*, or 100 metres; the *Kilometre*, or 1000 metres; and the *Myriametre*, or 10,000 metres. Descending below the Metre, we have *Decimetre*, meaning $\frac{1}{10}$ th of a Metre; *Centimetre*, representing $\frac{1}{100}$ th of a Metre; and *Millimetre*, indicating $\frac{1}{1000}$ th of a Metre.

In like manner, if we take the *Gramme* as the unit, we have first above *Decagramme*, then *Hectogramme*, *Kilogramme*, and *Myriagramme*; and below we have *decigramme*, *centigramme*, and *milligramme*. Or if we use the Litre as the unit, we have first above *Decalitre*, *Hectolitre*, *Kilolitre*, and *Myrialitre*; and below, *Decilitre*, *Centilitre*, and *Millilitre*—these prefixes, *deci*, *centi*, *milli*, *deca*, etc., being uniformly employed to express the subdivisions and multiples of whatever quantity we take as the unit or standard of measure.*

MEASURES OF LENGTH.

10 millimetres (marked m. m.)	= 1 centimetre,	c. m.
10 centimetres	= 1 decimetre,	d. m.
10 decimetres	= 1 metre,	M.
10 metres	= 1 decametre,	D. M.
10 decametres	= 1 hectometre,	H. M.
10 hectometres	= 1 kilometre,	K. M.
10 kilometres	= 1 myriametre,	M. M

Muslins, cloths, cassimeres, etc. are measured by the metre

* *Deci* is an abbreviation of the Latin word *Decimus*, meaning "the tenth;" *Centi* is a contraction of *Centesimus*, meaning "the one-hundredth;" and *Milli* is from *Millesimus*, meaning "the one-thousandth." *Deca* is a Greek word, signifying 10; *Hecto*, 100; *Kilo*, 1000; and *Myria*, 10,000.

long distances, such as canals, roads, etc., by the kilometre; very minute distances by the millimetre. The decimetre is very nearly 4 inches; the centimetre very nearly $\frac{1}{10}$ ths of an inch.

EXAMPLES FOR PRACTICE.

1. How many centimetres in 4.5 metres? *Ans.* 450 c. m.

2. How many millimetres in 7.85 metres?

Ans. 7850 m. m.

3. How many centimetres in 69 decametres?

Ans. 69,000 c. m.

4. Write 4 metres, 5 decimetres, 6 centimetres, 7 millimetres.

Ans. 4.567M.

5. Write 3 decametres, 2 metres, 3 decimetres, and 8 centimetres.

Ans. 32.38M.

NOTE 1.—It will be observed from the foregoing examples that numbers in the Metric System are written precisely as in Decimals. The operations of addition, subtraction, multiplication, and division are also performed as in Decimals.

6. Write 4 hectometres, 5 metres, 3 centimetres, 7 millimetres.

Ans. 405.037M.

NOTE 2.—Whenever any denomination is wanting, its place must be supplied with a cipher.

7. Write 8 kilometres, 3 decametres, 4 centimetres, 7 millimetres.

Ans. 8030.047M.

8. Reduce 5.365 metres to millimetres. *Ans.* 5365 m. m.

9. Reduce 7.8 hectometres to decimetres. *Ans.* 7800 d. m.

10. Reduce 4567 centimetres to metres. *Ans.* 45.67M.

11. Reduce 48.7 metres to hectometres. *Ans.* .487 H. M.

MEASURES OF SURFACE.

ART. 6. The SQUARE METRE is the unit of surface. It is a SQUARE of which each side is a metre. Since the metre is

equal to 10 decimetres, it follows that the *square* metre is equal to 100 square decimetres; also, as the decimetre equals 10 centimetres, then the square decimetre is equal to 100 square centimetres, etc. Hence, we have the following

TABLE.

100 sq. millimetres (marked m. m. ²)	= 1 sq. centimetre, c. m. ² .
100 " centimetres	= 1 " decimetre, d. m. ²
100 " decimetres	= 1 " metre, M ² .
100 " metres	= 1 " decametre, D. M ² .
100 " decametres	= 1 " hectometre, H. M ² .
100 " hectometres	= 1 " kilometre, K. M ² .
100 " kilometres	= 1 " myriametre, M. M ² .

EXAMPLES FOR PRACTICE.

1. Reduce 43.75 M². to c. m.². *Ans.* 437500 c. m.².
2. Reduce 567.8 d. m.². to m. m.². *Ans.* 567800 m. m.².
3. Reduce 346785 c. m.². to M². *Ans.* 34.6785 M².
4. Reduce 52.73 K. M². to D. M². *Ans.* 527300 D. M².
5. Reduce 7854 D. M². to K. M². *Ans.* .7854 K. M².

The above Table is adapted to the measurement of *all* surfaces, but for measuring *land* only *three* of the above denominations are employed; these are the square metre, square decametre, and square hectometre. The square decametre = 100 square metres, for the sake of brevity is called an **ARE**;^{*} the square metre, being the $\frac{1}{100}$ of a square decametre, is called a **CENTIARE**; and the square hectometre = the square of 100 metres = 10,000 square metres, is called a **HECTARE**. The **Are** is the unit of *Land Measure*. The **Centiare** is $\frac{1}{100}$ of the **Are**, and the **Hectare** is equal to 100 **Ares**. Hence, for *Land Measure* exclusively we have the following

^{*} *Are*, from the Latin word *Area*, meaning "space" or "surface."

TABLE.

100 centiares (marked C. A.)	= 1 are,	A.
100 ares	= 1 hectare,	H. A.

NOTE.—The are equals 119.6 square yards very nearly, and the hectare is about $2\frac{1}{2}$ acres.

EXAMPLES FOR PRACTICE.

1. Reduce 42.5 ares to centiares. *Ans.* 4250 C. A.
2. Reduce 37.85 hectares to ares. *Ans.* 3785 A.
3. Reduce 75659 centiares to ares. *Ans.* 756.59 A.
4. Reduce 84237 centiares to hectares. *Ans.* 8.4237 H. A.

SOLID OR CUBIC MEASURE.

ART. 7. The volume or solid content of a body is measured by the CUBIC METRE, regarded as the unit. The linear metre being equal to 10 decimetres, it follows that the *Cubic Metre* is equal to $10 \times 10 \times 10 = 1000$ cubic decimetres. In like manner, the linear decimetre being equal to 10 centimetres, then the *cubic decimetre* = $10 \times 10 \times 10 = 1000$ cubic centimetres. Hence, we have the following

TABLE.

1000 cubic millimetres (m. m ³ .)	= 1 cubic centimetre, c. m ³ .
1000 " centimetres	= 1 " decimetre, d. m ³ .
1000 " decimetres	= 1 " metre, M ³ .
1000 " metres	= 1 " decametre, D. M ³ .
1000 " decametres	= 1 " hectometre, H. M ³ .
1000 " hectometres	= 1 " kilometre, K. M ³ .
1000 " kilometres	= 1 " myriametre, M. M ³ .

The foregoing Table is used for measuring the volume of boxes, bales, etc., and indeed is amply sufficient for the measurement of all cubical bodies; but for measuring firewood or lumber in piles we use only *three* of the above denominations—viz.: the cubic decimetre, cubic metre, and cubic decametre

For the sake of brevity the cubic metre is called the **STERE**;* the cubic decimetre is termed the **DECISTERE**, and the cubic decametre is called the **DECASTERE**. Hence, for measuring *wood* and masses of lumber, we have the following

TABLE.

10 decisteres (marked d. s.)	= 1 stere,	S.
10 steres	= 1 decastere,	D. S.

NOTE.—The **STERE** is about 35.3166 cubic feet, and 3 $\frac{1}{4}$ steres about equal a cord.

EXAMPLES FOR PRACTICE.

1. Reduce 95.6 steres to decisteres. *Ans.* 956 d. s.
2. Reduce 256 decasteres to decisteres. *Ans.* 25600 d. s.
3. Reduce 75845 decisteres to decasteres.
Ans. 758.45 D. S.

WEIGHTS.

ART. 8. The **GRAMME** is the unit of weight. It is equal to 15.432 grains, Troy. A cubical vessel, each of whose edges is one centimetre in length, filled with distilled water at the temperature of 39.2° Fahrenheit, will contain a gramme in weight. For determining weights we have the following

TABLE.

10 milligrammes (marked m. g.)	= 1 centigramme,	c. g.
10 centigrammes	= 1 decigramme,	d. g.
10 decigrammes	= 1 gramme,	G.
10 grammes	= 1 decagramme,	D. G.
10 decagrammes	= 1 hectogramme,	H. G.
10 hectogrammes	= 1 kilogramme,	K. G.
10 kilogrammes	= 1 myriagramme,	M. G.

* *Stere*, from the Greek word *Stereos*, meaning "solid."

The gramme and its subdivisions are employed in weighing very small articles, in compounding medicines, and in philosophical experiments.

For the ordinary purposes of trade and business the kilogramme is more frequently used than any other weight. It is about $2\frac{1}{2}$ pounds Avoirdupois.

For weighing heavy articles, such as iron, hay, etc., two other denominations are in very frequent use; these are the *quintal* and the *tonneau*. The quintal is 100 kilogrammes, equivalent to about 220.46 pounds Avoirdupois, and the tonneau is 1000 kilogrammes, equivalent to about 2204.6 pounds. The tonneau is therefore a little more than $1\frac{1}{10}$ tons. In the newspapers giving the "prices current" the word "kilogramme" is contracted into *kilo*, and hectogramme is more commonly written simply *hecto*.

EXAMPLES FOR PRACTICE.

1. Reduce 75679 grammes to kilogrammes.

Ans. 75.679 K. G.

2. Reduce 37.5 kilogrammes to grammes. *Ans.* 37500G.

3. Reduce 4783 centigrammes to decagrammes.

Ans. 4.783 D. G.

4. Reduce 18 quintals to grammes. *Ans.* 1800000G.

5. At 35 cents a kilogramme, what would be the cost of 7 kilogrammes, 9 hectogrammes of honey? *Ans.* \$2.765.

6. At 18 cents a gramme, what would 9 decagrammes, 8 decigrammes of quinine cost? *Ans.* \$16.344.

MEASURES OF CAPACITY.

ART. 9. The LITRE is the unit of capacity. It is equal to a cubical vessel each of whose *edges* is a decimetre, or $\frac{1}{10}$ of a metre. The litre contains about $\frac{2}{10}$ of a quart, Dry Measure. It is used both for liquid and dry measures.

TABLE.

10 millilitres (m. l.)	= 1 centilitre,	c. l.
10 centilitres	= 1 decilitre,	d. l.
10 decilitres	= 1 litre,	L.
10 litres	= 1 decalitre,	D. L.
10 decalitres	= 1 hectolitre,	H. L.
10 hectolitres	= 1 kilolitre,	K. L.
10 kilolitres	= 1 myrialitre,	M. L.

The litre is chiefly used in measuring *liquids*, as wine, molasses, vinegar, etc.; the hectolitre, for measuring wheat, rye, potatoes, etc.

EXAMPLES FOR PRACTICE.

1. Reduce 425 litres to millilitres. *Ans.* 425000 m. l.
2. Reduce 52.75 kilolitres to litres. *Ans.* 52750L.
3. Reduce 46824 litres to hectolitres. *Ans.* 468.24 H. L.
4. Write 3 hectolitres, 5 litres, 7 centilitres.
Ans. 305.07L.
5. Write 8 kilolitres, 9 decalitres, 6 decilitres.
Ans. 8090.6L.

PROMISCUOUS EXAMPLES.

1. What would be the cost of 7 metres and 5 decimetres of cassimere at \$2.25 per metre? *Ans.* \$16.875.
2. If one piece of muslin measure 27 metres, 3 decimetres, how much would 15 pieces measure? *Ans.* 409.5M.
3. At 18 cents a litre, what would be the cost of 7 decalitres and 4 litres of wine? *Ans.* \$13.32.
4. At 12 cents a litre, what would be the cost of 9 litres, 3 decilitres, and 8 centilitres of vinegar? *Ans.* \$1.1256.
5. If one hectare of land be worth \$25, what would 7 hectares and 23 ares be worth at the same rate? *Ans.* \$190.75.

6. A man bought 7 decasteres of wood at \$1.85 a stere; what did the wood cost? *Ans.* \$129.50.

7. If I sell 5 steres and 7 decisteres of wood at \$2.25 a stere, what should I receive for the whole? *Ans.* \$12.825.

8. If one kilogramme of sugar cost 23 cents, what would 9 kilogrammes and 7 hectogrammes cost at the same rate? *Ans.* \$2.231.

9. At \$2.25 per myriagramme, what would be the cost of 17 myriagrammes, 8 kilogrammes, and 5 hectogrammes of mutton? *Ans.* \$40.1625.

10. A physician put up 5 decagrammes of quinine into pills, each pill containing 8 centigrammes; how many pills were there? *Ans.* 625 pills.

11. How many feet in 48 metres?

Analysis.—In 1 metre there are 39.37 inches; then in 48 metres there are 48 times as many inches $= 39.37 \times 48 = 1889.76\text{in.} = 157.48\text{ft.}$ *Ans.* 157.48ft.

12. How many yards in 72 metres? *Ans.* 78.74yd

13. How many metres in 1 mile?

Analysis.—In 1 mile there are 5280 feet $= 63360$ inches. Then as 1 metre contains 39.37 inches, there will be as many metres in 63360 inches as 39.37 is contained times in 63360, which will be 1609.34 metres. *Ans.* 1609.34M.

14. How many metres in 75 yards? *Ans.* 68.55M.

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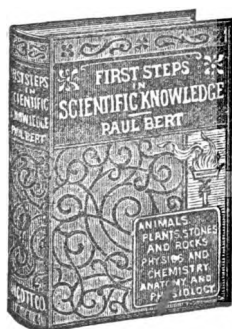
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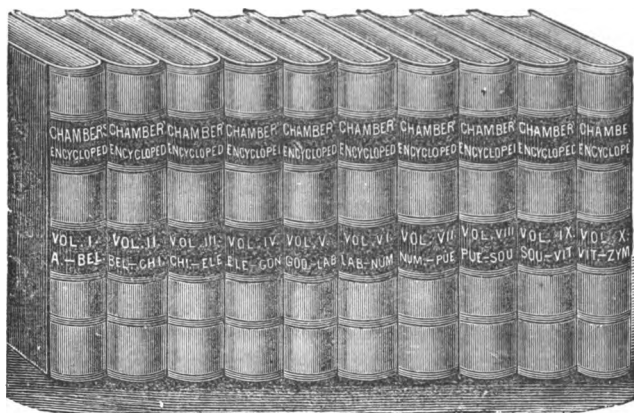
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